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AL-Mustaqbal university college

Class: 2nd

Subject: Digital Techniques

Lecturer: Rami Qays Malik

Lecture :6th – BOOLEAN ALGEBRA

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BOOLEAN ALGEBRA

INTRODUCTION:

The Boolean algebra was created by the English mathematician George Boole (1815-1864). The Boolean algebra codifies rules of relationship between mathematical quantities to one of two possible values: true or false, 1 or 0. So, all arithmetic operations performed with Boolean quantities have but one of two possible outcomes: either 1 or 0. There are three basic Boolean arithmetic operations:

- Boolean addition which is equivalent to the OR logic function, as well as parallel switch contacts;
- Boolean multiplication, which is equivalent to the AND function as well as series switch contacts;
- Boolean complementation which is equivalent to the NOT logic function..

1. BOOLEAN ADDITION:

As we have already said, Boolean addition is equivalent to the OR logic function. Therefore, we have the following relationships::

0 + 0 = 0

1 + 0 = 1

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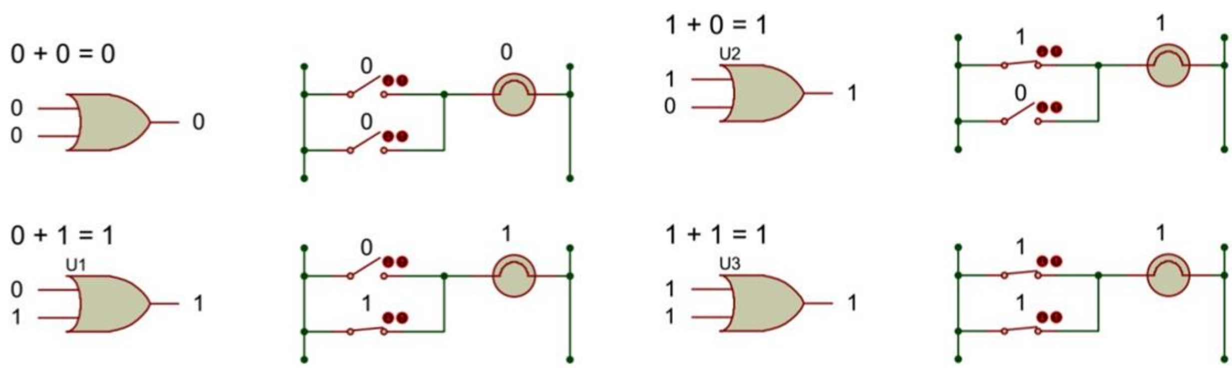
0 + 1 = 1

1 + 1 = 1



2. BOOLEAN MULTIPLICATION:

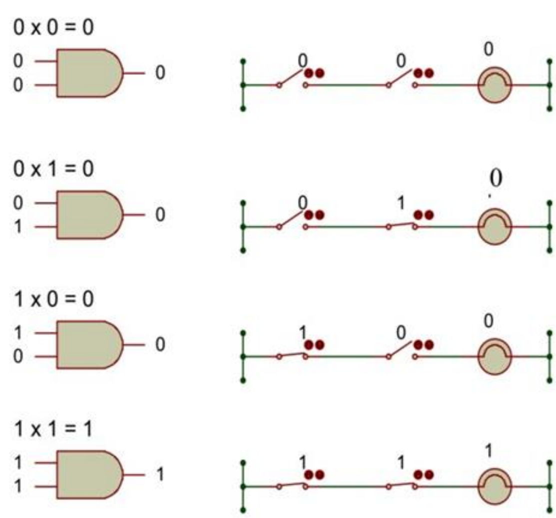
The Boolean multiplication is equivalent to the AND logic function:

0 x 0 = 0

0 x 1 = 0

1 x 0 = 0

1 x 1 = 1



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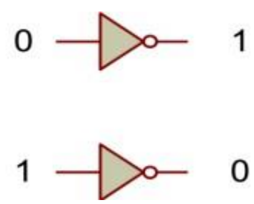
3. BOOLEAN COMPLEMENTATION:

The Boolean complementation is equivalent to the NOT logic function.

/0 = 1

/1 = 0



4. BOOLEAN ALGEBRAIC IDENTITIES:

An identity is a statement that is true for all possible values of its variables. There are two groups of Boolean algebraic identities: additive identities and multiplicative identities.

4.1 ADDITIVE IDENTITIES

If A is a Boolean variable, then the following statements are always true.

A + 0 = A

A + 1 = 1

A + A = A

A + /A = 1

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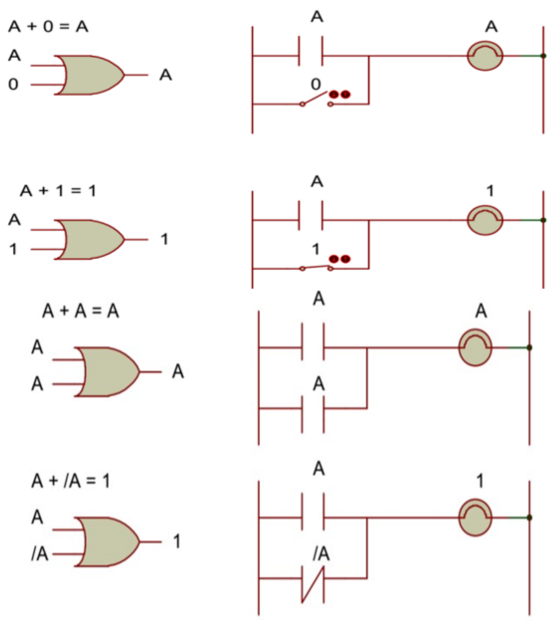
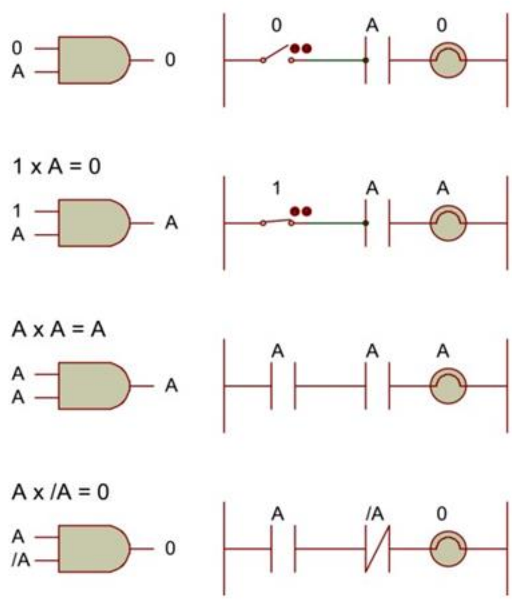


4.2 MULTIPLICATIVE IDENTITIES:

A being a Boolean variable, the following statements are always true.



0 x A = 0

1 x A = A

A x A = A

A x/A = 0

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Remark 1: Double complementation: complementing a variable twice results in the original Boolean value.

5. Boolean algebraic properties:

Let us consider three Boolean Variables A, B and C. The following properties are true.

A + B = B + A

A x B = B x A

A + (B + C) = (A + B) + C

A(B.C) = (A.B)C

A(B + C) = A.B + A.C

6. Boolean rules for simplification:

There are several rules for Boolean algebra intended to be used in reducing complex Boolean expressions to their simplest forms. The simplification of the Boolean expressions of logic circuits brings many advantages:

- Higher operating speed (less delay time from input signal transition to output signal transition).

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- Less power consumption (few IC used).
- Less cost.
- Greater reliability.

Rule 1: $A + AB = A$

$A + AB = A\,(1 + B)$

$= A\,(1)$

$= A$

Rule 2: $A + \bar{A}B = A + B$

$A + \bar{A}B = A + AB + \bar{A}B$ (Apply the previous rule to expand A term to A + AB)

$= A + B\left(A + \bar{A}\right)$ (Factorizing B)

$= A + B(1)$ (Applying identity $A + \bar{A} = 1$)

$= A + B$

Rule 3: $(A + B)(A + C) = A + BC$

$(A + B)(A + C) = A.A + A.C + A.B + B.C$ (Distributing terms)

$= A + AC + AB + BC$ (Applying identity AA = A)

$= A + AB + BC$ (Applying A + AC = A)

$= A + BC$ (Applying A + AB = A)

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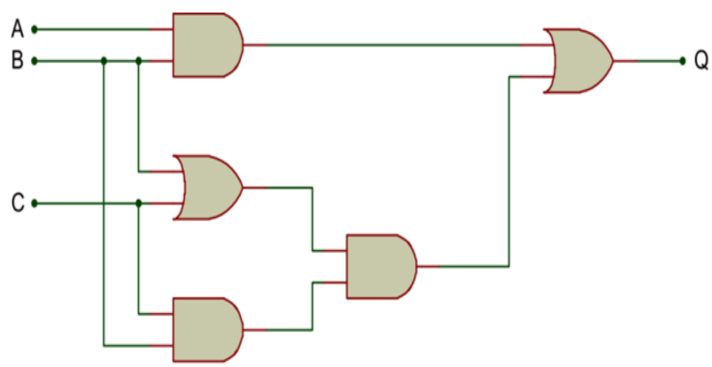
7. Circuit simplification example:

Let us consider the following logic circuit.



1. Write the Boolean expression of the output Q:

$$Q = AB + BC(B + C))$$

2. Reduce this expression to its simplest form using the rules of Boolean algebra.

$$Q = AB + BCB + BCC$$

$= AB + BC + BC$ (Using the identity A.A =A)

$= AB + BC$ (Identity A.A = A)

$$Q = B(A + C)$$

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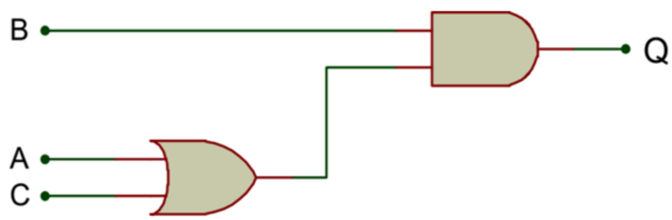
3. Generate the schematic diagram of the simplest expression



Remark 2

To convert Boolean expression to a gate circuit, you should evaluate the expression using standard order of operation:

- Multiplication before addition,
- Operation within parenthesis before anything else.
- Operation within parenthesis before anything else.