Biomedical Engineering
Engineering Analysis
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Lecture (5)

Inverse Fourier

1) The function is **not** even and **not** odd.

$$f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t}\, d\omega$$

2) The function is **even**.

$$f(t) = \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} F(\omega).\cos(\omega t)\, d\omega$$

3) The function is **odd**

$$f(t) = \frac{2}{\sqrt{2\pi}} \int_{0}^{\infty} F(\omega).\sin(\omega t)\, d\omega$$

Example 1. Find inverse F.T.

$$F(\omega) = \delta(\omega) + 3e^{-4|\omega|}$$

Sol.

$$f(t) = 1 + 3\left[\frac{4}{t^2 + 16}\right]$$

Example 2. Find inverse F.T. for

a) $F(\omega) = (i\omega)^2\, e^{-|\omega|}$

Sol.

$$f(t) = \frac{d^2}{dt^2}\left[\frac{1}{t^2 + 1}\right]$$

b) $F(\omega) = \frac{e^{-a|\omega|}}{(i\omega)}$

Sol.

$$f(t) = \int_{-\infty}^{t} \frac{a}{t^2 + a^2}\, dt$$

c) $F(\omega) = \frac{e^{-a|\omega|}}{(\omega)}$

Sol.

$$F(\omega) = \frac{e^{-a|\omega|}}{(\omega)} \times \frac{i}{i} = i \int_{-\infty}^{t} \frac{a}{t^2 + a^2}\, dt$$

Example 3. Find $F(\omega) = \frac{\sin \omega}{\omega^2}$

Sol.

$$F(\omega) = \frac{\sin \omega}{\omega^2} = \frac{i}{i\omega}\left[\frac{\sin \omega}{\omega}\right]$$

$$f(t) = i \int_{-\infty}^{t} U(t)dt \quad , |t| < 1$$

Example 4. Find $F(\omega) = \frac{e^{-3i\omega}}{\omega^2+1}$

Sol.

$$F(\omega) = \frac{e^{-3i\omega}}{\omega^2 + 1} = e^{-3i\omega}\left[\frac{1}{\omega^2 + 1}\right]$$

$$e^{-3i\omega} \to u(t-3), \quad \frac{1}{\omega^2 + 1} \to e^{-|t|}$$

$$f(t) = e^{-|t-3|}\, u(t-3)$$

Example 5. Find inverse F.T. for $F(\omega) = e^{-ai\omega}\left[\frac{sin^2(\omega)}{\omega^2}\right]$

Sol.

$$e^{-ai\omega} \to u(t-a), \quad \frac{sin^2(\omega)}{(\omega)^2} \to 1 - \frac{|t|}{2} \quad , |t| < 2$$

$$f(t) = \left[1 - \frac{|t-a|}{2}\right] u(t-a), |t-a| < 2$$

Laplace Transformation

$$F(s) = \int_0^{\infty} F(t).\, e^{-st}\, dt$$

Example. Convert the function f(t)= c

Sol.

$$F(s) = \int_0^{\infty} c.e^{-st}\ dt = \left[\frac{C.e^{-st}}{-s}\right]_0^{\infty}$$

$$= \left[\frac{C.e^{-s\infty}}{-s}\right] + \left[\frac{C.e^{0}}{s}\right] = \frac{C}{s}$$

Example. $F(t) = e^{-at}$

Sol.

$$F(s) = \int_0^{\infty} e^{-at}.e^{-st}\ dt = \int_0^{\infty} e^{-t(a+s)}\ dt = \left[\frac{e^{-t(a+s)}}{-(a+s)}\right]_0^{\infty}$$

$$= \frac{e^{-\infty(a+s)}}{-(a+s)} + \frac{e^{0}}{a+s} = \frac{1}{a+s}$$

F(t)	F(s)
C	C/s
e^{-at}	$\frac{1}{a+s}$
e^{at}	$\frac{1}{-a+s}$
t^n	$\frac{n!}{s^{n+1}}$
$t^{1/4}$	$\frac{\Gamma(n+1)}{s^{n+1}} = \frac{\Gamma(\frac{3}{2})}{s^{3/2}}$
$\sqrt{t}$	
Sin at	$\frac{a}{s^2+a^2}$

Cos at	$\frac{s}{s^2 + a^2}$
Sinh at	$\frac{a}{s^2 - a^2}$
Cosh at	$\frac{s}{s^2 - a^2}$

Example. Convert function f(t)= $\sqrt{t} + \sin(4t) + 3e^{2t}$

Sol.

$$F(s) = \frac{\Gamma(\frac{3}{2})}{s^{3/2}} + \frac{4}{s^2 + 16} + 3\,\frac{1}{S - 2}$$

Example. $F(t) = e^{2t-3}$

Sol.

$$F(t) = e^{2t}.e^{-3} = e^{-3}\,\frac{1}{s - 2}$$

Example. $F(t) = (t + 1)^2$

Sol.

$$F(t) = t^2 + 2t + 1 = \frac{2}{s^3} + 2\,\frac{1}{s^2} + \frac{1}{s}$$

Example. $F(t) = sin^2(t)$

Sol.

$$F(t) = \frac{1}{2} - \frac{1}{2}\cos 2t = F(s) = \frac{\frac{1}{2}}{s} - \frac{1}{2}\,\frac{s}{s^2 + 4}$$

Example. $F(t) = \sin(t + \frac{\pi}{4})$

Sol.

$F(t) = sint.\cos\frac{\pi}{4} + \cos(t)\,.\sin\frac{\pi}{4}$

$$F(s) = \frac{1}{\sqrt{2}} \cdot \frac{1}{s^2+1} + \frac{1}{\sqrt{2}} \frac{s}{s^2+1}$$

Example. $F(t) = \sin(t) \cdot \cos(t)$

Sol.

$F(t) = \frac{1}{2} \sin 2t$

$F(s) = \frac{1}{2} \frac{2}{s^2+4}$

Differentiation	Integration	Even/Odd Identities
$(cu)' = cu'$ (c constant) $(u + v)' = u' + v'$ $(uv)' = u'v + uv'$ $(x^n)' = nx^{n-1}$ $(e^x)' = e^x$ $(e^{ax})' = a\,e^{ax}$ $(\sin x)' = \cos x$ $(\cos x)' = -\sin x$ $(\sinh x)' = \cosh x$ $(\cosh x)' = \sinh x$	$\int uv'dx = uv - \int u'v\,dx \;\; (by\ parts)$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \;\; (n \neq -1)$ $\int \frac{1}{x}\,dx = \ln\lvert x\rvert + c$ $\int e^{ax}\,dx = \frac{1}{a}\,e^{ax} + c$ $\int sinx\,dx = -cosx + c$ $\int cosx\,dx = sinx + c$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$ $\csc(-\theta) = -\csc\theta$ $\sec(-\theta) = \sec\theta$ $\cot(-\theta) = -\cot\theta$

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Θ	π	Sin Θ	Cos Θ	tan Θ	Sec Θ	Csc Θ	Cot Θ
0	0	0	1	0	1	---	---
30	π/6	1/2	√3/2	1/√3	2/√3	2	√3
45	π/4	1/√ 2	1/√2	1	√2	√2	1
60	π/3	√3/2	1/2	√3	2	2/√3	1/√3

90	$\pi/2$	1	0	---	---	1	0
180	π	0	-1	0	-1	---	---
270	$3\pi/2$	-1	0	---	---	-1	0
360	2π	0	1	0	1	---	---