Lecture10

Third Stage

conservation of probability current density

By

Eman Ahmed Abdulmunem Abdulkadhim

Probability density function

In probability theory, a probability density function (PDF), or density of a continuous random variable, is a function whose value at any given sample (or point) in the sample space (the set of possible values taken by the random variable) can be interpreted as providing a relative likelihood that the value of the random variable would be close to that sample.

In other words, while the absolute likelihood for a continuous random variable to take on any particular value is 0 (since there is an infinite set of possible values to begin with), the value of the PDF at two different samples can be used to infer, in any particular draw of the random variable, how much more likely it is that the random variable would be close to one sample compared to the other sample.

In a more precise sense, the PDF is used to specify the probability of the random variable falling within a particular range of values, as opposed to taking on any one value. This probability is given by the integral of this variable's PDF over that range—that is, it is given by the area under the density function but above the horizontal axis and between the lowest and greatest values of the range. The probability density function is nonnegative everywhere, and its integral over the entire space is equal to 1.

The Basics of Probability Density Functions (PDFs)

PDFs are used to gauge the risk of a particular security, such as an individual stock or ETF. They are typically depicted on a graph, with a normal bell curve indicating neutral market risk, and a bell at either end indicating greater or lesser risk/reward. A bell at the right side of the curve suggests greater reward, but with lesser likelihood, while a bell on the left indicates lower risk and lower reward.

The Nature of the Probability Density Function of Quantum Mechanical Analysis

In 1926-27 Erwin Schrödinger formulated wave mechanics which came to be the preferred formulation for quantum physics. Schrödinger cast his theory in terms of a wave function. This stemmed from his background in optics and his captivation by Louis de Brogliem's notion that particles have a wave aspect. The immediate question was what was the nature of Schrödinger's wave function. Max Born asserted that the squared magnitude of the wave function is the probability density function for the system under analysis. Neils Bohr and Werner Heisenberg concurred with this interpretation of the wave function and it became a key element of what came to be known as the Copenhagen Interpretation of quantum physics.

What is argued below is that the squared magnitude of the wave function is a probability density function but not of the nature it is given in the Copenhagen Interpretation. Instead Schrödinger's time independent equation gives the proportion of the time the system spends in the the various states in its periodic cycle. The system cycles through the allowed states moving relatively slowly through an allowed state and then relatively rapidly to the next allowed state.

The nature of the wave function must be of one sort for all quantum mechanical systems, so to establish the above alternative to the the Copenhagen Interpretation it suffices to establish it for one significant case. The easy case is for harmonic oscillators and this has been done in Harmonic Oscillators.

The rapidly fluctuating function is the quantum mechanical probability density and the heavy line is the corresponding classical concept. The classical concept is the proportion of time spent at each possible location. It represent the probability of finding the particle at the various possible locations at any randomly specified time. There is a close relationship between a spatial average of the QM probability density and the classical concept except at the end points for the oscillator.

However harmonic oscillators are not the most natural example and the case of two body interactions is used instead, but in the form of a particle moving in a potential energy function field.

An Example of a Probability Density Function (PDF)

As indicated previously, PDFs are a visual tool depicted on a graph based on historical data. A neutral PDF is the most common visualization, where risk is equal to reward across a spectrum.

Someone willing to take limited risk will only be looking to expect a limited return and would fall on the left side of the bell curve below. An investor willing to take higher risk looking for higher rewards would be on the right side of the bell curve. Most of us, looking for average returns and average risk would be at the center of the bell curve.

What is the physical meaning of the probability current density in quantum mechanics?

In quantum mechanics particles don't have precisely defined positions but are more or less spread out in space according to a probability distribution giving the likelihood of seeing the particle at a given position when it is observed or measured.

For a moving particle this distribution must flow like a fluid whose total mass (or charge if the particle is an electron, or intensity if a photon) as obtained by integration of the distribution remains fixed but whose geometric centroid gives the overall direction of motion. As such it behaves like a current having varying density in space. For a moving charge it is in fact an electrical current. For a moving mass it is the particle's momentum in distributed form.

Applications of Probability Density Function

PDF, i.e. the probability density function has many applications in different fields of study such as statistics, science and engineering. Some of the important applications of the probability density function are listed below:

- In Statistics, it is used to calculate the probabilities associated with the random variables.
- The probability density function is used in modelling the annual data of atmospheric NOx temporal concentration
- It is used to model the diesel engine combustion.

For more maths concepts, keep visiting BYJU'S and get various maths related videos to understand the concept in an easy and engaging way.