اسم التدريسي	رامي قيس مالك
اسم المادة	تقنيات رقمية
عنوان المحاضرة	**De Morgan’s theorem**
رقم المحاضرة	**7**
الايميل الجامعي للتدريسي	**rami.qays@mustaqbal-college.edu.iq**

AL-Mustaqbal university college

Class: 2nd

Subject: Digital Techniques

Lecturer: Rami Qays Malik

Lecture :7th – De Morgan's theorem

E-mail : rami.qays@mustaqbal-college.edu.iq

DE MORGAN'S THEOREM

INTRODUCTION:

De Morgan's theorem may be thought in terms of breaking a long bar symbol. When a long bar is broken, the operation directly implies the changes from addition to multiplication or vice versa, and the broken bar pieces remains over the individual variables.

$$\overline{AB} = \overline{A} + \overline{B}$$

$$\overline{A + B} = \overline{A}..\overline{B}$$

REMARK 1:

When multiple layers of bar exist in an expression, you may only break one bar at a time.

Example 1:

Let us simplify the following expressions:

$$\overline{A + \overline{BC}} = \overline{A}.\overline{\overline{BC}} \quad \text{(The superior bar broken)}$$

$$= \overline{A}.BC$$

$$\overline{A + \overline{B} + \overline{C}} = \overline{A}.\overline{\overline{B}}.\overline{\overline{C}}$$

$$= \overline{A}.BC$$

AL-Mustaqbal university college

Class: 2nd

Subject: Digital Techniques

Lecturer: Rami Qays Malik

Lecture :7th – De Morgan's theorem

E-mail : rami.qays@mustaqbal-college.edu.iq

2. CONVERTING TRUTH TABLE INTO BOOLEAN EXPRESSION:

We can convert truth table into Boolean expression using one of the following methods:

- Sum of products (SOP)
- Product of sums (POS)

2.1 SUM OF PRODUCTS:

Boolean expressions may be generated from truth table quite easily using the following steps:

- Determine which rows of the table have an output of 1;
- Write one product for each raw;
- Sum all the product terms.

This creates a Boolean expression representing the truth table as a whole.

AL-Mustaqbal university college

Class: 2nd

Subject: Digital Techniques

Lecturer: Rami Qays Malik

Lecture :7th – De Morgan's theorem

E-mail : rami.qays@mustaqbal-college.edu.iq

Example 2:

Let's consider a logic circuit having the following truth table:

A	B	C	Q	
0	0	0	0	Row 1
0	0	1	0	Row 2
0	1	0	0	Row 3
0	1	1	1	Row 4
1	0	0	0	Row 5
1	0	1	1	Row 6
1	1	0	1	Row 7
1	1	1	1	Row 8

The rows 4, 6, 7 and 8 have an output of 1, each raw gives us a product. By summing those products, we obtain the following Boolean expression which is that of the output Q.

$$Q = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC$$

2.2PRODUCT OF SUMS:

Boolean expression may be also generated from truth table quite easily by determining which rows of the table have an output of 0, writing one sum term for each row and finally multiplying all the terms.

AL-Mustaqbal university college

Class: 2nd

Subject: Digital Techniques

Lecturer: Rami Qays Malik

Lecture :7th – De Morgan's theorem

E-mail : rami.qays@mustaqbal-college.edu.iq

Example 3:

Let us consider a logic circuit having the following truth table.

A	B	C	Q	
0	0	0	0	Row 1
0	0	1	1	Row 2
0	1	0	1	Row 3
0	1	1	1	Row 4
1	0	0	1	Row 5
1	0	1	1	Row 6
1	1	0	1	Row 7
1	1	1	0	Row 8

The rows 1 and 8 have an output of 0; each row gives us a sum. The product of those sums gives us a Boolean expression which is that of the output of the logic circuit. In fact, we have:

$$\overline{Q} = \overline{A}.\overline{B}.\overline{C} + ABC$$

$$\overline{\overline{Q}} = Q$$

$$= \overline{\left(\overline{A}..\overline{B}..\overline{C}\right) + ABC}$$

$$= \overline{\left(\overline{A}..\overline{B}..\overline{C}\right)}\,\overline{A.B.C}$$

$$= \left(\overline{\overline{A}} + \overline{\overline{B}} + \overline{\overline{C}}\right)\left(\overline{A} + \overline{B} + \overline{C}\right)$$

$$= (A + B + C)\left(\overline{A} + \overline{B} + \overline{C}\right)$$

$$Q = (A + B + C).(\overline{A} + \overline{B} + \overline{C})$$

AL-Mustaqbal university college

Class: 2nd

Subject: Digital Techniques

Lecturer: Rami Qays Malik

Lecture :7th – De Morgan's theorem

E-mail : rami.qays@mustaqbal-college.edu.iq

In reality for each row having an output of 0, we should notice that we have but the inverted output product ($\overline{Q}$). By inverting that output ($\overline{\overline{Q}}$), we obtain a sum using De Morgan's theorem. Finally, the product of all those sums gives us the output of the logic circuit.

Remark:

Generally, the sum of products is more used than the product of sums to convert a truth table into Boolean expression. However, when a few number of rows have an output of 0, it is preferable to use the POS than to use the SOP.

Exercise 3.4:

Assuming that $A \oplus B = \overline{A}.B + A.\overline{B}$, proof that $\overline{A \oplus B} = A.B + \overline{A}..\overline{B}$