**Al-Mustaqbal University College**

**Department of Medical Instrumentation Techniques Engineering**

# LECTURE THREE

## Inverse of trigonometric function

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# Inverse of trigonometric function:

**Definition:** If $y = f(x) and \frac{dy}{dx} = \acute{y}\ ; then$:

- $y = \sin^{-1} x \ \rightarrow x = \sin y$
- $y = \cos^{-1} x \ \rightarrow x = \cos\ y$
- $y = \tan^{-1} x \ \rightarrow x = \tan y$
- $y = \cot^{-1} x \ \rightarrow x = \cot y$
- $y = \sec^{-1} x \ \rightarrow x = \sec\ y$
- $y = \csc^{-1} x \ \rightarrow x = \csc y$

## Derivative of the inverse trigonometric function:

- $y = \sin^{-1} x \ \rightarrow \acute{y} = \frac{1}{\sqrt{1-x^2}}$
- $y = \cos^{-1} x \ \rightarrow \acute{y} = \frac{-1}{\sqrt{1-x^2}}$
- $y = \tan^{-1} x \ \rightarrow \acute{y} = \frac{1}{1+x^2}$
- $y = \cot^{-1} x \ \rightarrow \acute{y} = \frac{-1}{1+x^2}$
- $y = \sec^{-1} x \ \rightarrow \acute{y} = \frac{1}{x\sqrt{x^2-1}}$
- $y = \csc^{-1} x \ \rightarrow \acute{y} = \frac{-1}{x\sqrt{x^2-1}}$

**Examples:**

$$1.\ y = \sin^{-1}(x^2)\ ,\quad y' = \frac{1}{\sqrt{1-(x^2)^2}} * 2x \quad = \quad \frac{2x}{\sqrt{1-x^4}}$$

**Example 2.**

Find $f'(2\sqrt{3})$, where $f(x) = x \tan^{-1}(x/2)$

**Sol:**

$$f'(x) = 1 * \tan^{-1}(x/2) + x * \frac{1}{1+(x/2)^2} * 1/2$$

$$= \tan^{-1}(x/2) + \frac{x}{2(1+(x/2)^2)}$$

$$f'(2\sqrt{3}) = \tan^{-1}(2\sqrt{3}/2) + \frac{2\sqrt{3}}{2(1+(2\sqrt{3}/2)^2)} = \pi/3 + \sqrt{3}/4$$

**Example 3.**

$Y = x^2 \tan^{-1}(\sqrt{x})$

$$\frac{dy}{dx} = x^2 . \frac{1}{1+x^2} . \frac{1}{2\sqrt{x}} + 2x \tan^{-1}(\sqrt{x})$$

**Example4:** If $y = \sin^{-1} t\ , x = \cos^{-1} t,\ find\ \frac{dy}{dx}, \frac{d^2y}{dx^2}$

Sol:

$$y = \sin^{-1} t \ \rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-t^2}}$$

$$x = \cos^{-1} t \rightarrow \frac{dx}{dt} = \frac{-1}{\sqrt{1-t^2}}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{\sqrt{1-t^2}}}{\frac{-1}{\sqrt{1-t^2}}} = -1$$

$$\frac{d^2y}{dx^2} = 0$$

## H.w

**1.** $y = \sin^{-1}(\frac{x-1}{x+1})$ $find$ $\frac{dy}{dx}$

2. If $y = \frac{\sqrt{x^2-4}}{x^2} + \frac{1}{2}\sec^{-1}(\frac{1}{2}x)$

3. $y = \sin^{-1}\sqrt{2}\,t$

**4.** $y = \cos^{-1} x^2$