# Al-Mustaqbal University CollegeDepartment of Medical InstrumentationTechniques Engineering

# First class

**Lecture Four**

**Exponential function and logarithmic function .**

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## The Logarithmic function:

The logarithmic was discovered by Noble man John Napier (1550-1617)

$$y = f(x) = \log_b x \rightarrow x = b^y$$

Where **y** is the logarithm, **x** is the number, **b** is the base.

If $b = 10$, we write $y = \log x$ $or$ $y = \log x$

If $b = e = 2.7183$ , we write

$y = \log_e x$ $or$ $\log x = y \rightarrow y = \ln x$

Where (ln) is read natural logarithm.

## Derivative of the natural logarithm:

$If \ \ y = f(x) = \ln x \ \ then:$ $\quad \dfrac{dy}{dx} = \acute{f}(x) = \dfrac{1}{x}$

**Example**: Find $\frac{dy}{dx}$ of the following functions:

1- $y = \ln(x^3 + 2x^2 - 1)$

Sol:

$\frac{dy}{dx} = \frac{1}{x^3+2x^2-1} \cdot (3x^2 + 4x)$

2- $y = \ln(x^{-2} + \sin^2 3x)$

Sol:

$$\frac{dy}{dx} = \frac{1}{x^{-2}+\sin^2 3x} \cdot -2x^{-3} + 2\sin(3x)\cos(3x) \cdot 3$$

3- $y = \sin^{-1}(\ln x) \cdot \ln(sin^{-1}\ 3x)$

Sol:

$$\frac{dy}{dx} = \sin^{-1}(\ln x) \cdot \frac{1}{\sin^{-1} 3x} \cdot \frac{3}{\sqrt{1-(3x)^2}} + \ln(\sin^{-1} 3x) \cdot \frac{1}{\sqrt{1-(\ln x)^2}} \cdot \frac{1}{x}$$

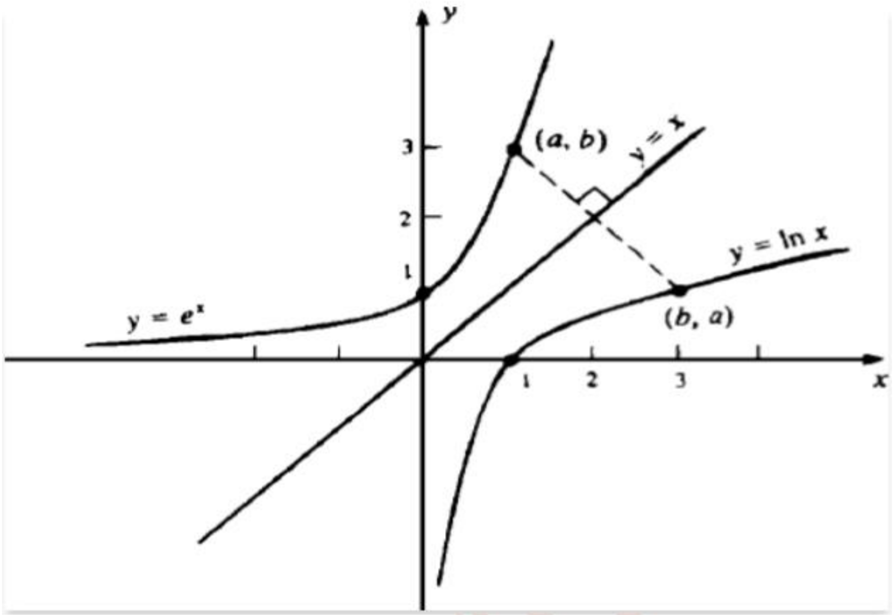
## The Exponential function:

**Definition:** the exponential function is defined as an inverse of natural logarithm, and denoted by: **exp** or **e** , that is:

For $-\infty < x < \infty \ we\ define\ \ y = f(x) = e^x$

$then \qquad x = \ln y, \ \ 0 < y < \infty$

# Exponential Functions and Logarithm Functions




| The natural logarithm function | Exponent function |
|---|---|
| $\ln x = \int_1^x \frac{1}{t}\,dt \quad x > 0$<br>properties:<br>1 . $\ln ax = \ln a + \ln x$<br>2 . $\ln \frac{a}{x} = \ln a - \ln x$<br>3 . $\ln 1 = 0$<br>4 . $\ln \frac{1}{x} = -\ln x$<br>5 . $\ln x^a = a \ln x$<br>Derivative :<br>$y = \ln u$ , $y' = \frac{1}{u} * \frac{du}{dx}$ | $y = \ln^{-1} x \rightarrow x = \ln y \rightarrow y = e^x$<br>properties:<br>1 . $e^{x1} * e^{x2} = e^{x1+x2}$<br>2 . $e^{-x} = \frac{1}{e^x}$<br>3 . $\frac{e^{x1}}{e^{x2}} = e^{x1-x2}$<br>4 . $(e^{x1})^{x2} = e^{x1*x2} = (e^{x2})^{x1}$<br>Derivative :<br>$y = e^u$ , $y' = e^u * \frac{du}{dx}$ |

Inverse equations for $e^x$ and $\ln x$:

$e^{\ln x} = x$ , $\ln(e^x) = x$

**Example:** Simplify the following expressions.

1- $e^{\ln 2} = 2$

2- $\ln e^{\sin x} = \sin x$

3- $e^{\ln(x^2+1)} = x^2 + 1$

4- $\ln e^{-1.3} = -1.3$

5- $\ln \frac{e^{2x}}{5} = \ln e^{2x} - \ln 5 = 2x - \ln 5$

6- $e^{\ln 2 + 3\ln x} = e^{\ln 2} . e^{3\ln x} = 2. e^{\ln x^3} = 2\, x^3$

7- $e^{2x + \ln x} = e^{2x} . e^{lnx} = x\, e^{2x}$

**Example**: Solve for y if: $\ln(y-1) - \ln y = 2x$

Sol:

$\ln(y-1) - \ln y = 2x \rightarrow \ \ln \frac{y-1}{y} = 2x$ (Take $exp$ for both sides)

$e^{\ln \frac{y-1}{y}} = e^{2x} \rightarrow \ \frac{y-1}{y} = e^{2x}$

$y - 1 = y\, e^{2x} \quad \rightarrow \ y - ye^{2x} = 1$

$y(1 - e^{2x}) = 1 \quad \rightarrow \therefore \ y = \frac{1}{1 - e^{2x}}$

## Derivative of the exponential function:

$$\text{If } \boldsymbol{y = e^x} \quad \boldsymbol{then} \quad \frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \boldsymbol{e^x}$$

Now, if $u = u(x) \quad then\; y = e^u$

$$\frac{\boldsymbol{du}}{\boldsymbol{dx}} = \boldsymbol{e^u}.\frac{\boldsymbol{du}}{\boldsymbol{dx}}$$

**Example:** Find $\frac{dy}{dx}$ of the following functions.

1- $y = e^{x^2+\sin 2x}$

$$\frac{dy}{dx} = e^{x^2+\sin 2x}\,.(2x + 2\cos 2x)$$

2- $y = e^{(\tan^{-1} 2x)+\ln x}$

$$\frac{dy}{dx} = e^{(\tan^{-1} 2x)+\ln x}.\left(\frac{2x}{1+4x^2} + \frac{1}{x}\right)$$

3- $y = \tan^{-1}(e^{2x})$

$$\frac{dy}{dx} = \frac{1}{1+e^{4x^2}}\,.e^{2x}.2 = \frac{2\,e^{2x}}{1+e^{4x^2}}$$

## The Function $\boldsymbol{a^x}$:

**Definition:** for $a > 0, we\ define\ a^x = e^{x\ln a}$

If $y = a^x\ then$

$$\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \boldsymbol{a^x \ln a}$$

Now if $u = u(x)$ $then\ y = a^x$

$$\frac{dy}{dx} = a^x \cdot \ln a \frac{du}{dx}$$

**Example**: Find $\frac{dy}{dx}$ of the following functions.

1- $y = 2^{\sin^2 2x}$

$$\frac{dy}{dx} = 2^{\sin^2 2x} . \ln 2\ (2 \sin 2x \cos 2x\ .2)$$

2- $y = 3^{\tan^{-1} 2x}$

$$\frac{dy}{dx} = 3^{\tan^{-1} 2x} . \ln 3 \frac{2}{1 + 4x^2}$$

**Example**

$y = \log_{10}(3x - 1) \Rightarrow y' = \frac{1}{\ln 10} * \frac{1}{3x-1} * 3 = \frac{3}{(\ln 10)(3x-1)}$

$y = \log_2 (8t)^{\ln 2} = \ln 2 \log_2 8t \Rightarrow y' = \ln 2 \frac{1}{\ln 2} * \frac{1}{8t} * 8 = \frac{1}{t}$

**H.W**

1. $, Find\ \frac{dy}{dx}\ , y = x^{\cos x}$
2. Solve for x if $2^x = 4^{x-1}$
3. Find $\frac{dy}{dx}$ $e^{\sec x}. \sec e^x$
4. $y = (x+1)^x \quad = e^{x \ln(x+1)}$