**Lecture9**

**Third Stage**

# ***probability current density***

**By**

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## *The Probability Current*

In quantum mechanics, the probability current (sometimes called probability flux) is a mathematical quantity describing the flow of probability. Specifically, if one thinks of probability as a heterogeneous fluid, then the probability current is the rate of flow of this fluid. It is a real vector that changes with space and time. Probability currents are analogous to mass currents in hydrodynamics and electric currents in electromagnetism. As in those fields, the probability current is related to the probability density function via a continuity equation. The probability current is invariant under gauge transformation.

The concept of probability current is also used outside of quantum mechanics, when dealing with probability density functions that change over time, for instance in Brownian motion and the Fokker–Planck equation.

## *What is the Probability Density Function?*

The Probability Density Function(PDF) defines the probability function representing the density of a continuous random variable lying between a specific range of values. In other words, the probability density function produces the likelihood of values of the continuous random variable. Sometimes it is also called a probability distribution function or just a probability function. However, this function is stated in many other sources as the function over a broad set of values.

Often it is referred to as cumulative distribution function or sometimes as probability mass function(PMF). However, the actual truth is PDF (probability density function ) is defined for continuous random variables, whereas PMF (probability mass function) is defined for discrete random variables.

## *Probability Density Function Formula*

In the case of a continuous random variable, the probability taken by X on some given value x is always 0. In this case, if we find P(X = x), it does not work. Instead of this, we must calculate the probability of X lying in an interval (a, b). Now, we have to figure it for P(a< X< b), and we can calculate this using the formula of PDF. The Probability density function formula is given as,

$$\mathbf{P(a < X < b) = \int_a^b f(x)\, dx}$$

Or

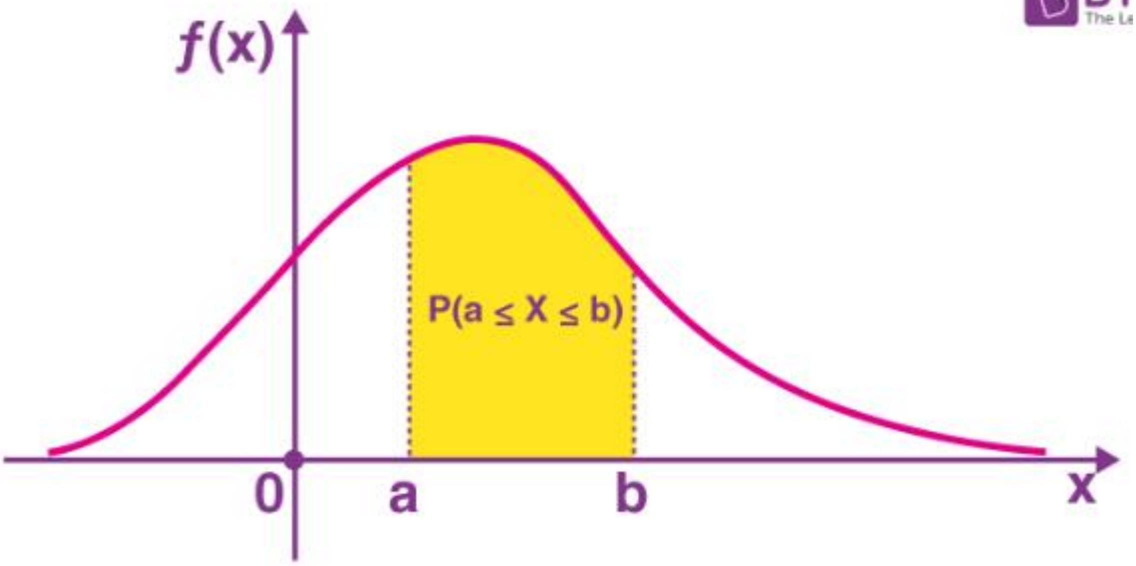
$$\mathbf{P(a \leq X \leq b) = \int_a^b f(x)\, dx}$$

This is because, when X is continuous, we can ignore the endpoints of intervals while finding probabilities of continuous random variables. That means, for any constants a and b,

$P(a \leq X \leq b) = P(a < X \leq b) = P(a \leq X < b) = P(a < X < b)$.

## *Probability Density Function Graph*

The probability density function is defined as an integral of the density of the variable density over a given range. It is denoted by f (x). This function is positive or non-negative at any point of the graph, and the integral, more specifically the definite integral of PDF over the entire space is always equal to one. The graph of PDFs typically resembles a bell curve, with the probability of the outcomes below the curve. The below figure depicts the graph of a probability density function for a continuous random variable x with function f(x).



## Probability Density Function Properties

Let x be the continuous random variable with density function f(x), and the probability density function should satisfy the following conditions:

- For a continuous random variable that takes some value between certain limits, say a and b, the PDF is calculated by finding the area under its curve and the X-axis within the lower limit (a) and upper limit (b). Thus, the PDF is given by P(x)=$\int_a^b fx\, dx$
- The probability density function is non-negative for all the possible values, i.e. f(x)$\geq$ 0, for all x.
- The area between the density curve and horizontal X-axis is equal to 1, i.e. $\int_{\infty}^{\infty} fx\, dx$ =1
- Due to the property of continuous random variables, the density function curve is continued for all over the given range. Also, this defines itself over a range of continuous values or the domain of the variable.