Ministry of Higher Education and Scientific Research
Al-Mustaqbal University College
Department of Technical Computer Engineering

measurement and instrumentation

2st Stage

Lecturer: Ali Rashid

2021-2022

Example (3.2)

A fifty readings in volt were taken to measure a voltage drope across a circuit. From the frequency distribution of measurement, shown in table below calculate the arithmetic mean.

Voltage readings (volt)	Number of readings (frequency)
199.7	1
199.8	4
199.9	12
200.0	19
200.1	10
200.2	3
200.3	1

Solution

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i f_i}{\sum_{i=1}^{n} f_i}$$

$$= \frac{199.7 * 1 + 199.8 * 4 + \ldots\ldots\ldots\ldots. + 200.3 * 1}{1 + 4 + 12 + 19 + 10 + 3 + 1}$$

$$= \frac{9999.6}{50}$$

$$= 199.992\ volts$$

3.3.2 The median

When the number of readings in the data set is very large, the calculation of the mean value become tedious, and it is more convenient to use the median value. This being a close approximation to the mean value.

The median is given by the middle value the set of readings is arranged in ascending or descending order.

- For odd number of readings, the median is the middle value when the readings are arranged in ascending or descending order.
- For even number of readings, the median is the average of the two middle numbers when the reading are arranged in ascending or descending order.

Example (3.3)

A set of readings in Kg were recorded when measuring a mass of a particular body. The readings were

5.4 , 4.7 , 5.3 , 5.1 , 4.9

Determine the median for these readings.

Solution:

If the readings are arranged in ascending order as shown below,

4.7 , 4.9 , 5.1 , 5.3 , 5.4

And since the number of readings is odd,

$\therefore$ the median is 5.1 kg.

Example (3.4)

If the reading of Example (3.3) were:

5.4 , 4.7 , 5.3 , 5.1 , 4.9 , 5.5 calculate the median for these readings.

Solution:

The readings will be arranged in ascending order as follows:

4.7 , 4.9 , 5.1 , 5.3 , 5.4 , 5.5

Since the number of readings is even,

∴ the median = $= \frac{5.1+5.3}{2} = \frac{10.4}{2} = 5.2\ kg$

3.3.3 The mode

The mode of a set of readings, is the value which occurs with great frequency. The mode may not be unique or may not exist.

Example (3.5)

Find the mode for the following set of readings:

2 , 2 , 3 , 4 , 7 , 8

Solution

The mode is 2

Example (3.6)

Find the mode for the following set of readings

1.1 , 2.2 , 2.3 , 2.4

Solution

The set has no mode

Example (3.7)

Find the mode for the following set of readings

2 , 3 , 4 , 4 , 4 , 5 , 6 , 7 , 7 , 7 , 8

Solution

The set of readings has two modes 4 and 7.

3.3.4 The standard Deviation and average deviation

The deviation of a reading from the value is a measure of error in measurement. The deviation from the mean can be expressed as:

$$d_i = x_i - \bar{x} \text{.............} (3.8)$$

Where,

$\bar{x}$ = the mean value of the set of readings.

X_i = the i_{th} value of the readings.

d_i = the deviation of the i_{th} value of readings from the mean.

i = 1 , 2 , ……… , n.

It should be noted that the deviation from the mean may be positive or negative and the algebraic sum of these deviation is always equal to zero.

Example (3.8)

The following measurements in mA were taken of a current in a circuit:

21.5, 22.1 , 21.3 , 21.7

For the measurement, calculate the deviation and the algebraic sum of deviation.

Solution

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{21.5 + 22.1 + 21.3 + 21.7}{4} = 21.65mA$$

$$d_i = x_i - \bar{x}$$

$$\therefore d_1 = x_i - \bar{x} = 21.5 - 21.65 = -0.15\ mA$$

$$d_2 = x_2 - \bar{x} = 22.1 - 21.65 = 0.45\ mA$$

$$d_3 = x_3 - \bar{x} = 21.3 - 21.65 = -0.35\ mA$$

$$d_4 = x_4 - \bar{x} = 21.7 - 21.65 = 0.05\ mA$$

$$\sum_{i=1}^{4} d_i = d_1 + d_2 + d_3 + d_4$$

$$= -0.15 + 0.45 - 0.35 + 0.05 = 0$$

The average deviation is defined as the mean of the absolute value of the deviations of readings, and it indicates the precision of the instrument. A highly precise instrument gives a low average deviation of readings. The average deviation may be expressed as:

$$D = \frac{\sum_{i=1}^{n} |d_i|}{n} \text{.....................} (3.9)$$

Or

$$D = \frac{|d_1| + |d_2| + \dots\dots\dots. + |d_n|}{n} \dots\dots\dots\dots (3.10)$$

<u>Example (3.9)</u>

For the measurements given in Example (3.8), determine the average deviation.

<u>Solution</u>:

$$D = \frac{|d_1| + |d_2| + |d_3| + |d_4|}{4}$$

$$= \frac{|-0.15| + |0.45| + |-0.35| + |0.05|}{4}$$

The root mean square of the deviations from the mean is known as the "standard deviation", and is a very useful in the analysis random errors in the measurement.

The standard deviation is expressed as:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (d_i)^2}{n-1}}$$

Substituting Eq. (3.8) into Eq. (3.11), we get:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \underline{x})^2}{n-1}} \quad \text{......................} (3.12)$$

Where,

σ= the standard deviation (reading sigma)

N= the number of readings.

If the number of readings is very large (exceeds 30 readings), the standard deviation is given by:

$$s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \underline{x})^2}{n}} \quad \text{......................} (3.13)$$

Example (3.10)

For the set of reading given in Example (3.8), calculate the standard deviation,

Solution:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

n=4

$\bar{x}$= 21.65

$$\therefore \sigma = \sqrt{\frac{(21.5-21.65)^2+(22.1-21.65)^2+(21.3-21.65)^2+(21.7-21.65)^2}{4-1}}$$

$$= \sqrt{\frac{0.35}{3}} = 0.34\,mA$$

3.3.5 The variance

The deviation of readings can alternatively be exbressed by the variance, which is the square of the standard deviation, I.e

$$V = \sigma^2 = \sqrt{\frac{\sum_{i=1}(x_i - \underline{x})^2}{n-1}} \quad \text{..........................} (3.14)$$

Where

$V = the\ variance$

The variance essentially gives the same information as can be obtained from the standard deviation, but the standard deviation, however has the advantages of being of the same units as variable.

<u>Example (3.11)</u>

For the set of readings given in Example (3.8), calculate the variance.

<u>Solution</u>:

$$V = \sigma^2$$

From Example (3.10), $\sigma = 0.34\,mA$

$$\therefore V = (0.34)^2 = 0.1165\,mA^2$$