



Mathematical and Statistics

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First Stage

Lecter One

Measures of Central Tendency

Measures of Central Tendency

It is the metrics that attempt to describe the point of data collection (observations) and its idea goes back to the English scholar Francis Galton. Central tendency measures are used to summarize data numerically since they are considered typical or ideal values for data. Also, these metrics are used to describe a data set or to compare it with other data sets. There are many types of measures of central tendency as follows:

- 1) The Mean
- 2) The Median
- 3) The Mode

1- The Mean

The mean represents the average value of the dataset. It can be calculated as the sum of all the values in the dataset divided by the number of values. In general, it is considered as the arithmetic mean. Some other measures of mean used to find the central tendency are as follows:

The Mean

The Arithmetic Mean

The Geometric Mean

The Harmonic Mean

not tabulated Data

Tabulated Data

A. The Arithmetic Mean

It is a value which a set of data are collected around it. It is denoted by the symbol (\bar{X}) and calculated as follows:

1. Arithmetic Mean for not tabulated Data

If there is a set of data ($x_1, x_2, x_3, x_4, \dots, \dots, x_n$), The Arithmetic Mean is calculated as :

$$\bar{X} = \frac{\sum x_i}{n}, \bar{X} = \frac{x_1 + x_2 + x_3 + x_4 + \dots + x_n}{n}$$

Example: Find the arithmetic mean for this data: 500, 20, 40, 60, 100, 200, 50

Solution: $\bar{X} = \frac{\sum x_i}{n}$

$$\bar{X} = \frac{500 + 20 + 40 + 60 + 100 + 200 + 50}{7} = \frac{970}{7} = 138.571$$

2) Arithmetic Mean for Tabulated Data

Tabulated data are the data set in frequency distribution table, and for each class there is an upper and lower limit:

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$x_i = \frac{\text{upper limit} + \text{lower limit}}{2} \text{ and } f_i: \text{frequency of class}$$

Example : Find the arithmetic mean for this data:

| class | frequency |
|-------|-----------|
| 20-25 | 4 |
| 25-30 | 8 |
| 30-35 | 16 |
| 35-40 | 8 |
| 40-45 | 4 |

solution:

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$x_i = \frac{\text{upper limit} + \text{lower limit}}{2}$$

$$x_1 = \frac{20 + 25}{2} = \frac{45}{2} = 22.5$$

$$x_2 = \frac{25 + 30}{2} = \frac{55}{2} = 27.5$$

$$\bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1300}{40} = 32.5$$

| class | f_i | x_i | $f_i x_i$ |
|-----------|-----------------|-------|-----------------------|
| 20-25 | 4 | 22.5 | 90 |
| 25-30 | 8 | 27.5 | 220 |
| 30-35 | 16 | 32.5 | 520 |
| 35-40 | 8 | 37.5 | 300 |
| 40-45 | 4 | 42.5 | 170 |
| summation | $\sum f_i = 40$ | | $\sum f_i x_i = 1300$ |

B- The Geometric Mean

The geometric mean is used to calculate average values in case data are ratios, as is the case with population growth rates, and it is calculated in two cases:

1. The Geometric Mean for not tabulated Data

The geometric mean of a set of values $(x_1, x_2, x_3, x_4, \dots, x_n)$ is defined as the square root of the product of these values, and is denoted by the symbol \bar{X}_g

$$\log \bar{X}_g = \frac{1}{n} \sum \log x_i$$

Example: Find the value of the geometric mean for these values: 6, 4.2, 6, 4.2, 3.7, 4.8

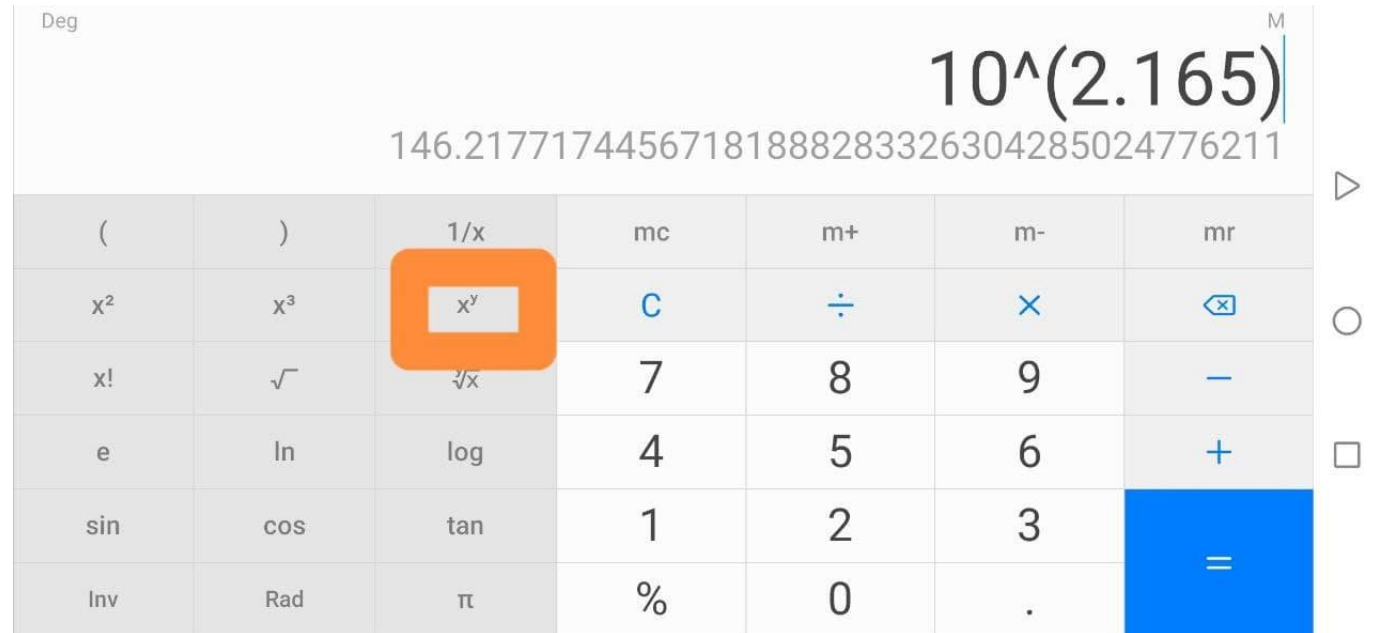
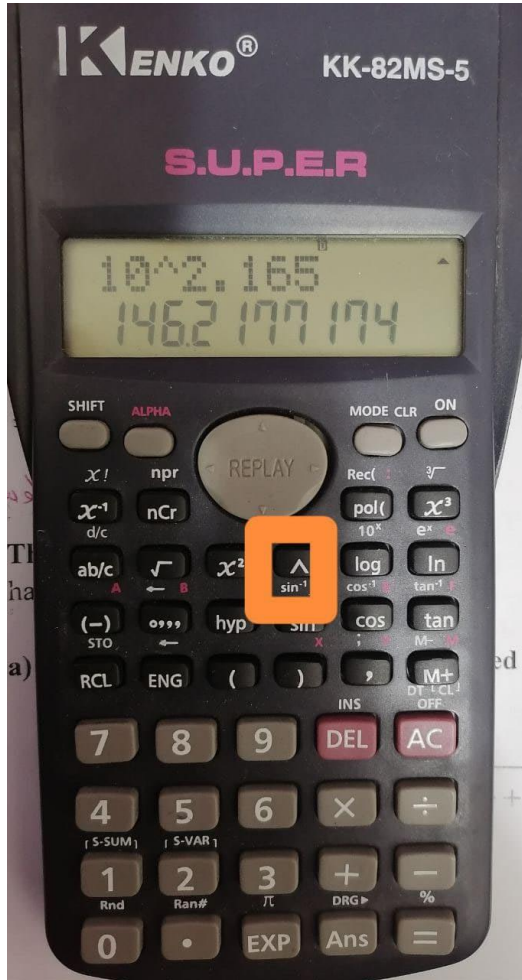
Solution:

$$\text{Log } \bar{X}_g = \frac{1}{6} \Sigma (\text{Log } 6 + \text{Log } 4.2 + \text{Log } 6 + \text{Log } 4.2 + \text{Log } 3.7 + \text{Log } 4.8)$$

$$\text{Log } \bar{X}_g = \frac{1}{6} (0.77 + 0.62 + 0.77 + 0.62 + 0.56 + 0.68)$$

$$\text{Log } \bar{X}_g = 0.67$$

$$\bar{X}_g = 10^{0.67} = 4.677$$



a) The Geometric Mean for Tabulated Data

To calculate the value of the geometric mean for tabulated data used this equation:

$$\log \bar{X}_g = \frac{1}{\sum f_i} \sum f_i \log x_i$$

x_i : central of class and f_i : frequency of class

Example: Find the geometric mean for data:

| class | frequency |
|---------|-----------|
| 60-80 | 22 |
| 80-100 | 38 |
| 100-120 | 45 |
| 120-140 | 35 |
| 140-160 | 20 |
| Total | 160 |



| class | frequency | x_i | $\log x_i$ | $f_i \log x_i$ |
|---------|-----------|-------|------------|----------------|
| 60-80 | 22 | 70 | 1.845 | 40.59 |
| 80-100 | 38 | 90 | 1.954 | 74.25 |
| 100-120 | 45 | 110 | 2.041 | 91.85 |
| 120-140 | 35 | 130 | 2.114 | 73.99 |
| 140-160 | 20 | 150 | 2.176 | 43.52 |
| Total | 160 | | | 324.2 |

$$\log \bar{X}_g = \frac{1}{\sum f_i} \sum f_i \log x_i$$

$$\log \bar{X}_g = \frac{1}{160} 324.2$$

$$\log \bar{X}_g = 2.02625$$

$$\bar{X}_g = 106.23$$

C-The Harmonic Mean

The harmonic mean is calculated in two cases:

A. The Harmonic Mean for not tabulated Data

Its calculated by using this equation:

$$\bar{X}_h = \frac{1}{\frac{1}{n} \sum \frac{1}{x_i}} = \frac{n}{\sum \frac{1}{x_i}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots \dots \dots + \frac{1}{x_n}}$$

Example: Find the Harmonic Mean for data:18, 37, 25, 46, 77, 20

Solution:

$$\begin{aligned} \bar{X}_h &= \frac{n}{\sum \frac{1}{x_i}} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots \dots \dots + \frac{1}{x_n}} \\ &= \frac{6}{\frac{1}{18} + \frac{1}{37} + \frac{1}{25} + \frac{1}{46} + \frac{1}{77} + \frac{1}{20}} = 31.1315 \end{aligned}$$

B. The Harmonic Mean for tabulated Data

Its calculated by using this equation: $\overline{X}_h = \frac{\sum f_i}{\sum \frac{f_i}{x_i}}$

Example: Calculate the harmonic mean for the following data

Solution:

| class | frequency | f/x |
|-------|-----------|--------------------|
| 1 | 2 | 2 |
| 3 | 4 | 1.332 |
| 5 | 6 | 1.2 |
| 7 | 8 | 1.144 |
| 9 | 10 | 1.111 |
| 11 | 12 | 1.092 |
| sum | 42 | $\sum f/x = 7.879$ |

| | | | | | | |
|-----------|---|---|---|---|----|----|
| class | 1 | 3 | 5 | 7 | 9 | 11 |
| frequency | 2 | 4 | 6 | 8 | 10 | 12 |

$$\overline{X}_h = = \frac{\sum f_i}{\sum \frac{f_i}{x_i}} = \frac{42}{7.879} = 5.330$$

2- The Median

It is the value in which a set of values is mediated after ascending or descending order and symbolized by the symbol (M). It is calculated in two cases:

❖ Case of not tabulated data

The group values are arranged in ascending or descending order, and the median is the value that falls exactly in the middle.

- If n is odd number then order of median $\frac{n+1}{2}$
- If n is even number then order of median $\frac{\text{value } \frac{n}{2} + (\frac{n}{2} + 1)}{2}$

Example: Find the value of the Median from the data:

13, 50, 7, 15, 47, 12, 5

Solution: The data is arranged in **ascending** or **descending** order:

In descending order: 50, 47, 15, 13, 12, 7, 5

In ascending order: 5, 7, 12, 13, 15, 47, 50

$$n = 7, \text{ order of median } \frac{n+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$$

The median is (13)

| | | | | | | | |
|--------|---|---|----|----|----|----|----|
| median | 5 | 7 | 12 | 13 | 15 | 47 | 50 |
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 |

Example: Find the value of the Median from the data:

20, 12, 42, 15, 65, 30

Solution:

In ascending order: 12, 15, 20, 30, 42, 65

$$n = 6, \frac{n}{2} = \frac{6}{2} = 3$$

$$\frac{n}{2} = 3$$

$$\left(\frac{n}{2} + 1\right) = 3 + 1 = 4$$

$$M = \frac{30 + 20}{2} = 25$$

| | | | | | | |
|--------|----|----|----|----|----|----|
| median | 12 | 15 | 20 | 30 | 42 | 65 |
| n | 1 | 2 | 3 | 4 | 5 | 6 |

