

- Domain and Range -

Domain (Df) is the values of (x).

Range (Rf) is the values of (y).

ex1/ $y = x + 3$

$D_f = \mathbb{R}$

$y = x + 3 \Rightarrow x = y - 3 \Rightarrow R_f = \mathbb{R}$

• \mathbb{R} \because $\forall y \in \mathbb{R}$ $\exists x \in \mathbb{R}$ $y = x + 3$

ex2/ $y = \frac{3x}{x-2}$

$D_f = \mathbb{R} \setminus \{2\}$

$R_f :$ $3x = y(x-2)$

$3x - yx = -2y$

$x(3-y) = -2y$

$x = \frac{-2y}{3-y}$

$\therefore R_f = \mathbb{R} \setminus \{3\}$

$0 = \frac{-2y}{3-y}$ $\Rightarrow y = 0$ $\Rightarrow x = 0$ $\Rightarrow (0,0) \in \mathbb{R}^2$

$$\text{ex 3/ } y = \frac{x+2}{x-3}$$

$$D_f = \mathbb{R} / \{3\}$$

$$R_f: y = \frac{x+2}{x-3} \Rightarrow x+2 = yx - 3y$$

$$\cancel{xxxx} \quad 2 + 3y = yx - x$$

$$2 + 3y = x(y-1)$$

$$x = \frac{2+3y}{y-1} \Rightarrow R_f = \mathbb{R} / \{1\}$$

$$\text{ex 4/ } y = \frac{2x+5}{3x-1}$$

$$D_f \Rightarrow 3x-1 \neq 0 \Rightarrow 3x=1 \Rightarrow x = \frac{1}{3}$$

$$= D_f = \mathbb{R} / \left\{ \frac{1}{3} \right\}$$

$$R_f: 3xy - y = 2x + 5$$

$$3xy - 2x = 5 + y$$

$$x(3y-2) = 5+y$$

$$x = \frac{5+y}{3y-2}$$

$$3y-2 \neq 0 \Rightarrow 3y=2 \Rightarrow y = \frac{2}{3}$$

$$R_f = \mathbb{R} / \left\{ \frac{2}{3} \right\}$$

$$\text{ex 5 / } y = \frac{1}{x-2}$$

$$D_f \Rightarrow \mathbb{R} / \{2\}$$

$$yx - 2y = 1 \Rightarrow yx = 1 + 2y$$

$$x = \frac{1+2y}{y}, \quad R_f = \mathbb{R} / \{0\}$$

$$\text{ex 5 / } y = \frac{2x}{x-1}$$

$$D_f = \mathbb{R} / \{1\}$$

$$R_f: yx - y = 2x$$

$$yx - 2x = y$$

$$x(y-2) = y$$

$$x = \frac{y}{y-2}$$

$$R_f = \mathbb{R} / \{2\}$$

$$\text{ex 6 / } y = \sqrt{x+4}$$

$$x+4 \geq 0 \Rightarrow x \geq -4$$

$$\text{DF: } \{x : x \geq -4\}$$

$$\text{RF: } y^2 = x+4$$

$$y^2 - 4 = x \Rightarrow x = y^2 - 4$$

$$(y-2)(y+2)$$

$$\text{either } y=2 \text{ or } y=-2$$

$$\text{RF: } \mathbb{R} \setminus \{+2, -2\}$$

$$\text{ex 7 / } y = \sqrt{9-x^2}$$

$$9-x^2 \geq 0 \Rightarrow (3-x)(3+x) \geq 0$$

$$\text{DF: } \{x : -3 \leq x \leq 3\}$$

$$\text{RF: } y = \sqrt{9-x^2}$$

$$y^2 = 9-x^2$$

$$x^2 = 9-y^2 \Rightarrow x = \pm\sqrt{9-y^2}$$

$$9-y^2 \geq 0$$

$$(3-y)(3+y) \geq 0$$

$$\text{RF: } \{y : -3 \leq y \leq 3\}$$

$$\text{ex 8/ } y = \sqrt{\frac{x+1}{x-1}}$$

$$\mathbb{R} \quad x-1 \neq 0, x \neq 1, \frac{x+1}{x-1} \geq 0$$

$$x+1 \geq 0 \Rightarrow x \geq -1$$

$$x-1 \geq 0 \Rightarrow x \geq +1$$

$$D_f = \mathbb{R} / \{-1, +1\}$$

$$R_f: y^2 = \frac{x+1}{x-1} \Rightarrow y^2(x-1) = x+1$$

$$y^2x - y^2 = x+1 \Rightarrow y^2x - x = y^2 + 1$$

$$x(y^2 - 1) = y^2 + 1$$

$$x = \frac{y^2 + 1}{y^2 - 1}$$

$$y^2 - 1 \neq 0, y^2 \neq 1 \Rightarrow R_f \circ \mathbb{R} / \{-1, +1\}$$

$$\text{ex 9/ } y = f(x) = \frac{1}{x}$$

$$x \neq 0 \Rightarrow D_f \{ \mathbb{R} / \{0\} \}$$

$$R_f: x = \frac{1}{y} \Rightarrow R_f \{ \mathbb{R} / \{0\} \}$$