Ministry of Higher Education and Scientific Research
Al-Mustaqbal University College
Air Conditioning and Refrigeration Technologies

Heat Transfer Laboratory

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Experiment No. (5)

Two-Dimensional Heat Transfer

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Experiment (5)

أسم التجربة: أنتقال الحرارة باتجاهيين.

Experimental Title: Two-Dimensional Heat Transfer.

Objective: To study the steady-state heat transfer process in two-dimensional flat plate**.**

Theoretical Part:

The steady-state temperature in a two dimensional Cartesian coordinate system obeys, when thermal conductivity is constant and no heat is generated , the Laplace equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \qquad (5.1)$$

The solution of this equation, *T(x,y)*, can be differentiated and combined with Fourier's equation to yield the components of the vector of heat transfer rate. These components are

$$q_x = -kA_x \frac{\partial T}{\partial x} \qquad (5.2)$$

$$q_y = -kA_y \frac{\partial T}{\partial y} \qquad (5.3)$$

where A_x is the area normal to q_x and A_y is the area normal to q_y.

A number of methods of solving the Laplace equation are available, including analytical, numerical, graphical and analog techniques.

Numerical Method of Analysis

An immense number of analytical solutions for conduction heat-transfer problems have been accumulated in the literature over the past 150 years. Even so, in many practical situations the geometry or boundary conditions are such that an analytical solution has not been obtained at all, or if the solution has been developed, it involves such a complex series solution that numerical evaluation becomes exceedingly difficult. For such situations the most fruitful approach to the problem is one based on finite-difference techniques, the basic principles of which we shall outline in this section.

Consider a two-dimensional body that is to be divided into equal increments in both the *x* and *y* directions, as shown in Figure (5.1). The nodal points are designated as shown, the *m* locations indicating the *x* increment and the *n* locations indicating the *y* increment. We wish to establish the temperatures at any of these nodal points within the body using equation (5.1) as a governing condition. Finite differences are used to approximate differential increments in the temperature and space coordinates; and the smaller we choose these finite increments, the more closely the true temperature distribution will be approximated.

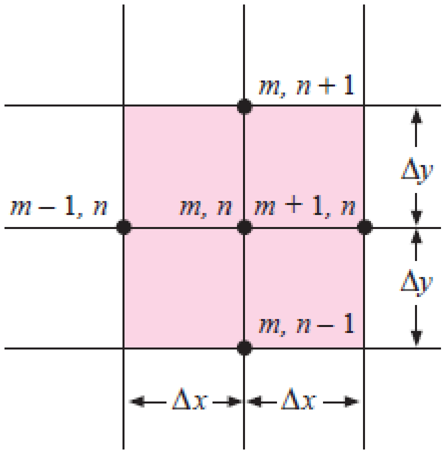


Figure (5.1): Sketch illustrating nomenclature used in two-dimensional numerical analysis of heat conduction.

The temperature gradients may be written as follows:

$$\left.\frac{\partial T}{\partial x}\right]_{m+1/2,n} \approx \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right]_{m-1/2,n} \approx \frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial y}\right]_{m,n+1/2} \approx \frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

$$\left.\frac{\partial T}{\partial y}\right]_{m,n-1/2} \approx \frac{T_{m,n} - T_{m,n-1}}{\Delta y}$$

$$\left.\frac{\partial^2 T}{\partial x^2}\right]_{m,n} \approx \frac{\left.\frac{\partial T}{\partial x}\right]_{m+1/2,n} - \left.\frac{\partial T}{\partial x}\right]_{m-1/2,n}}{\Delta x} = \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2}$$

$$\left.\frac{\partial^2 T}{\partial y^2}\right]_{m,n} \approx \frac{\left.\frac{\partial T}{\partial y}\right]_{m,n+1/2} - \left.\frac{\partial T}{\partial y}\right]_{m,n-1/2}}{\Delta y} = \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{(\Delta y)^2}$$

Thus the finite-difference approximation for the above Equations becomes,

$$\frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2} + \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{(\Delta y)^2} = 0$$

If $\Delta x = \Delta y$, then

$$T_{m+1,n} + T_{m-1,n} + T_{m,n+1} + T_{m,n-1} - 4T_{m,n} = 0 \qquad (5.4)$$

Since we are considering the case of constant thermal conductivity, the heat flows may all be expressed in terms of temperature differentials. Equation (5.4) states very simply that the net heat flow into any node is zero at steady-state conditions. In effect, the numerical finite-difference approach replaces the continuous temperature distribution by fictitious heat-conducting rods connected between small nodal points that do not generate heat.

Experimental part:

The goal of this experiment is to investigate two-dimensional steady-state conduction in an aluminum plate (30cm * 30cm * 5cm thick) as shown in Figure (5.2), subjected to the following temperature boundary conditions:

1- Two edges are heated using thermally bounded electrical resistance strip heaters (constant heat flux boundary condition).

2- The other two edges are cooled using thermally bonded heat exchanger plates supplied with cooling water from a chiller (constant temperature boundary condition).

3- The bottom face is insulated with glass wool.

4- The top face is also insulated from the surroundings by an air gap trapped underneath a glass plate. Temperature will also be measured using 16 thermocouples inserted into tiny holes drilled into the plate on a (6 cm) grid as shown in Figure (5.3).

Table (5.1): Properties of Aluminum cylinder.

Density (ρ) (kg/m^3)	**Specific heat at constant pressure (C_p) (kJ/kg.°C)**	**Thermal conductivity (k) (W/m.°C)**
2900	900	130

Figure (5.2) the experimental Apparatus

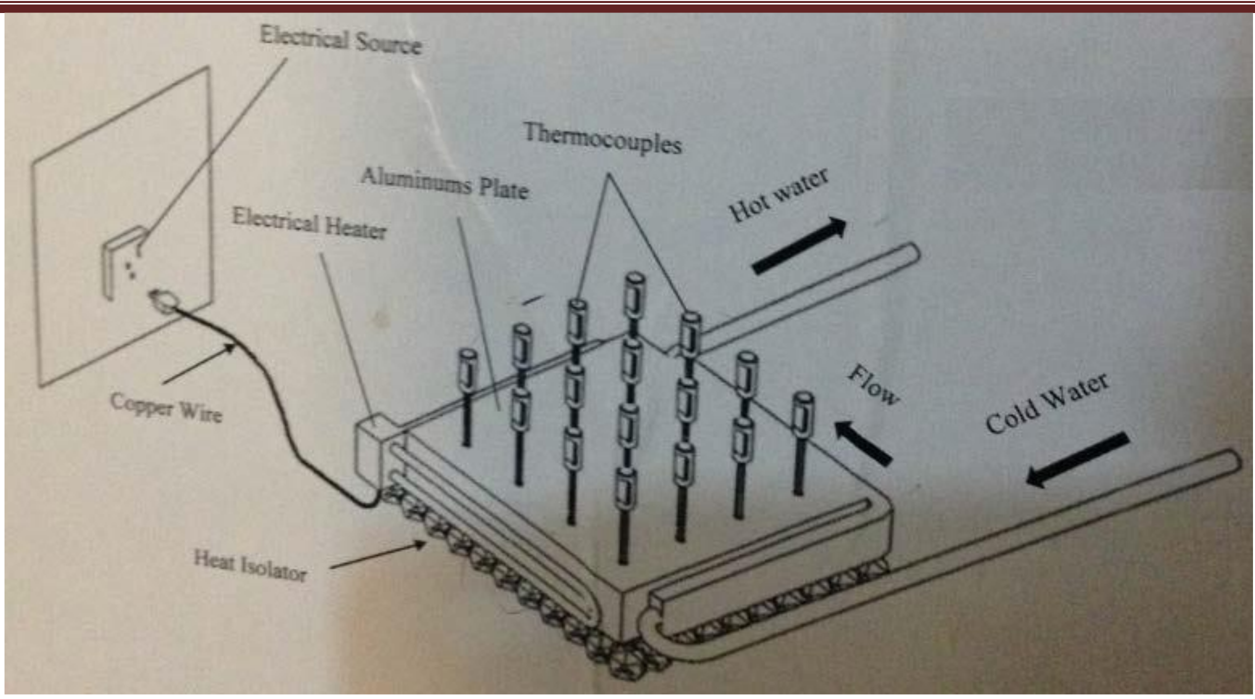
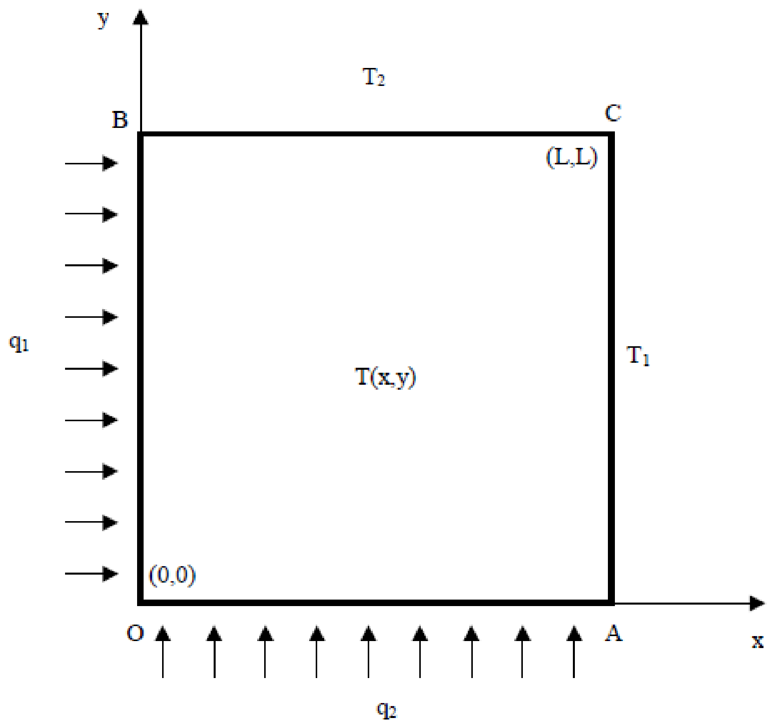


Figure (5.3) schematic diagram of the experimental Apparatus

You will be given the freedom to select your own temperature and heat flux boundary conditions for which you will record thermocouple readings. Because the plate has a substantial thermal inertia, it takes time to reach steady-state in response to the boundary conditions. This allows us the opportunity to study the transient behavior of the plate by recording temperatures from all 16 thermocouples.



We can solve for the time dependent temperature T(x,y,t) subject to the following initial conduction:

$$T(x, y, 0) = T_o$$

$$q(0, y, t) = -k\frac{\partial T(0, y, t)}{\partial x} = q_1$$

$$q(x, 0, t) = -\frac{k\partial T(x, 0, t)}{\partial y} = q_2$$

$$T(L, y, t) = T_1$$

$$T(x, L, t) = T_2$$

where *k* is the thermal conductivity of the aluminum plate. (T_1) and (T_2) are assumed to be equal to the chiller setting and (q_1) and (q_2) may be estimated by noting the electrical power supplied by each electrical heater. To is known from the thermocouples. Therefore, all four boundary conditions and initial conditions are completely determined and can be used to calculate the temperature distribution within the plate.

Procedure:

1. Record the initial plate temperature by cycling through all the thermocouples. This will provide the initial condition for your numerical simulation.

2. Set the chiller temperature to (20 ºC) and start the flow through the heat exchangers. This will set the boundary conditions T_1 and T_2 (assuming that the water temperature does not increase very much during its travel across the plate). Go immediately to the next step.

3. Set the varices to provide any desired value of q_1 and q_2. You may find that a setting between 70 and 80 V is suitable. Record the voltage V, and determine the power P using $P= V^2/R$ where R is the resistance of the heater (57.5 ohms). The heat flux is obtained using q = P/A where A is the area of the edge.

4. You will find that the plate takes a while to reach steady state. After steady state record the temperature from all 16 thermocouples by cycling through them as rapidly as possible.

5. Determine the temperature in the internal nodes by using the equation (5.4).

Observation Table:

Sr. No	V Volt	I Amp	Node Temperatures															
			T_1 ºC	T_2 ºC	T_3 ºC	T_4 ºC	T_5 ºC	T_6 ºC	T_7 ºC	T_8 ºC	T_9 ºC	T_{10} ºC	T_{11} ºC	T_{12} ºC	T_{13} ºC	T_{14} ºC	T_{15} ºC	T_{16} ºC
1																		
2																		
3																		
4																		
5																		

Discussion:

1- Comparing the results from the equation with the readings from the experiment.

2- Discusses the effect of changing the applied heat on the temperature distribution.