Engineering Mathematics

K. A. Stroud
Formerly Principal Lecturer
Department of Mathematics
Coventry University
United Kingdom

with additions by

Dexter J. Booth
Principal Lecturer
School of Computing and Mathematics
University of Huddersfield
United Kingdom

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Useful background information

Symbols used in the text

$=$	is equal to	$\rightarrow$	tends to
$\approx$	is approximately equal to	$\neq$	is not equal to
$>$	is greater than	$\equiv$	is identical to
$\geq$	is greater than or equal to	$<$	is less than
$n!$	factorial $n = 1 \times 2 \times 3 \times \ldots \times n$	$\leq$	is less than or equal to
$\|k\|$	modulus of k, i.e. size of k irrespective of sign	∞	infinity
$\sum$	summation	$\lim\limits_{n \to \infty}$	limiting value as $n \to \infty$

Useful mathematical information

1 Algebraic identities

$(a+b)^2 = a^2 + 2ab + b^2$ $\quad$ $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$(a-b)^2 = a^2 - 2ab + b^2$ $\quad$ $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

$a^2 - b^2 = (a-b)(a+b)$ $\quad$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

2 Trigonometrical identities

(a) $\sin^2\theta + \cos^2\theta = 1$; $\sec^2\theta = 1 + \tan^2\theta$; $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

(b) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$

$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

(c) Let $A = B = \theta$ $\therefore$ $\sin 2\theta = 2\sin\theta\cos\theta$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

(d) Let $\theta = \frac{\phi}{2}$ $\therefore$ $\sin\phi = 2\sin\frac{\phi}{2}\cos\frac{\phi}{2}$

$$\cos\phi = \cos^2\frac{\phi}{2} - \sin^2\frac{\phi}{2} = 1 - 2\sin^2\frac{\phi}{2} = 2\cos^2\frac{\phi}{2} - 1$$

$$\tan\phi = \frac{2\tan\frac{\phi}{2}}{1 - 2\tan^2\frac{\phi}{2}}$$

(e) $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos D - \cos C = 2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

(f) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

(g) Negative angles: $\sin(-\theta) = -\sin\theta$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

(h) Angles having the same trigonometrical ratios:

(i) Same sine: θ and $(180^\circ - \theta)$

(ii) Same cosine: θ and $(360^\circ - \theta)$, i.e. $(-\theta)$

(iii) Same tangent: θ and $(180^\circ + \theta)$

(i) $a\sin\theta + b\cos\theta = A\sin(\theta + \alpha)$

$a\sin\theta - b\cos\theta = A\sin(\theta - \alpha)$

$a\cos\theta + b\sin\theta = A\cos(\theta - \alpha)$

$a\cos\theta - b\sin\theta = A\cos(\theta + \alpha)$

$$\text{where } \begin{cases} A = \sqrt{a^2 + b^2} \\ \alpha = \tan^{-1}\dfrac{b}{a} \quad (0^\circ < \alpha < 90^\circ) \end{cases}$$

3 Standard curves

(a) *Straight line*

Slope, $m = \dfrac{dy}{dx} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Angle between two lines, $\tan\theta = \dfrac{m_2 - m_1}{1 + m_1 m_2}$

For parallel lines, $m_2 = m_1$

For perpendicular lines, $m_1 m_2 = -1$

Equation of a straight line (slope $= m$)

(i) Intercept c on real y-axis: $y = mx + c$

(ii) Passing through (x_1, y_1): $y - y_1 = m(x - x_1)$

(iii) Joining (x_1, y_1) and (x_2, y_2): $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

(b) *Circle*

Centre at origin, radius r: $x^2 + y^2 = r^2$

Centre (h, k), radius r: $(x - h)^2 + (y - k)^2 = r^2$

General equation: $x^2 + y^2 + 2gx + 2fy + c = 0$

with centre $(-g, -f)$: radius $= \sqrt{g^2 + f^2 - c}$

Parametric equations: $x = r\cos\theta,\ y = r\sin\theta$

(c) *Parabola*

Vertex at origin, focus $(a, 0)$: $y^2 = 4ax$

Parametric equations : $x = at^2,\ y = 2at$

(d) *Ellipse*

Centre at origin, foci $\left(\pm\sqrt{a^2 + b^2}, 0\right)$: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

where $a =$ semi-major axis, $b =$ semi-minor axis

Parametric equations: $x = a\cos\theta,\ y = b\sin\theta$

(e) *Hyperbola*

Centre at origin, foci $\left(\pm\sqrt{a^2+b^2}, 0\right)$: $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$

Parametric equations: $x = a\sec\theta,\ y = b\tan\theta$

Rectangular hyperbola:

Centre at origin, vertex $\pm\left(\dfrac{a}{\sqrt{2}}, \dfrac{a}{\sqrt{2}}\right)$: $xy=\dfrac{a^2}{2}=c^2$

i.e. $xy = c^2$ where $c = \dfrac{a}{\sqrt{2}}$

Parametric equations: $x = ct,\ y = c/t$

4 Laws of mathematics

(a) *Associative laws* – for addition and multiplication

$a+(b+c)=(a+b)+c$

$a(bc)=(ab)c$

(b) *Commutative laws* – for addition and multiplication

$a+b=b+a$

$ab=ba$

(c) *Distributive laws* – for multiplication and division

$a(b+c)=ab+ac$

$\dfrac{b+c}{a}=\dfrac{b}{a}+\dfrac{c}{a}$ (provided $a\neq 0$)

First-order differential equations

Learning outcomes

When you have completed this Programme you will be able to:

- Recognize the order of a differential equation
- Appreciate that a differential equation of order n can be derived from a function containing n arbitrary constants
- Solve certain first-order differential equations by direct integration
- Solve certain first-order differential equations by separating the variables
- Solve certain first-order homogeneous differential equations by an appropriate substitution
- Solve certain first-order differential equations by using an integrating factor
- Solve Bernoulli's equation

Introduction

1

A *differential equation* is a relationship between an independent variable, x, a dependent variable y, and one or more derivatives of y with respect to x.

e.g. $$x^2\frac{dy}{dx} = y\sin x = 0$$

$$xy\frac{d^2y}{dx^2} + y\frac{dy}{dx} + e^{3x} = 0$$

Differential equations represent dynamic relationships, i.e. quantities that change, and are thus frequently occurring in scientific and engineering problems.

The *order* of a differential equation is given by the highest derivative involved in the equation.

$x\frac{dy}{dx} - y^2 = 0$ is an equation of the 1st order

$xy\frac{d^2y}{dx^2} - y^2\sin x = 0$ is an equation of the 2nd order

$\frac{d^3y}{dx^3} - y\frac{dy}{dx} + e^{4x} = 0$ is an equation of the 3rd order

So that $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = \sin 2x$ is an equation of the ………..order.

2

second

2

second

Because

In the equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = \sin 2x$, the highest derivative involved is $\frac{d^2y}{dx^2}$.

Similarly:

(a) $x\frac{dy}{dx} = y^2 + 1$ is a ....order equation

(b) $\cos^2 x\frac{dy}{dx} + y = 1$ is a ....order equation

(c) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2$ is a ....order equation

(d) $(y^3 + 1)\frac{dy}{dx} - xy^2 = x$ is a ....order equation