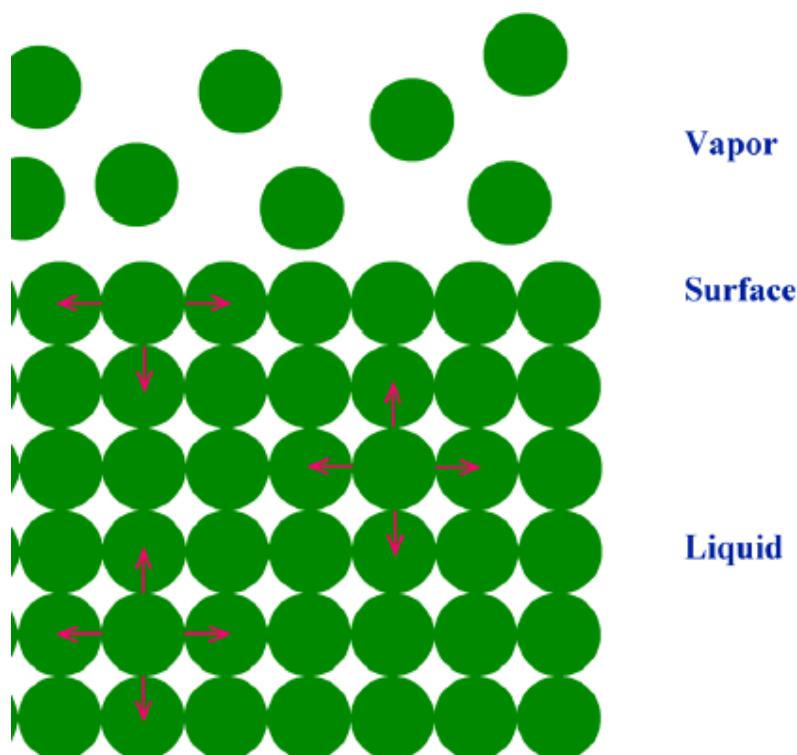


## Surface tension

Surface tension is a fundamental property by which the gas–liquid interfaces are characterized. The zone between a gaseous phase and a liquid phase looks like a surface of zero thickness. The surface acts like a membrane under tension. Let us consider a liquid in contact with its vapor, as illustrated in Fig



A molecule in the bulk liquid is subjected to attractive forces from all directions by the surrounding molecules. It is practically in a uniform field of force. But for the molecule at the surface of the liquid, the net attraction towards the bulk of the liquid is much greater than the attraction towards the vapor phase, because the molecules in the vapor phase are more widely dispersed.

This indicates that the molecules at the surface are pulled inwards. This causes the liquid surfaces to contract to minimum areas, which should be compatible

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with the total mass of the liquid. The droplets of liquids or gas bubbles assume spherical shape, because for a given volume, the sphere has the least surface area.

If the area of the surface is to be extended, one has to bring more molecules from the bulk of the liquid to its surface. This requires expenditure of some energy because work has to be done in bringing the molecules from the bulk against the inward attractive forces. The amount of work done in increasing the area by unity is known as the *surface energy*. If the molecules of a liquid exert large force of attraction, the inward pull will be large. Therefore, the amount of work done will be large. Surface energy is the amount of work done per unit area extended. Its unit is J/m<sup>2</sup> (which is equivalent to N/m).

**Exercise:** Explain why the drops of a liquid or gas bubbles tend to assume spherical shape.

**Solution:** Among all three dimensional bodies with a given surface area, the sphere has the largest volume. In other words, for a given volume, the area will be minimal when the body has spherical shape.

### **Definition Of Surface Tension**

The surface tension  $\gamma$  is the magnitude  $F$  of the force exerted parallel to the surface of a liquid divided by the length  $L$  of the line over which the force acts:

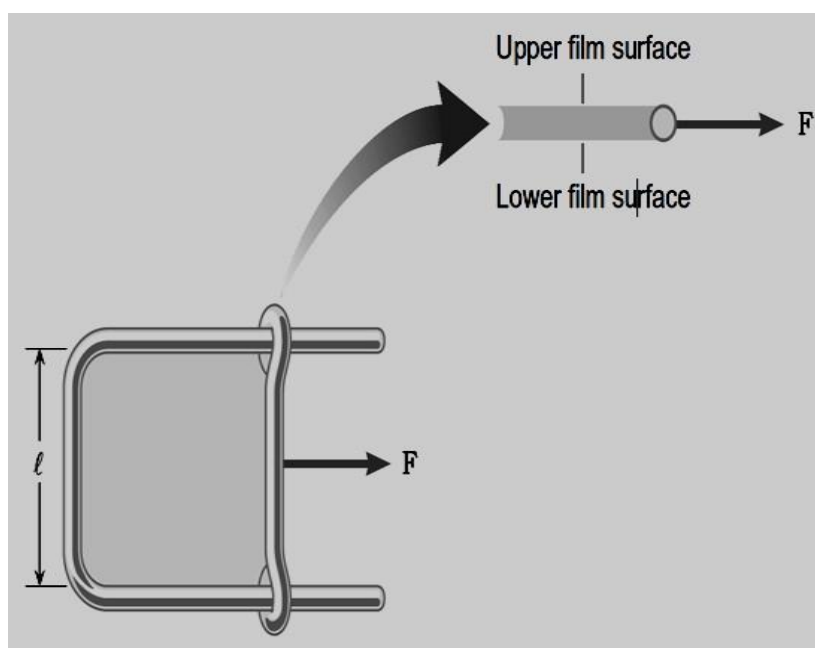
$$\gamma = F / L$$

SI Unit of Surface Tension: N/m

For the specific case illustrated in Figure 3, there is an upper surface and a lower surface, as the blow-up drawing indicates. Thus, the force  $F$  acts along a

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total length of  $L = 2l$ , where  $l$  is the length of the slider. Example 1 deals with a demonstration of the effects of surface tension that you can try yourself.



This apparatus, consisting of a C-shaped wire frame and a wire slider, can be used to measure the surface tension of a liquid.

**EXAMPLE 1** • Floating a Needle on the Surface of Water A needle has a length of 3.2 cm. When placed gently on the surface of the water ( $\gamma = 0.073 \text{ N/m}$ ) in a glass, this needle will float if it is not too heavy. What is the weight of the heaviest needle that can be used in this demonstration?

**Reasoning** As the end view in Figure 4 shows, three forces act on the needle, its weight  $W$  and the two forces  $F_1$  and  $F_2$  due to the surface tension of the water. The forces  $F_1$  and  $F_2$  result from the surface tension acting along the length of the needle on either side. According to Equation 1, they have the same magnitude  $F_1 = F_2 = \gamma L$ , where  $\gamma = 0.073 \text{ N/m}$  is the surface tension of water and  $L$  is the length of the needle.  $F_1$  and  $F_2$  are each tangent to the indented water surface that is formed when the needle presses on the surface, with the

result that each acts at an angle  $\theta$  with respect to the vertical. The needle floats in equilibrium. Therefore, the net force  $\Sigma F$  acting on the needle is zero. In the vertical direction this means that the sum of the vertical components of  $F_1$ ,  $F_2$ , and  $W$  equals zero.

**Solution** Applying the fact that the net force acting on the needle is zero we have

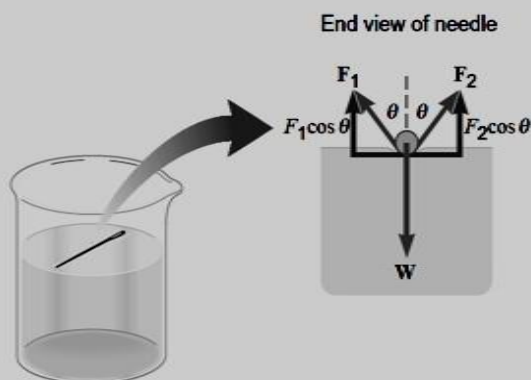
$$\Sigma F = 0$$

$$-W + \underbrace{(\gamma L) \cos \theta}_{\text{Vertical component of } F_1} + \underbrace{(\gamma L) \cos \theta}_{\text{Vertical component of } F_2} = 0$$

$$W = 2(\gamma L) \cos \theta$$

In other words, the sum of the vertical components of  $F_1$  and  $F_2$  balances the weight of the needle. The forces due to the surface tension will balance the largest weight when they point completely vertically and  $\theta = 0^\circ$ . Therefore, the weight of the heaviest needle that can be used in this demonstration is

$$|W = 2(\gamma L) \cos \theta = 2(0.073 \text{ N/m})(0.032 \text{ m}) \cos 0^\circ = 4.7 \times 10^{-3} \text{ N}$$



A needle can float on a water surface, because the surface tension of the water can lead to forces strong enough to support the needle's weight.

### Equivalence between surface tension and surface energy

The natural tendency of the surface of a liquid is to contract to minimize the surface area. Therefore, if we attempt to increase the area, work will be required. Consider a thin film of liquid, ABCD, contained in a rectangular wire-frame, as shown in Fig below.

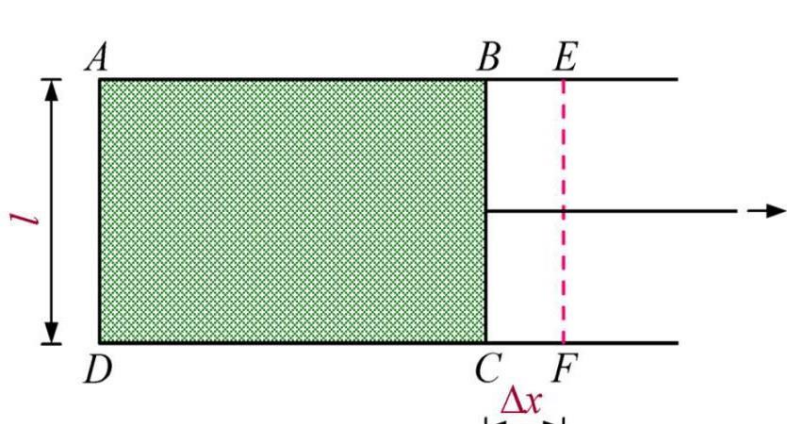


Fig. above Illustration of the work done in increasing the surface area. The boundary BC (length =  $l$ ) is movable. Imagine now that the film is stretched by moving the boundary BC by  $\Delta x$  to the new position EF. If  $\gamma$  be the surface tension, the force acting on the film is  $2\gamma l$ , because the film has two surfaces. The work done in stretching the film is,

$$W = 2\gamma l \Delta x = \gamma \Delta A$$

where  $\Delta A = 2l\Delta x$  is the change in total area on the two sides of the film. Therefore

$$\gamma = \frac{W}{\Delta A}$$

This depicts the equivalence between surface tension and surface energy.

**Measurement of Surface and Interfacial Tensions**

- Capillary Rise Method.
- The DuNoüy Ring Method
- Drop weight method (Stalagmometer) , bubble pressure, pendent drop, sessile drop, Wilhelmy plate, and oscillating drop,

The choice of the method for measuring surface and interfacial tension depend on:

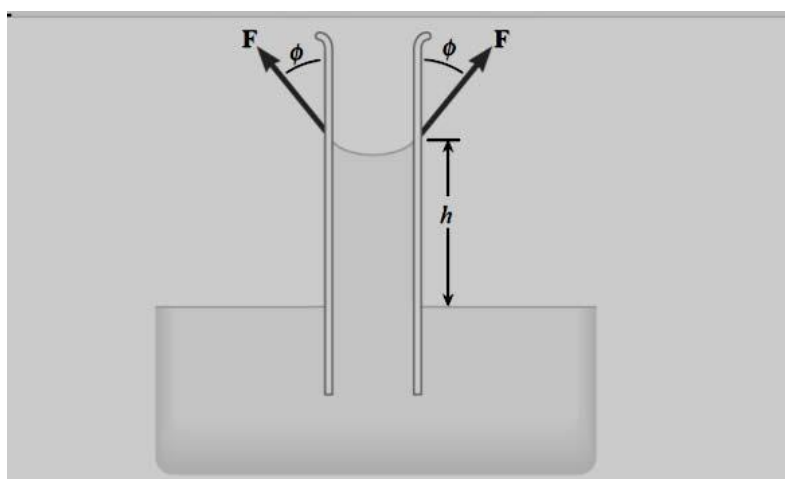
- Whether surface or interfacial tension is to be determined.
- The accuracy desired
- The size of sample.

**Capillary rise method:**

We have seen that surface tension arises because of the intermolecular forces of attraction that molecules in a liquid exert on one another. These forces, which are between like molecules, are called cohesive forces. A liquid, however, is often in contact with a solid surface, such as glass. Then additional forces of attraction come into play. They occur between molecules of the liquid and molecules of the solid surface and, being between unlike molecules, are called adhesive forces. Consider a tube with a very small diameter, which is called a capillary. When a capillary, open at both ends, is inserted into a liquid, the result of the competition between cohesive and adhesive forces can be observed. For instance, Figure 5 shows a glass capillary inserted into water. In this case, the adhesive forces are stronger than the cohesive forces, so that the water molecules are attracted to the glass more strongly than to each other. The result

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is that the water surface curves upward against the glass. It is said that the water “wets” the glass. The surface tension leads to a force  $F$  acting on the circular boundary between the water and the glass. This force is oriented at an angle  $\phi$ , which is determined by the competition between the cohesive and adhesive forces. The vertical component of  $F$  pulls the water up into the tube to a height  $h$ . At this height the vertical component of  $F$  balances the weight of the column of water of length  $h$ .



*Fig 5. Water rises in a glass capillary due to the surface tension of the water and the fact that the water wets the glass surface.*

Figure 6 shows a glass capillary inserted into mercury, a situation in which the adhesive forces are weaker than the cohesive forces. The mercury atoms are attracted to each other more strongly than they are to the glass. As a result, the mercury surface curves downward against the glass and the mercury does not “wet” the glass. Now, in contrast to the situation illustrated in Figure 5, the surface tension leads to a force  $F$ , the vertical component of which pulls the mercury down a distance  $h$  in the tube. The behavior of the liquids in both Figures 5 and 6 is called **capillary action**.

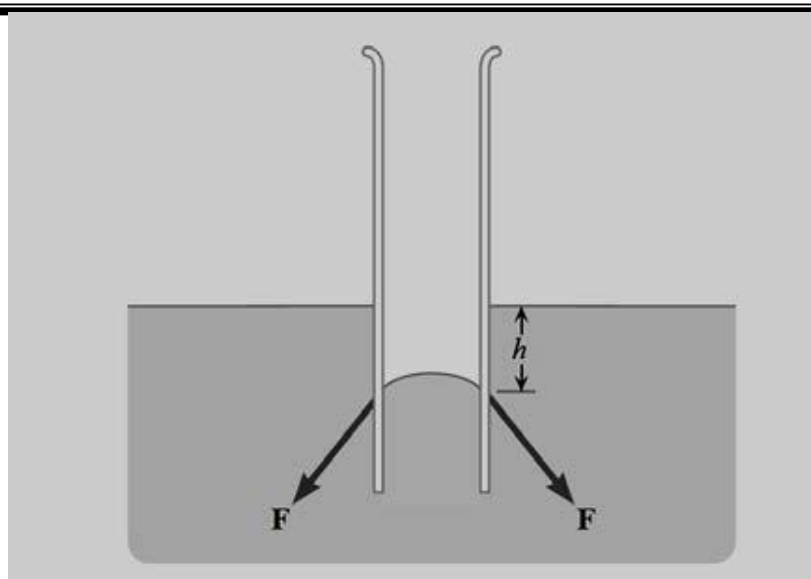


Figure 6 Mercury falls in a glass capillary due to the surface tension of the mercury and the fact that the mercury does not wet the glass surface.

- When a capillary tube is placed in a liquid contained in a beaker, the liquid generally rises up the tube a certain distance.
- By measuring this rise in a capillary, it is possible to determine the surface tension of the liquid. It is not possible, however, to obtain interfacial tensions using the capillary rise method.
- Because of the surface tension, the liquid continues to rise in the tube, but because of the weight of the liquid, the upward movement is just balanced by the downward force of gravity.

$$\gamma = \frac{1}{2} r h \rho g$$

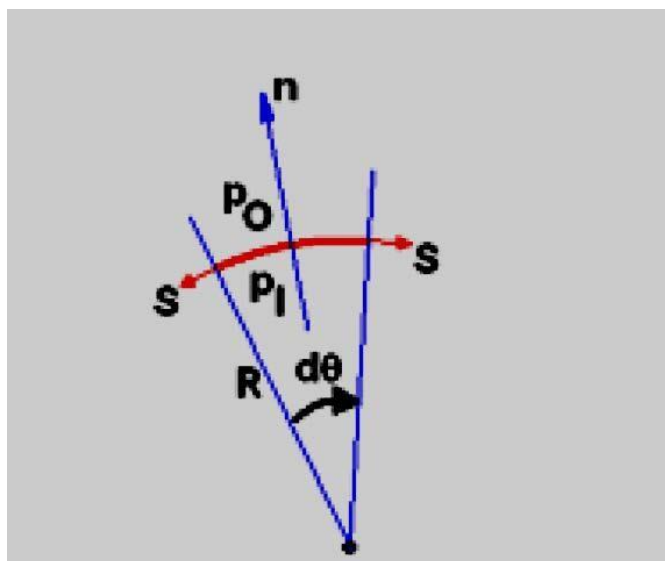
**Example:** A sample of chloroform rose to a height of 3.67 cm at 20°C in a capillary tube having an inside radius of 0.01 cm. What is the surface tension of chloroform at this temperature? The density of chloroform is 1.476 g/cm<sup>3</sup>.

$$\gamma = \frac{1}{2} \times 0.01 \text{ cm} \times 3.67 \text{ cm} \times 1.476 \text{ g/cm}^3 \times 981 \text{ cm/sec}^2$$

$$\gamma = 26.6 \text{ g/sec}^2 = 26.6 \text{ dynes/cm}$$



### Pressure difference across curved surface tension



One common manifestation of surface tension is the difference in pressure it causes across a curved surface. For simplicity we consider first a liquid surface which is curved in only one plane but is flat in a direction perpendicular to that plane. A small section of such a surface is sketched in Figure 7. The pressures in the different fluids on either side of this interface are denoted by  $p^o$  (on the outside of curve) and  $p^i$  (on the inside of the curve). Now consider all the forces acting on a small section of the surface of length,  $ds$ , and unit dimension normal to the sketch. The radius of curvature of the surface is denoted by  $R$  as indicated so that the angle subtended at the center of curvature,  $d\theta$ , is given by  $ds = R d\theta$ . Now consider all the forces in the direction  $n$  normal to the center of the fluid element. By the result described in static forces the net force in the outward direction due to forces near and at a fluid surface that we will describe in this and some linked sections.  $p^o$  and  $p^i$  will be  $2R \sin d\theta (p^i - p^o)$ . Opposing this will be the components of the surface tension forces  $S$  acting on the two ends of the section of surface which yield an inward force (in the negative  $n$  direction) equal to  $2 \sin d\theta S$ . Thus in equilibrium

$$2R \sin d\theta (p_I - p_O) = 2 \sin d\theta S$$

$$p_I - p_O = S/R$$

Thus the surface tension causes a greater pressure inside the surface and the difference is the surface tension divided by the radius of curvature (so that a flat surface yields no pressure difference). Note that it makes no difference whether the liquid is on the outside or on the inside. Now consider a spherical surface, specifically the spherical drop or bubble shown in Figure 7. If the drop is cut in half as shown the force imposed by surface tension on the remaining half will be  $2\pi RS$ . Opposing that will be the pressure difference ( $p^I - p^O$ ) acting on the projected area  $\pi R^2$  and therefore, in equilibrium,

$$2\pi RS = \pi R^2 (p_I - p_O)$$

so that, in the case of a spherical surface,

$$p_I - p_O = 2S/R$$

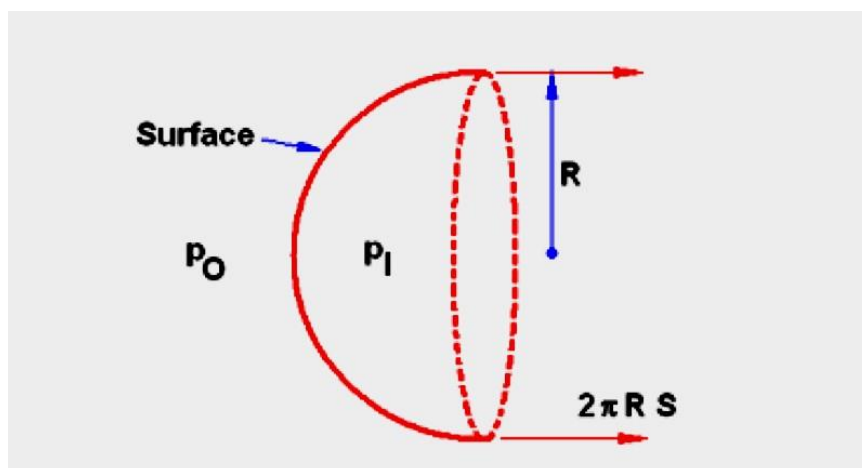


Figure 7: Half of a spherical drop of radius,  $R$ , (red) with surface tension,  $S$ , and pressures,  $p^O$  and  $p^I$  on the outside and inside respectively.

An appropriate way to visualize this is that, on the surface, the curvatures in each of the two perpendicular directions contribute equally to the pressure difference caused by the surface tension. Indeed, a general three-dimensional surface will have two principal radii of curvature,  $R_1$  and  $R_2$ , and it can be shown that the resulting pressure difference in this general case is given by

$$p_I - p_O = S \left[ \frac{1}{R_1} + \frac{1}{R_2} \right]$$

In the special case of the cylindrical surfaces,  $R_1 = R$  and  $R_2 = \infty$  and in the special case of the sphere,  $R_1 = R_2 = R$ .