Al-Mustaqbal University College

Department of Medical Instrumentation Technologies

Mathematics II / Second Stage

By

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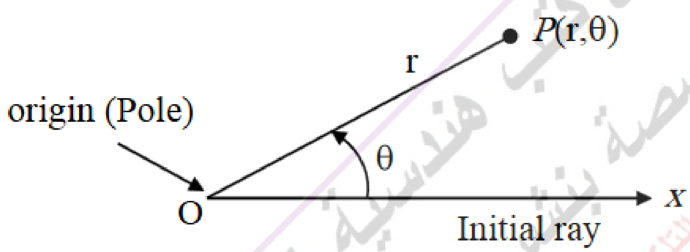
Polar coordinates

To define polar coordinate, we first fix an **origin O** (called the **pole**) and **initial ray** from O. Then each point P can be located by assigning to it a **polar coordinate pair** (r,θ) in which r gives the directed distance from O to P and θ gives the directed angle from the initial ray to ray OP.



***Graphing in polar coordinates*:**

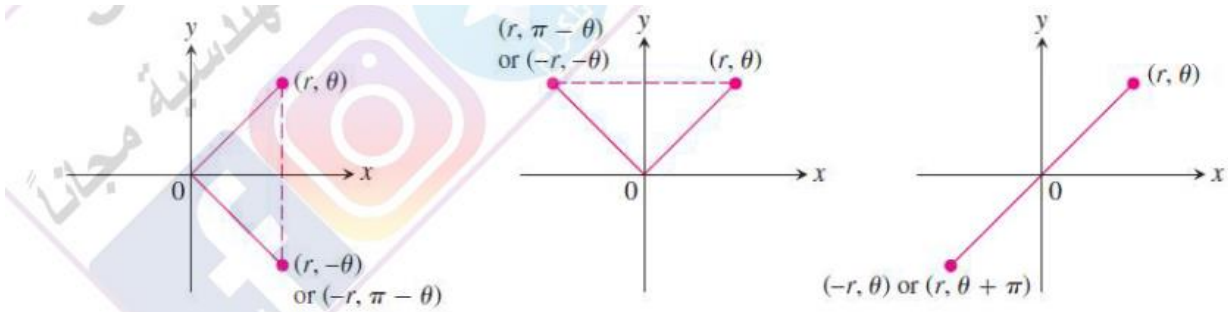
Symmetry:

Symmetry tests for polar graphs:

1. symmetry about the $x-$ axis: If the point (r,θ) lies on the graph, the point $(r,-\theta)$ or $(-r,\pi-\theta)$ lies on the graph.

2. symmetry about the $y-$ axis: If the point (r,θ) lies on the graph, the point $(r,\pi-\theta)$ or $(-r,-\theta)$ lies on the graph.

3. symmetry about the origin: If the point (r,θ) lies on the graph, the point $(-r,\theta)$ or $(r,\theta-\pi)$ lies on the graph.



(a) About the x-axis (b) About the y-axis (c) About the origin

The Complex Number

Definition. **A complex number** is an object of the form

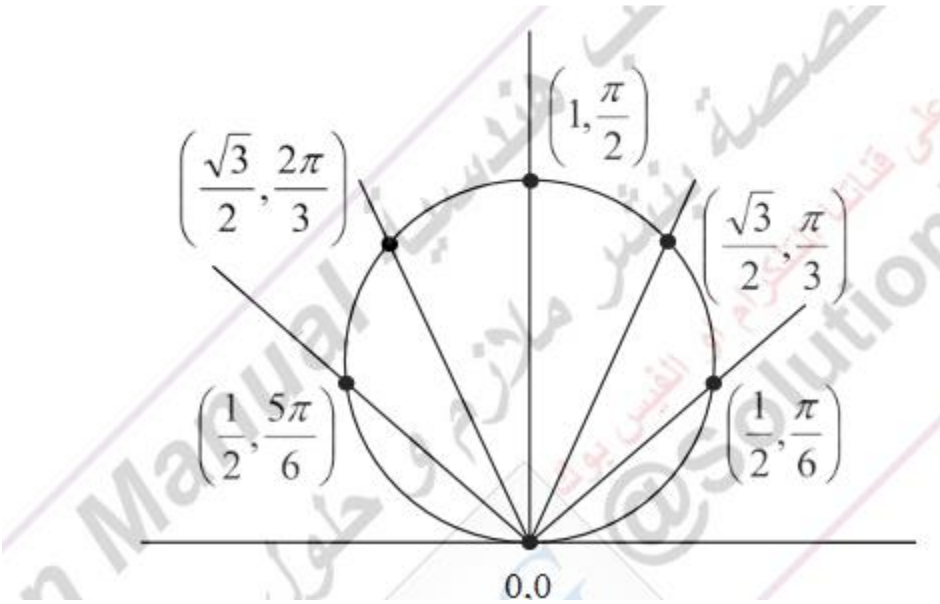
$$a + bi,$$

where a and b are real numbers and $i^2 = -1$.

Example: Sketch the graph of the equation $r = \sin\theta$ in polar coordinates

Solution:

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$r = \sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0



Ex2) Draw $r=a(1-\cos\theta)$, where (a) is any positive number?

Sol)

1- check the symmetry:

a) About origin point, $r=a(1+\cos\theta) \longrightarrow -r=a(1+\cos\theta)$ change
b) About x-axis, $r=a(1+\cos\theta) \longrightarrow r=a(1+\cos(-\theta))$ unchanged
c) About y-axis, $r=a(1+\cos\theta) \longrightarrow -r=a(1+\cos(-\theta))$ change

symmetry about x-axis only.

2- Make the table between (θ) and (r) :

θ	0	60($\pi/3$)	90($\pi/2$)	120($2\pi/3$)	180(π)
r	2a	1.5 a	a	0.5 a	0

Ex3) Graph the Curve $r^2 = 4\cos\theta$.

Sol)

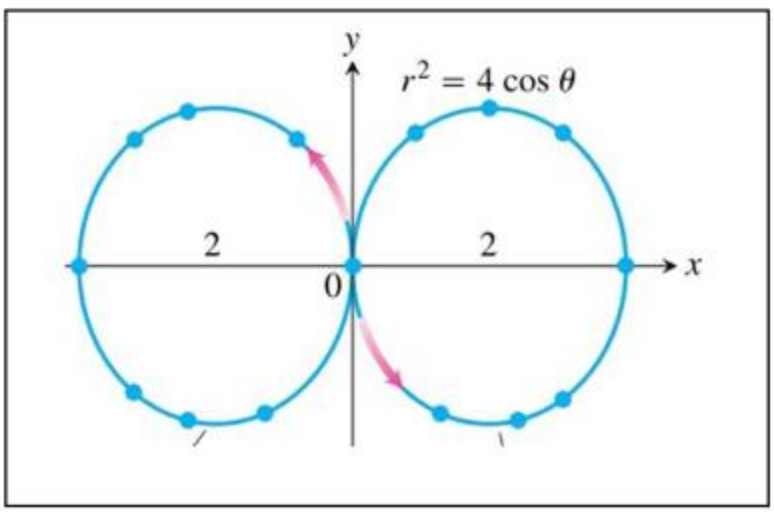
1- check the symmetry:

a) About origin point, $r^2 = 4\cos\theta$ $(-r)^2 = 4\cos\theta$ unchanged
b) About x-axis, $r^2 = 4\cos\theta$ $r^2 = 4\cos(-\theta)$ unchanged
c) About y-axis, $r^2 = 4\cos\theta$ $(-r)^2 = 4\cos(-\theta)$ unchanged

symmetry about origin point, x-axis, and y-axis.

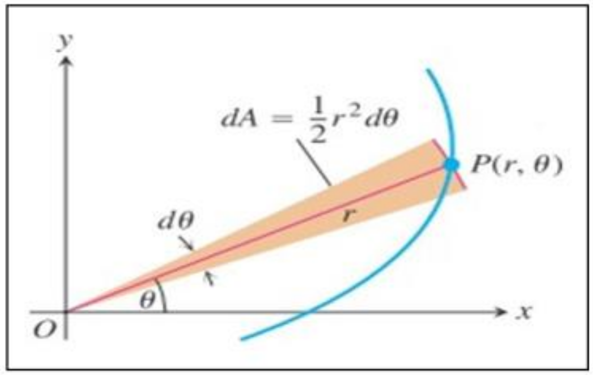
2- Make the table between (θ) and (r) :

θ	0	30(π /6)	45(π/4)	60(π/3)	90(π/2)
r	$\pm$ 2	$\pm$ 1.9	$\pm$ 1.7	$\pm$ 1.4	0



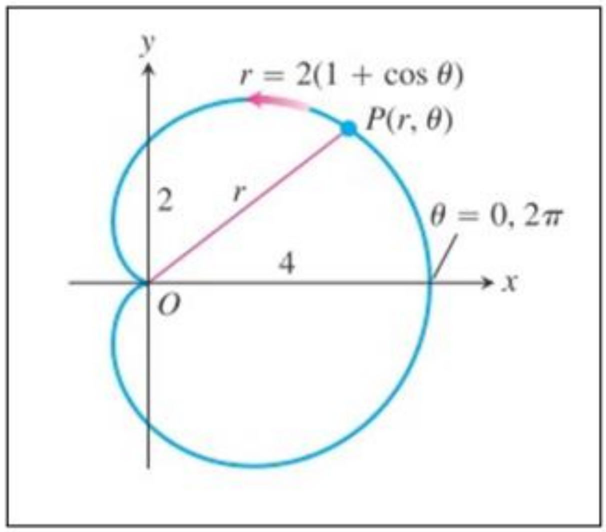
Plane Area In Polar System

$A = \int_{\theta 1}^{\theta 2} \frac{1}{2} r^2 \, d\theta$



Ex1) find the area of the region that is bounded by the curve r=1+cos θ

Sol)



$A = \int_{\theta 1}^{\theta 2} \frac{1}{2} r^2 \, d\theta$

$A = \int_{\theta 1}^{\theta 2} \frac{1}{2} r^2 \, d\theta$

$A = 2 \int_{0}^{180°} \frac{1}{2} (1 + \cos\theta)^2 \, d\theta$

$A = \int_{0}^{180°} (1 + 2cos\theta + cos\theta 2) \, d\theta$

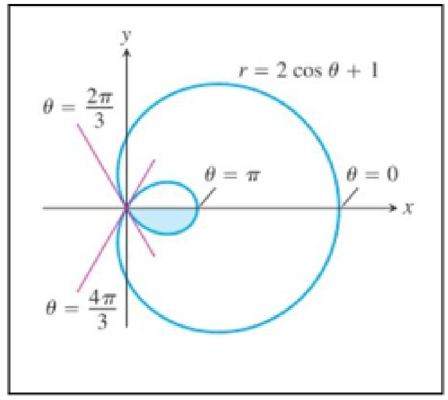
$A = \int_{0}^{180°} (1 + 2cos\theta + \frac{1+cos2\theta}{2}) \, d\theta$

$A = \int_{0}^{180°} (\frac{3}{2} + 2cos\theta + \frac{cos2\theta}{2}) \, r^2 \, d\theta$

$A = [\frac{3}{2}\,\theta + 2sin\theta + \frac{sin2\theta}{4}]_0^{\pi} = \frac{3}{2}\pi$

Ex2) find the area of the smaller loop of the curve r= 1+2cos θ

Sol) it is symmetry about x-axis



θ	0	60	90	120	180
r	3	2	1	0	-1

$A = \int_{\theta 1}^{\theta 2} \frac{1}{2}\, r^2 \, d\theta$

$A = 2 \int_{\frac{2\pi}{3}}^{\pi} \frac{1}{2}(1+2\cos\theta)^2 \, d\theta$

A=1.7 square unit