Al-Mustaqbal University College

Department of Medical Instrument Technologies

Mathematics II / Second Stage

By

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Infinite Series المتسلسلات غير المنتهية

A sequence of the form :

$a_1 + a_2 + a_3 + a_4 + \cdots + a_n + \cdots$

is called an infinite Series. The term

the nth term of the Series.

The Sequence $\{S_n\}$ defined by :

$S_1 = a_1$

$S_2 = a_1 + a_2 = S_1 + a_2$

$S_3 = a_1 + a_2 + a_3 = S_2 + a_2$

$\vdots$

$S_n = a_1 + a_2 + a_3 + \cdots + a_n = \sum_{k=1}^{n} a_k$

is the Sequence of partial sums of the Series and the number S_n being the nth partial Sum.

If the sequence of partial sum S_n converges to a limiting value (L), then the series is **convergent** and its Sum is (L). Hence,

$$a_1+a_2+a_3+a_4+\cdots+a_n+\cdots=\sum_{n=1}^{\infty} a_n = L$$

If S_n of series does not converge, then the Series **divergent** and there is no sum.

* Any Series can be expressed by:

$$\sum_{n=1}^{\infty} a_n \left(\sum_{k=1}^{\infty} a_k\right) \text{ or Simply } \sum a_n \left(\sum a_k\right)$$

The following examples shows how the "partial fraction technique" can be used to compute the sums for some series. When this sum can be done, then the series is convergent.

Ex) Determine whether the following series is convergent or no? ① $\sum \frac{2}{n(n+1)}$

Sol.) $\sum \frac{2}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \Rightarrow A(n+1) + Bn = 2 \rightarrow$

if $n=-1 \rightarrow B=-2$
$n=0 \rightarrow A=2 \quad \rightarrow$

$$\sum \frac{2}{n(n+1)} = \sum \left(\frac{2}{n} - \frac{2}{n+1}\right) \rightarrow$$

$$S_n = \underbrace{(2-\cancel{1})}_{n=1} + \underbrace{(\cancel{1}-\cancel{\tfrac{2}{3}})}_{n=2} + \underbrace{(\cancel{\tfrac{2}{3}}-\cancel{\tfrac{2}{4}})}_{n=3} + \cdots + \underbrace{(\cancel{\tfrac{2}{n-1}}-\cancel{\tfrac{2}{n}})}_{n=n-1} + \underbrace{(\cancel{\tfrac{2}{n}}-\tfrac{2}{n+1})}_{n=n}$$

$\therefore S_n = 2 - \frac{2}{n+1}$

$\therefore \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(2 - \frac{2}{n+1}\right) = 2 \rightarrow \therefore$ the series is convergent

EX) Discuss the convergence of this series : $\sum \frac{4}{(4n-3)(4n+1)}$?

Sol.) $\frac{4}{(4n-3)(4n+1)} = \frac{A}{4n-3} + \frac{B}{4n+1} \rightarrow A(4n+1) + B(4n-3) = 4$

if $n = 3/4 \rightarrow A = 1$ and if $n = -1/4 \rightarrow B = -1 \rightarrow$

$\therefore \sum \frac{4}{(4n-3)(4n+1)} = \sum \left(\frac{1}{4n-3} - \frac{1}{4n+1} \right)$

$$S_n = \underbrace{(1-\tfrac{1}{5})}_{n=1} + \underbrace{(\tfrac{1}{5}-\tfrac{1}{9})}_{n=2} + \underbrace{(\tfrac{1}{9}-\tfrac{1}{13})}_{n=3} + \cdots + \left(\frac{1}{4n-3} - \frac{1}{4n+1}\right)$$

$$= 1 - \frac{1}{4n+1} \rightarrow \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(1 - \frac{1}{4n+1}\right) = 1 \rightarrow$$

$\therefore$ the series is convergent

Power Series

- An expression of the form :

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n + \cdots \quad \text{—— (1)a}$$

is a power series centered at $x = 0$.

- An expression of the form :

$$\sum_{n=0}^{\infty} a_n (x-a)^n = a_0 + a_1(x-a) + a_2(x-a)^2 + a_3(x-a)^3 + \cdots + a_n(x-a)^n + \cdots \quad \text{(1)b}$$

is a power series centered at $x = a$. The term $a_n(x-a)^n$ is the nth term ; the number a is the center.

$* \sim\sim\sim *$

Power Series can be used for representing the functions whose possess a derivatives of all orders at the center (zero for eq. 1a and a for eq. 1b) and continuous on its domains, only .

$* \sim\sim\sim *$

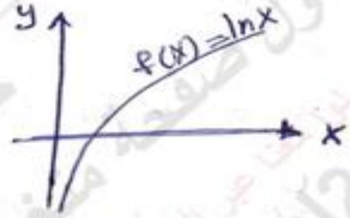
Ex1 Check whether the following functions can be represented by Power series : $\sin x$, $\cos x$, e^{ax}, $\ln x$ & $\sqrt{x}$?

Sol.1
① $f(x) = \sin x \rightarrow f'(x) = \cos x \rightarrow f'(0) = 1$
② $f(x) = \cos x \rightarrow f'(x) = -\sin x \rightarrow f'(0) = 0$
③ $f(x) = e^{ax} \rightarrow f'(x) = a e^{ax} \rightarrow f'(0) = a$

then the above functions can be represented by Power Series

(4) $f(x) = \ln x \rightarrow f'(x) = \frac{1}{x} \rightarrow f'(0) = \frac{1}{0} = \underline{\underline{\infty}}$

the function $\ln x$ can not be represented by power series because neither $f(x)$ nor its derivative $f'(x)$ exist at $x = 0$.



(5) $f(x) = \sqrt{x} \rightarrow f'(x) = \frac{1}{2\sqrt{x}} \rightarrow f'(0) = \frac{1}{0} = \underline{\underline{\infty}}$

the function $\sqrt{x}$ cannot be represented by power series since $f(x) = \sqrt{x}$ have no derivative @ $x = 0$.

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