

**Al-Mustaqbal University College**  
**Department of Medical Instrumentation**  
**Techniques Engineering**



**Subject: Mathematics (I)**

**First Class**

**Lecture: Suhair Hussein Talib**

# Lecture one: Limit

## ✚ Limit and Continuity:

When  $f(x)$  close to the number  $L$  as  $x$  close to the number  $a$ , we write

$$f(x) \rightarrow L \text{ as } x \rightarrow a \quad \text{means: } \lim_{x \rightarrow a} f(x) = L$$

**Example 1:** Let  $f(x) = 2x + 5$  evaluate  $f(x)$  at  $x = 1$

Sol:

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (2x + 5) = 2 * 1 + 5 = 7$$

## ✚ The Limit Laws:

If  $L$ ,  $M$ ,  $C$ , and  $k$  are real numbers and  $\lim_{x \rightarrow c} f(x) = L$  and  $\lim_{x \rightarrow c} g(x) = M$

1. Sum Rule:  $\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$

2. Difference Rule:  $\lim_{x \rightarrow c} (f(x) - g(x)) = L - M$

3. Constant Multiple Rule:  $\lim_{x \rightarrow c} k \cdot f(x) = k \cdot L$

4. Product Rule:  $\lim_{x \rightarrow c} (f(x) \cdot g(x)) = L \cdot M$

5. Quotient Rule:  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}$ ,  $M \neq 0$

6. Power Rule:  $\lim_{x \rightarrow c} [f(x)]^n = L^n$

7. Root Rule:  $\lim_{x \rightarrow c} \sqrt[n]{f(x)} = \sqrt[n]{L}$ , n is a positive integer.

## Indeterminate quantities

$\frac{0}{0}$	,	$\frac{\infty}{\infty}$	,	$0 \cdot \infty$	,	$\infty - \infty$	,	$\infty^0$	,	$0^0$
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**Example 2:** Evaluate the following limits

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \frac{0}{0} = \infty, \text{ then the solution, } x \neq 1$$

Sol:

$$\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \lim_{x \rightarrow 1} \frac{(x - 1)(x^2 + x + 1)}{x - 1} = \lim_{x \rightarrow 1} x^2 + x + 1 = 1^2 + 1 + 1 = 3$$

**Example 3:**  $\lim_{h \rightarrow 0} \frac{1}{h} \left( \frac{1}{x+h} - \frac{1}{x} \right), h \neq 0$

Sol:

$$\lim_{h \rightarrow 0} \left[ \frac{1}{h} \left( \frac{1}{x+h} - \frac{1}{x} \right) \right] = \lim_{h \rightarrow 0} \left[ \frac{1}{h} \left( \frac{x - x - h}{x(x+h)} \right) \right] = \lim_{h \rightarrow 0} \left[ \frac{1}{h} \left( \frac{-h}{x(x+h)} \right) \right]$$

$$\lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = -\frac{1}{x(x+0)} = -\frac{1}{x^2}$$

**Example 4 :**  $\lim_{x \rightarrow -2} \sqrt{4x^2 - 3} = \sqrt{\lim_{x \rightarrow -2} (4x^2 - 3)} = \sqrt{4 * (-2)^2 - 3} =$

$\sqrt{16 - 3} = \sqrt{13}$   Root Rule

**Example 5:** Evaluate the following limits if they exist.

1)  $\lim_{x \rightarrow -1} \frac{\sqrt{2+x}-1}{x+1}$  ,  $x \neq -1$  ,  $x \neq -2$

Sol:

$$\lim_{x \rightarrow -1} \frac{\sqrt{2+x}-1}{x+1} * \frac{\sqrt{2+x}+1}{\sqrt{2+x}+1} = \lim_{x \rightarrow -1} \frac{2+x-1}{x+1(\sqrt{2+x}+1)}$$

$$\lim_{x \rightarrow -1} \frac{x+1}{x+1(\sqrt{2+x}+1)} = \lim_{x \rightarrow -1} \frac{1}{(\sqrt{2+x}+1)} = \frac{1}{(\sqrt{2-1}+1)} = \frac{1}{2}$$

## ✚ Limits of trigonometric functions.

$\lim_{x \rightarrow 0} \sin x = 0$	$\lim_{x \rightarrow 0} \cos x = 1$	$\lim_{x \rightarrow 0} \tan x = 0$
$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$	$\lim_{x \rightarrow 0} \frac{\sin ax}{ax} = 1$	$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$
$\lim_{x \rightarrow 0} \frac{\tan ax}{ax} = 1$	$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$	

### Examples 6

$$1. \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{5x} = \frac{1}{5} \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{x} = \frac{1}{5} \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{x} * \frac{2}{2}$$

$$= \frac{2}{5} \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{2x} = \frac{2}{5} * 1 = \frac{2}{5}$$

$$2. \quad \lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \rightarrow 0} \frac{\sin 2x \frac{2x}{2x}}{\sin 3x \frac{3x}{3x}} = \lim_{x \rightarrow 0} \frac{2x \frac{\sin 2x}{2x}}{3x \frac{\sin 3x}{3x}}$$

$$= \frac{2}{3} \quad \lim_{x \rightarrow 0} \frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}} = \frac{2}{3} \quad \frac{\lim_{x \rightarrow 0} \frac{\sin 2x}{2x}}{\lim_{x \rightarrow 0} \frac{\sin 3x}{3x}} = \frac{2}{3} * \frac{1}{1} = \frac{2}{3}$$

$$3. \quad \lim_{\theta \rightarrow 0} \frac{\theta}{\sin \theta} = \lim_{\theta \rightarrow 0} \frac{1}{\frac{\sin \theta}{\theta}} = \frac{1}{\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}} = \frac{1}{1} = 1$$

$$4. \quad \lim_{\theta \rightarrow 0} \frac{\sin^2 \theta}{\theta^2} = \left( \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \right)^2 = (1)^2 = 1$$

$$\text{or} \quad \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} * \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1 * 1 = 1$$

### Limits of infinity:

1- If k is constant, then  $\lim_{x \rightarrow \infty} k = k$

2-  $\lim_{x \rightarrow \infty} \frac{1}{x} = 0$ ,  $\lim_{x \rightarrow -\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

3-  $\lim_{x \rightarrow 0} \frac{1}{x} = \infty$

### Examples 7:

1.  $\lim_{x \rightarrow \infty} \frac{x}{2x+3} = \lim_{x \rightarrow \infty} \frac{1}{2+\frac{3}{x}} = \frac{1}{2+0} = \frac{1}{2}$

2.  $\lim_{x \rightarrow \infty} \frac{2x^2+3x+5}{5x^2-4x+1} = \lim_{x \rightarrow \infty} \frac{2+\frac{3}{x}+\frac{5}{x^2}}{5-\frac{4}{x}+\frac{1}{x^2}} = \frac{2+0+0}{5-0+0} = \frac{2}{5}$

3.  $\lim_{x \rightarrow \infty} \frac{2x^2+1}{3x^2-2x^2+5x-2} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x}+\frac{1}{x^3}}{3-\frac{2}{x}+\frac{5}{x^2}-\frac{2}{x^3}} = 0$

4.  $\lim_{x \rightarrow \infty} \frac{2x^2+1}{3x^2-2x^2+5x-2} = \lim_{x \rightarrow \infty} \frac{\frac{2}{x}+\frac{1}{x^3}}{3-\frac{2}{x}+\frac{5}{x^2}-\frac{2}{x^3}} = 0$

## H. w:

1. If  $f(x) = \frac{x^2-3x+2}{x-2}$ ,  $x \neq 2$ ; find  $\lim_{x \rightarrow 2} f(x)$ .
2.  $\lim_{x \rightarrow -2} \sqrt{4x^2 - 3} = \sqrt{\lim_{x \rightarrow -2} (4x^2 - 3)}$
3.  $\lim_{t \rightarrow 0} \frac{\tan t \sec 2t}{3t}$
4.  $\lim_{x \rightarrow \infty} \frac{11x+2}{2x^3-1}$
5.  $\lim_{x \rightarrow \infty} \frac{2x^2+1}{3x^2-2x^2+5x-2}$