Ministry of Higher Education and Scientific Research
Al-Mustaqbal University College
Department of Technical Computer Engineering

measurement and instrumentation

2st Stage

Lecturer: Ali Rashid

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Chapter Three

Measurement Errors and Their Analysis

3.1 Introduction

No measurement can be made with perfect accuracy, but it is important in any measurement system to quantify the maximum error in order to reducing it from the instrument output readings. The error is the failure of instrument in exactly specifying the value of the quantity to be measured, and hence the departure of the measured value from the true value.

By proper analysis of the measurement data, the error can be predicted and avoided or eliminated from the instrument output reading.

3.2 Classification of errors

Errors may come from different sources, and are usually classified under three main categories:

1- Gross errors

The gross errors are mistakes or blunders include

(i) Misreading of instrument.

(ii) Incorrect adjustment of instrument.

(iii) Improper application of instrument.

(iv) Computational mistakes.

2- Systematic Errors.

Systematic errors in the output of many instruments are due to factors inherent in the manufacture of the instrument. This type of errors may be reduced or corrected and can sub-divided into:

a- Instrumental errors

These are defects or shortcoming of instruments that may arise due to:

(i) Tolerances in the components of the instrument.

(ii) use the instrument components over a period of time.

(iii) Error in calibration.

(iv) Using unsuitable or defective elements in the structure or instrument.

b- Environmental errors

The environmental errors are introduced by a physical affects that influence the instrument, the quantity to be measured and the experimental list. An Example of the physical effects are the temperature, pressure, electromagnetic fields,, humidity, ….etc.

c- Observation errors

The observational errors pertain to habits of the observer, such as:

(i) Imperfect technique.

(ii) Control tubes.

(iii) Peculiarities in making observation, … and others.

d- System disturbance due to measurement

Disturbance of the measured system by the act of measurement is one source of the systematic errors. In general, the process of measurement always disturbs the system being measured. The magnitude of disturbance varies from system to another and is affected by the type of the instrument used for measurement.

As an Example of the temperature of a hot water contained in a beaker by a mercury in glass thermometer as the thermometer is plunged into the water, a heat transfer would take place between the water and the thermometer. This heat transfer would lower the water temperature and hence disturb the physical quantity being measured (the water temperature).

Measurements in electrical circuits are prone to errors induced throw the loading effect on the circuit when instrument, are applied to make voltage and current measurement. To illustrate the loading effect consider the simple electric circuit shown in fig. (3.1) in this circuit, the voltage across the resistance R_2 is to be measured by a voltmeter with resistance R_s.

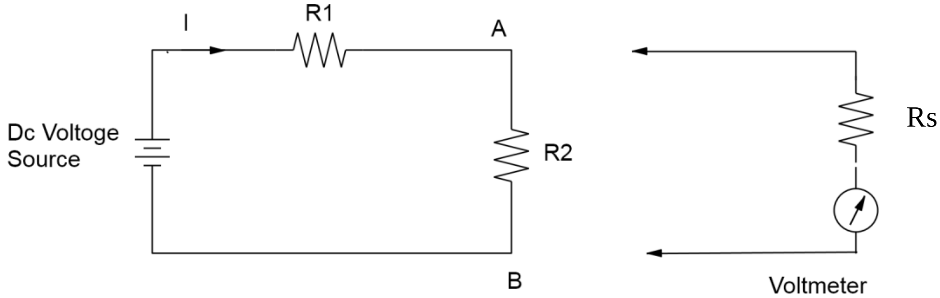

Fig. (3.1) loading of circuit by adding voltmeter

At the measurement process, the resistance R_1 will act as a shunt resistance across R_2, decreasing the resistance between the points A and B and so disturbing the circuit. Therefore, the voltage E_m measured by the voltmeter is not the value of the voltage E_o that exist prior to measurement, where:

$$E_0 = I.R_2$$

but, $I = \frac{V}{R_1+R_2}$

There,

$$E_0 = \frac{V}{R_1 + R_2}.R_2 \ldots\ldots\ldots\ldots\ldots(3.1)$$

When the voltmeter is added to the circuit at measurement, the resistance between the point, A and B will be R_{AB}, where:

$$R_{AB} = \frac{R_2 R_s}{R_2 + R_s}$$

Therefore, the voltage E_m measured by the voltmeter will be:

$$E_m = I . R_{AB}$$

In this case $I = \frac{V}{R_1 + R_{AB}}$

Therefore,

$$E_m = \frac{V}{R_1 + R_{AB}} . R_{AB}$$

$$= \frac{V}{R_1 + \frac{R_2 R_s}{R_2 + R_s}} \cdot \frac{R_2 R_s}{R_2 + R_s}$$

$$= \frac{V \cancel{(R_2 + R_s)}}{R_2 R_s + R_1(R_2 + R_s)} \cdot \frac{R_2 R_s}{\cancel{R_2 + R_s}}$$

$$\therefore\ E_m = \frac{V R_2 R_s}{R_2 R_s + R_1(R_2 + R_2)} \ldots\ldots\ldots\ldots (3.2)$$

Thus, from Eqs. (3.1) and (3.2), we get:

$$\frac{E_m}{E_0} = \frac{\frac{V R_2 R_s}{R_2 R_s + R_1(R_2 + R_s)}}{\frac{V R_2}{R_1 + R_s}}$$

$$= \frac{R_s(R_1 + R_2)}{R_2 R_s + R_1(R_2 + R_s)}$$

$$\therefore \frac{E_m}{E_0} = \frac{R_s(R_1 + R_2)}{R_1 R_2 + R_s(R_1 + R_2)} \ldots\ldots\ldots (3.3)$$

It is obvious that as R_1 gets larger, the ratio E_m/E_o get, closer to unity, showing that the design strategy should be to make R_1 as large as possible in order to minimize the disturbance of the measurement system.

e- Modifying inputs in measurement systems.

The variations of the environmental conditions away from the calibration conditions cause the characteristics of the measuring instruments to vary to same extent. These variations are described as modifying inputs to the system and are further sources of the systematic error. The environmental variations are considered as input to the environmental variations are considered as input to the measuring system because the effect on the system output is the same as if the value of the measure quantity (which is the real input) had changed by a certain amount.

In general it is very difficult to avoid the modifying inputs, because it is impractical impossible to control the environmental conditions surrounding the measurement system. But, the effect of the modifying input on the instrument output can be reduced with a proper analysis, careful instrument design or by using some techniques such as the method of opposing inputs, the using of high-gain feedback and using the signal filtering, ….etc.

3-Random errors

Random errors are accidental errors, whose magnitude and sign fluctuate in a manner that cannot be predicted from a knowledge of the measuring system and the conditions of measurement. The random errors are also known as the residual errors. Generally the random errors are minimized by employing a statistical analysis for a large number of readings.

3.3 Statistical Analysis

After the gross and systematic errors are removed or minimized, there remains random error in the final result of measurement. The outcome of measurement with random error can be predicted by statistical analysis.

The statistical analysis of experimental data is obtained by two forms of test

1- Single-sample test.

2- Multi-sample test.

In single- sample test, the measurement is done under identical conditions but at different time. Multi sample test involves repeated measurements of a given quantity using different test conditions, such as different instrument different ways of measuring the quantity and different observers. The collection of measured data is called the sample data. Many data may repeated a number of times. The number of repetition of a datum is called its frequency. The sample data may be represented by a graph known as a histogram. The statistical analysis is done by calculating some numbers called statistical descriptors, such as the arithmetic mean, the mode, the median, the standard deviation and the variance.

3.3.1 The arithmetic mean

The arithmetic mean of a number of readings gives the most probable value of the measured quantity.

The arithmetic mean of n readings is calculated as follows:

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n} \quad \text{...(3.4)}$$

Or

$$\bar{x} = \frac{x_1 + x_2 + \cdots x_n}{n} \ldots \ldots \ldots (3.5)$$

Where,

$\bar{x}$ = is *the mean value*

n = number of readings

x_1 , x_2 , … x_n = are the n readings of the measured quantity.

If the sample data has a frequency distribution, the arithmetic mean will be calculated as follows:

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i} \quad \text{...} (3.6)$$

Or

$$\bar{x} = \frac{x_1 f_1 + x_2 f_2 + \cdots + \cdots + f_1}{f_1 + f_2 + \cdots + \cdots + f_n} \text{..........} (3.7)$$

Where, n = number of groups of repeated data.

F_1 , F_2 , … f_n = are the frequencies of the measured quantity.

Example (3.1)

Five readings are taken to measure a resistance of nominal value of 10 Ω, the readings are:

9.8, 9.9, 10, 10.1 and 10.2 Ω Determine the arithmetic mean for these readings.

Solution

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{5} x_i}{n} = \frac{9.8 + 9.9 + 10 + 10.1 + 10.2}{5}$$

$$\therefore \bar{x} = \frac{50}{5} = 10\Omega$$