



**Laws of Derivatives:**

1-  $\frac{d}{dx} c = 0$  where c is constant.

➤  $\frac{d}{dx} 4 = 0$

➤  $\frac{d}{dx} \sqrt{2} = 0$

2-  $\frac{d}{dx} x^n = nx^{n-1}$

➤  $\frac{d}{dx} x^4 = 4x^3$

➤  $\frac{d}{dx} x^{-3} = -3x^{-4}$

➤  $\frac{d}{dx} \sqrt{x} = \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{-\frac{1}{2}}$

3- If U and V are two functions of x then:

a)  $\frac{d}{dx} (c * U) = c \frac{d}{dx} (U)$

➤  $\frac{d}{dx} 3x^3 = 3 \frac{d}{dx} x^3 = 3(3x^2) = 9x^2$

b)  $\frac{d}{dx} (U \pm V) = \frac{d}{dx} (U) \pm \frac{d}{dx} (V)$

➤  $\frac{d}{dx} (x^2 + x^3) = \frac{d}{dx} (x^2) + \frac{d}{dx} (x^3) = 2x + 3x^2$

c)  $\frac{d}{dx} (U * V) = U \frac{d}{dx} (V) + V \frac{d}{dx} (U)$

➤  $\frac{d}{dx} (x^2 * x^3) = x^2 \frac{d}{dx} (x^3) + x^3 \frac{d}{dx} (x^2) = x^2 * (3x^2) + x^3 * (2x)$

d)  $\frac{d}{dx} U^n = nU^{n-1} * \frac{d}{dx} U$

➤  $\frac{d}{dx} (x^5)^3 = 3(x^5)^2 * \frac{d}{dx} x^5 = 3(x^5)^2 * 5x^4$

e)  $\frac{d}{dx} \left( \frac{U}{V} \right) = \frac{V \frac{dU}{dx} - U \frac{dV}{dx}}{V^2}$

➤  $\frac{d}{dx} \left( \frac{x^2}{x^3} \right) = \frac{x^3 * (2x) - x^2 * 3x^2}{(x^3)^2}$

## Lecture (5)



**Example:** If  $y = x^3 + 4x^2 - 5x + 4$  find  $f'(2)$ .

Sol:  $f'(x) = 3x^2 + 8x - 5$

$$f'(2) = 3 * (2)^2 + 8 * (2) - 5$$

$$f'(2) = 23$$

**Example:** If  $y_1 = 3x^2 + 2x + 1$  ,  $y_2 = 5x + 8$  find  $f'(y_1 + y_2)_{(2)}$

Sol:  $f'(y_1 + y_2) = f'(y_1) + f'(y_2)$

$$= f'(3x^2 + 2x + 1) + f'(5x + 8)$$

$$= (6x + 2) + (5)$$

$$= 6x + 7$$

$$f'(y_1 + y_2)_{(2)} = 6 * (2) + 7$$

$$f'(y_1 + y_2)_{(2)} = 19.$$

### H.W:

1- If  $y = 3x^4 + 4x^3 - 2x + 6$  find  $f'(3)$ .

2- If  $y_1 = 5x^4 + 2x^3 - 1$  ,  $y_2 = 5x^3 - x + 2$  find  $f'(y_1 + y_2)_{(3)}$

3- If  $y = \frac{2x^2 - 6x}{4x}$  find  $f'(1)$ .



المشتقة الثانية ( Second and Higher Order Derivative )

$$f''(x) = \frac{d}{dx} \left( \frac{d}{dx} y \right)$$

**Example:** If  $y = x^4 + 2x^3 - 3x + 1$  find  $f''(1)$ .

Sol:  $f'(x) = 4x^3 + 2 * 3x^2 - 3 = 4x^3 + 6x^2 - 3$

$$f''(x) = 4 * 3x^2 + 6 * 2x = 12x^2 + 12x$$

$$f''(x) = 12(1)^2 + 12 = 24.$$

**Example:** If  $y = 2x^4 + 5x^3 - 3x^2 + 6$  find  $f''(-1)$ .

Sol:  $f'(x) = 8x^3 + 15x^2 - 6x$

$$f''(x) = 24x^2 + 30x - 6$$

$$f''(x) = 24(-1)^2 + 30(-1) - 6$$

$$= 24 - 30 - 6 = -12$$

**H.W:**

1- If  $y = 3x^4 + 4x^3 - 2x + 6$  find  $f''(4)$ .

2- If  $y_1 = 5x^4 + 2x^3 - 1$  ,  $y_2 = 5x^3 - x + 2$  find  $f''(y_1 - y_2)$ (1)

3- If  $y = 5x^3 + 4x - 7$  find  $f''(2)$ .



**(Implicit Differentiation) المشتقة الضمنية**

In some cases, it is difficult to solve  $y=f(x)$ , so to find  $\frac{d}{dx}$  for such cases, implicit differentiation will be use.

**Example:** Find  $\frac{d}{dx}$  of the following

1-  $y^2 + x^2 = 1$

**Sol:**  $2y \frac{d}{dx} + 2x = 0$

$$2y \frac{d}{dx} = -2x$$

$$\frac{d}{dx} = \frac{-2x}{2y}$$

2- Find the Implicit Differentiation of the function

$2y = x^2 + 3xy^2$  at  $x=1, y=2$

**Sol:**  $2 \frac{d}{dx} = 2x + (3x * 2y * \frac{d}{dx}) + (y^2 * 3)$

$$2 \frac{d}{dx} = 2x + (6xy \frac{d}{dx}) + (3y^2)$$

$$2 \frac{d}{dx} - 6xy \frac{d}{dx} = 2x + (3y^2)$$

$$(2 - 6xy) \frac{d}{dx} = 2x + (3y^2)$$

$$\frac{d}{dx} = \frac{2x + 3y^2}{2 - 6xy} = \frac{2 * (1) + 3 * (2)^2}{2 - (6 * (1) * (2))} = \frac{14}{-10}$$