## Couples

The moment produced by two equal, opposite, and noncollinear forces is called a couple. Couples have certain unique properties and have important applications in mechanics. Consider the action of two equal and opposite forces $\mathbf{F}$ and - $\mathbf{F}$ a distance $d$ apart, as shown in Figure. These two force s cannot be combined into a single force because their sum in every direction is zero. Their on ly effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as a in their plane is the couple $M$. This couple has a magnitude

$\mathrm{M}=\mathrm{F}(\mathrm{a}+\mathrm{d})-\mathrm{Fa}$
Or
$\mathrm{M}=\mathrm{Fd}$

## Equivalent Couples

Changing the values of $\mathbf{F}$ and $\mathbf{d}$ does not change a give n couple as long as the product $\mathbf{F d}$ remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plan e. Figure shows four different configurations of the same couple $\mathbf{M}$. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.


## Force-Couple Systems

The replacement of a force by a force and a couple is illustrated in Figure, where the given force $F$ acting at point $A$ is replaced by an equal force $F$ at some point $B$ and the counterclockwise couple $\mathrm{M}=\mathrm{Fd}$.


## Problem 1

The rigid structural member is subjected to a couple consisting of the two $100-\mathrm{N}$ forces. Replace this couple by an equivalent couple consisting of the two forces P and -P , each of which has a magnitude of 400 N . Determine the proper angle $\theta$.


Dimensions in millimeters


Solution
$\mathrm{M}=\mathrm{Fd}$
$\mathrm{M}=100(0.1)=10 \mathrm{~N} \cdot \mathrm{~m}$
The forces P and - P produce a counterclockwise couple
$\mathrm{M}=400(0.040) \cos \theta$
$10=400(0.040) \cos \theta$
$\theta=\cos ^{-1} \frac{10}{16}=51.3$

## Problem 2

Replace the horizontal $80 \cdot \mathrm{lb}$ force acting on the lever by an equivalent system consisting of n force at O and a couple.


Solution


We apply two equal and opposite $80 \cdot \mathrm{lb}$ forces at O and identify the counterclockwise couple
$\mathrm{M}=\mathrm{Fd}$
$M=80(9 \sin 60)=624 \mathrm{lb}-\mathrm{in}$.

## Problem 3

The indicated force- couple system is applied to a small shaft at the center of the rectangular plate. Replace this system by a single force and specify the coordinate of the point on the $y$ axis through which the line of action of this resultant force passes.


Solution


ศ $7 \mathrm{M}=\mathrm{Fd}: 375=5000 \mathrm{~d}$

$$
d=0.075 \mathrm{~m}
$$

$$
\therefore \quad y=-75 \mathrm{~mm}
$$

## Problem 4

The top view of a revolving entrance door is shown. Two persons simultaneously approach the door and exert forces of equal magnitudes as shown. If the resulting moment about the door pivot axis at O is $25 \mathrm{~N} . \mathrm{m}$, determine the force magnitude F .


Solution

$f+M_{0}=\Sigma F d$

$$
25=2 F\left(\cos 15^{\circ}\right)(0.8)
$$

$$
F=16.18 \mathrm{~N}
$$

