• Error Function.

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$

داله error الرئيسية

$$\operatorname{erf}_c(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\, du$$

Complementery

مكملة - للـerror تسمى

* داله error تحتوي على الكثير من الخواص وكل خاصية لديها برهان اليكم بعض الخواص اثباتها.

ex prove $\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t}\, t^{-\frac{1}{2}}\, dt$ *

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$

* الداله الرئيسية للـerror

نفرض ①

let $\boxed{u^2 = t}$ جذر $\boxed{u = \sqrt{t}}$

② نشتق $\rightarrow \frac{dt}{du} = 2u \rightarrow 2du\, u = dt$

$\boxed{dt = 2u\, du}$

$\boxed{du = \frac{dt}{2u}}$

$\boxed{du = \frac{dt}{2\sqrt{t}}}$

③ الحدود

$u: 0 \longrightarrow x$

$t: 0 \longrightarrow x^2$

$$\operatorname{erf}(x) = \frac{\cancel{2}}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot \frac{dt}{\cancel{2}\sqrt{t}}$$

$\sqrt{t} = t^{\frac{1}{2}}$

$$= \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot t^{-\frac{1}{2}} \cdot dt \quad \#$$

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commonly referred

② Prove : $erf(\infty) = 1$

Solution

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot t^{-\frac{1}{2}}\, dt$$

اثبات السؤال السابق

$$erf(\infty) = \frac{1}{\sqrt{\pi}} \underbrace{\int_0^{\infty} e^{-t} \cdot t^{-\frac{1}{2}}\, dt}_{\Gamma(n)}$$

تمثل دالة Gamma *

$n - 1 = -\frac{1}{2}$

$n = \frac{1}{2}$

$\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$= \frac{1}{\cancel{\sqrt{\pi}}} * \cancel{\sqrt{\pi}}$$

$$\boxed{erf(\infty) = 1} \quad \#$$

③ $erf(0) = 0$

$$erf(0) = \frac{2}{\sqrt{\pi}} \int_0^0 e^{-u^2}\, du = 0$$

④ $erf(x) + erf_c(x) = 1$

نأخذ الطرف الأيسر

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} \cdot du \, / \, erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} \cdot du$$

$$erf(x) + erf_c(x) = \frac{2}{\sqrt{\pi}} \left[\int_0^x e^{-u^2} du + \int_x^{\infty} e^{-u^2} \cdot du \right]$$

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2}\, du = erf(\infty) = 1$$

$\therefore erf(x) + erf_c(x) = 1 \quad \#$

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ex (3) prove the error function is an odd function

* Function $f(x)$ is said to be odd if $f(-x) = -f(x)$

$$f(-x) = -f(x)$$

$$\operatorname{erf}(-x) = -\operatorname{erf}(x)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2}\, du$$

$$\operatorname{erf}(-x) = \frac{2}{\sqrt{\pi}} \int_0^{-x} e^{-u^2}\, du$$

① نفرض
put $\boxed{u = -t}$, ② نشتق $\rightarrow \boxed{du = -dt}$

③ الحدود

$u:\ 0 \longrightarrow -x$

$t:\ 0 \longrightarrow x$

$u = -t$

$\cancel{-}x = \cancel{-}t$

$\boxed{t = x}$

$$\operatorname{erf}(-x) = -\frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

$$\therefore \operatorname{erf}(-x) = -\operatorname{erf}(x) \quad \#$$