Continuity of functions الاستمرارية

تكون الدالة مستمرة اذا كانت مستمرة في كل نقطة في مجالها.

- اذا كانت الدالة $f(x)$ مستمرة نقول ان الدالة $f(x)$ مستمرة في النقطة (c)، والعكس بالعكس.

Continuity test اختبار الاستمرارية

تكون الدالة $f(x)=y$ مستمرة في $x=c$ اذا، فقط اذا تحققت الشروط التالية:

① $f(c)$ موجودة

② $\lim_{x \to c} f(x)$ موجودة $\to$ $\lim_{x \to a^+} f(x) = R$, $\lim_{x \to a^-} f(x) = L$ $\to R = L$

③ $\lim_{x \to c} f(c) = f(c)$

ex/ Determine weather the following function are continous at $x = 2$

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & , \quad x \neq 2 \\ 3 & , \quad x = 2 \end{cases}$$

① $f(c) = f(2) = 3$

② $\lim_{x \to 2} f(x) \Rightarrow \lim_{x \to 2} \dfrac{x^2-4}{x-2} \Rightarrow \dfrac{2^2-4}{2-2} = \dfrac{0}{0}$

$\lim \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}} \Rightarrow 2+2=4$.

$f(c) \neq \lim_{x \to c} f(x)$

$3 \neq 4 \rightarrow$ the function is discontinuous at $x=2$

ex 2/

$$f(x) = \begin{cases} x^2 & , \quad x < 1 \\ x/2 & , \quad x \geq 1 \end{cases}$$

① $f(c) = f(1) = \frac{1}{2}$

② $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2 \Rightarrow = 1^2 = 1 = R$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x/2 \Rightarrow \frac{1}{2} = L$

$R \neq L$

∴ the function is dis continuous at $x = 1$

ex 3/ Test the continutiy of the following functions at given points.

$$f(x) = \begin{cases} x^2 - 1 & -1 \leq x < 0 \\ 2x & 0 \leq x < 1 \\ 1 & x = 1 \\ -2x - 4 & 1 < x < 2 \\ 0 & 2 \leq x \leq 3 \end{cases}$$

at $x = -1$, $x = 0$, $x = +1$, $x = 2$, $x = 3$

at $x=-1$

① $f(-1) = x^2-1 \Rightarrow =(-1)^2-1=0$

② $\lim\limits_{x\to -1} f(x) = x^2-1 = (-1^2)-1=0$

③ $f(-1)=\lim\limits_{x\to -1} f(x)$

$0=0$ ∴ the function is continuous at $x=-1$

at $x=0$

① $f(0)=2x=2(0)=0$

② $\lim\limits_{x\to 0^+} f(x)=2x \neq 2(0)=0=R$

$\lim\limits_{x\to 0^-} f(x)=x^2-1=(0)^2-1=-1=L$

∴ $R\neq L \Rightarrow 0\neq -1$

∴ the function is discontinuous at $x=0$

at $x=1$

① $f(1)=1$

② $\lim\limits_{x\to 1^+} f(x)=-2x+4=-2(1)+4=2=R$

$\lim\limits_{x\to 1^-} f(x)=2x=2(1)=2=L$

∴ $R=L$

③ $f(1)=\lim\limits_{x\to 1} f(x) \Rightarrow 1=1$ ∴ the function is continuous at $x=1$

at $x = 2$

① $f(2) = 0$

② $\lim_{x \to 2^+} f(x) = 0 = R$

$\lim_{x \to 2} f(x) = -2x + 4 \Rightarrow -2(2) + 4 \Rightarrow -4 + 4 = 0 = L$

$\therefore R = L \Rightarrow 0 = 0$

③ $\therefore \lim_{x \to 2} f(x) = f(2) \Rightarrow 0 = 0$

$\therefore$ the function is continuous at $x = 2$

at $x = 3$

① $f(3) = 0$

② $\lim_{x \to 3} f(x) = 0$

③ $f(3) = \lim_{x \to 3} f(x)$

$0 = 0$

$\therefore$ the function is continuous at $x = 3$

ex 4/ If. the Function $f(x) = x^2+3$ continuous at $x=1$?

① $f(1) = x^2+3 = 1^2+3 = 4$

② $\lim\limits_{x \to 1} f(x) = \lim (x^2+3)$
$= 1^2+3 = 4$

③ $f(3) = \lim\limits_{x \to 1} f(x) \Rightarrow 4 = 4$

∴ the fun. is cont. at $x=1$.

ex 5/ $f(x) = \frac{x}{x+1}$ at $x=3$ ~~test~~test

① $f(x) = f(3) = \frac{x}{x+1} = \frac{3}{3+1} = \frac{3}{4}$

② $\lim\limits_{x \to 3} f(x) = \lim\limits_{x \to 3} \frac{x}{x+1}$

$= \frac{\lim\limits_{x \to 3} x}{\lim\limits_{x \to 3} x+1} = \frac{3}{3+1} = \frac{3}{4}$

③ $f(3) = \lim\limits_{x \to 3} f(x) \Rightarrow \frac{3}{4} = \frac{3}{4}$

∴ the fun. is cont. at $x=3$

ex 6 / if the Function $f(x) = \frac{x^2+x-6}{x^2-4}$

continuous at $x = 2$?

① $f(2) = \frac{(2)^2+2-6}{(2)^2-4} = \frac{0}{0}$ غير معرفة

$\therefore$ the Function is discontinuous at $x = 2$

لعدم تحقق الشرط الاول.

ex 7 /

$$f(x) = \begin{cases} x+2 & 2 \leq x < 3 \\ 4 & x < 2 \end{cases}$$ test the cont— at $x = 2$

① $f(2) = x+2 = 2+2 = 4$

② $\lim_{x \to 2^+} f(x) = x+2 = 2+2 = 4 = R$

$\lim_{x \to 2^-} f(x) = 4 = L$

$\therefore R = L$

③ $f(2) = \lim_{x \to 2} f(x) = 4$

$\therefore$ the F is cont— at $x = 2$.