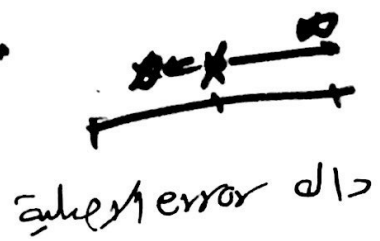


# Error Function.



$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$$

Complementary  
مكملة - لـ error function

\* دالة error تحتوي على الكثير من الخصائص وكل خاصية لديها برهان اليكيم بعض الخصائص اثباتها.

ex prove  $\text{erf}(x) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} t^{-\frac{1}{2}} dt$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

\* ازالة الجذر من error

نقطة

let  $u^2 = t$

نقطة

$$\frac{dt}{du} = 2u \rightarrow 2du = dt$$

$$dt = 2u du$$

$$du = \frac{dt}{2u}$$

$$du = \frac{dt}{2\sqrt{t}}$$

الحدود

$t: 0 \rightarrow x^2$   
 $u: 0 \rightarrow x$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot \frac{dt}{2\sqrt{t}}$$

$$\sqrt{t} = t^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot t^{-\frac{1}{2}} dt$$

#

② Prove :  $\text{erf}(\infty) = 1$

Solution

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du = \frac{1}{\sqrt{\pi}} \int_0^x e^{-t} \cdot t^{-\frac{1}{2}} dt$$

$$\text{erfc}(\infty) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-t} \cdot t^{-\frac{1}{2}} dt$$

أثبتنا السؤال السابق

$$= \frac{1}{\sqrt{\pi}} * \sqrt{\pi} \Gamma(n)$$

\* نقل  $n > 0$  Gamma

$$n-1 = -\frac{1}{2}$$

$$n = \frac{1}{2}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$\text{erfc}(\infty) = 1$

 #

③  $\text{erf}(0) = 0$

$$\text{erf}(0) = \frac{2}{\sqrt{\pi}} \int_0^0 e^{-u^2} du = 0$$

④  $\text{erf}(x) + \text{erfc}(x) = 1$

نصف الكون الاخرى

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du \quad \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$$

$$\text{erf}(x) + \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \left[ \int_0^x e^{-u^2} du + \int_x^{\infty} e^{-u^2} du \right]$$

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2} du = \text{erf}(\infty) = 1$$

... = 1 #

ex (2) Prove the error function is an odd function

\* Function  $f(x)$  is said to be odd if  $f(-x) = -f(x)$

$$f(-x) = -f(x)$$

$$\text{erf}(-x) = -\text{erf}(x)$$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

$$\text{erf}(-x) = \frac{2}{\sqrt{\pi}} \int_0^{-x} e^{-u^2} du$$

① نفرض

Put

$$u = -t$$

② نستنتج

$$du = -dt$$

③ الحدود

$$u: 0 \rightarrow -x$$

$$u = -t$$

$$-x = -t$$

$$t = x$$

$$t: 0 \rightarrow x$$

$$\text{erf}(-x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$\therefore \text{erf}(-x) = -\text{erf}(x) \quad \#$$