

## PROBLEMS & EXAMPLE CH.1

### EXAMPLE 1.1 THE ROCKET ENGINE

A rocket engine, Fig. E1.1, burns a stoichiometric mixture of fuel (liquid hydrogen) in oxidant (liquid oxygen). The combustion chamber is cylindrical, 75 cm long and 60 cm in diameter, and the combustion process produces 108 kg/s of exhaust gases. If combustion is complete, find the rate of reaction of hydrogen and of oxygen.

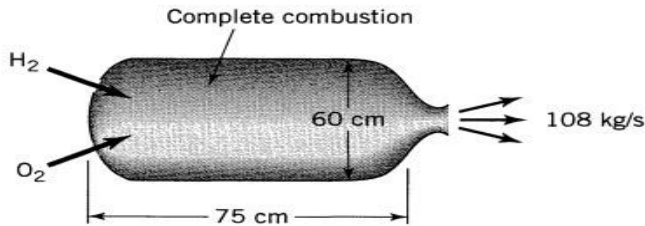


Figure E1.1

### SOLUTION

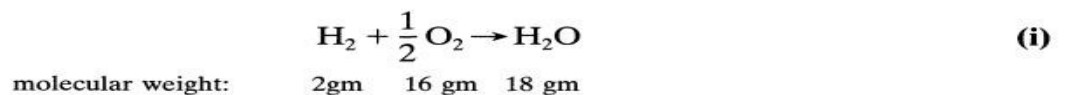
We want to evaluate

$$-r_{\text{H}_2} = \frac{1}{V} \frac{dN_{\text{H}_2}}{dt} \quad \text{and} \quad -r_{\text{O}_2} = \frac{1}{V} \frac{dN_{\text{O}_2}}{dt}$$

Let us evaluate terms. The reactor volume and the volume in which reaction takes place are identical. Thus,

$$V = \frac{\pi}{4} (0.6)^2 (0.75) = 0.2121 \text{ m}^3$$

Next, let us look at the reaction occurring.



Therefore,

$$\text{H}_2\text{O produced/s} = 108 \text{ kg/s} \left( \frac{1 \text{ kmol}}{18 \text{ kg}} \right) = 6 \text{ kmol/s}$$

So from Eq. (i)

$$\text{H}_2 \text{ used} = 6 \text{ kmol/s}$$

$$\text{O}_2 \text{ used} = 3 \text{ kmol/s}$$

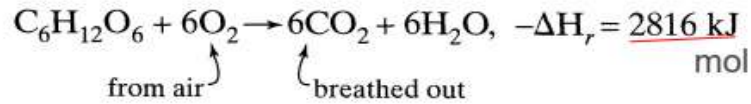
and the rate of reaction is

$$\underline{\underline{-r_{\text{H}_2}} = -\frac{1}{0.2121 \text{ m}^3} \cdot \frac{6 \text{ kmol}}{\text{s}} = 2.829 \times 10^4 \frac{\text{mol used}}{(\text{m}^3 \text{ of rocket}) \cdot \text{s}}}$$

$$\underline{\underline{-r_{\text{O}_2}} = -\frac{1}{0.2121 \text{ m}^3} \cdot 3 \frac{\text{kmol}}{\text{s}} = 1.415 \times 10^4 \frac{\text{mol}}{\text{m}^3 \cdot \text{s}}}$$

**EXAMPLE 1.2 THE LIVING PERSON**

A human being (75 kg) consumes about 6000 kJ of food per day. Assume that the food is all glucose and that the overall reaction is



Find man's metabolic rate (the rate of living, loving, and laughing) in terms of moles of oxygen used per  $\text{m}^3$  of person per second.

**SOLUTION**

We want to find

$$-r''_{\text{O}_2} = -\frac{1}{V_{\text{person}}} \frac{dN_{\text{O}_2}}{dt} = \frac{\text{mol O}_2 \text{ used}}{(\text{m}^3 \text{ of person})\text{s}} \quad \text{(i)}$$

Let us evaluate the two terms in this equation. First of all, from our life experience we estimate the density of man to be

$$\rho = 1000 \frac{\text{kg}}{\text{m}^3}$$

Therefore, for the person in question

$$V_{\text{person}} = \frac{75 \text{ kg}}{1000 \text{ kg/m}^3} = 0.075 \text{ m}^3$$

Next, noting that each mole of glucose consumed uses 6 moles of oxygen and releases 2816 kJ of energy, we see that we need

$$\frac{dN_{\text{O}_2}}{dt} = \left( \frac{6000 \text{ kJ/day}}{2816 \text{ kJ/mol glucose}} \right) \left( \frac{6 \text{ mol O}_2}{1 \text{ mol glucose}} \right) = 12.8 \frac{\text{mol O}_2}{\text{day}}$$

Inserting into Eq. (i)

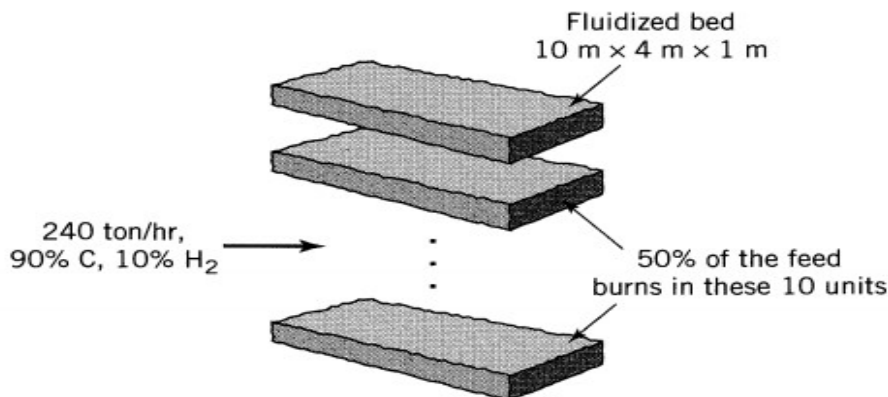
$$-r''_{\text{O}_2} = \frac{1}{0.075 \text{ m}^3} \cdot \frac{12.8 \text{ mol O}_2 \text{ used}}{\text{day}} \cdot \frac{1 \text{ day}}{24 \times 3600 \text{ s}} = \underline{\underline{0.002 \frac{\text{mol O}_2 \text{ used}}{\text{m}^3 \cdot \text{s}}}}$$

## PROBLEMS

### 1.1. Municipal waste water treatment plant. Home Work

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**1.2. Coal burning electrical power station.** Large central power stations (about 1000 MW electrical) using fluidized bed combustors may be built some day (see Fig. P1.2). These giants would be fed 240 tons of coal/hr (90% C, 10%



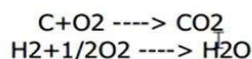
H<sub>2</sub>), 50% of which would burn within the battery of primary fluidized beds, the other 50% elsewhere in the system. One suggested design would use a battery of 10 fluidized beds, each 20 m long, 4 m wide, and containing solids to a depth of 1 m. Find the rate of reaction within the beds, based on the oxygen used.

**Sol.**

$$V_{\text{batteries}} := 10 \cdot 20 \text{ m} \cdot 4 \text{ m} \cdot 1 \text{ m} = (8 \cdot 10^5) \text{ L}$$

$$C_{\text{moles}} := 0.9 \cdot 240 \frac{\text{ton}}{\text{h}} \cdot 1000 \frac{\text{kg}}{\text{ton}} \cdot 1000 \frac{\text{gm}}{\text{kg}} \cdot \frac{1}{12 \frac{\text{gm}}{\text{mol}}} \cdot \frac{\text{h}}{3600 \text{ s}} = (5 \cdot 10^3) \frac{\text{mol}}{\text{s}}$$

$$H2_{\text{moles}} := 0.1 \cdot 240 \frac{\text{ton}}{\text{h}} \cdot 1000 \frac{\text{kg}}{\text{ton}} \cdot 1000 \frac{\text{gm}}{\text{kg}} \cdot \frac{1}{2 \frac{\text{gm}}{\text{mol}}} \cdot \frac{\text{h}}{3600 \text{ s}} = (3.333 \cdot 10^3) \frac{\text{mol}}{\text{s}}$$



So, the total moles of oxygen consumed are equal to carbon moles plus half amount of the hydrogen moles.

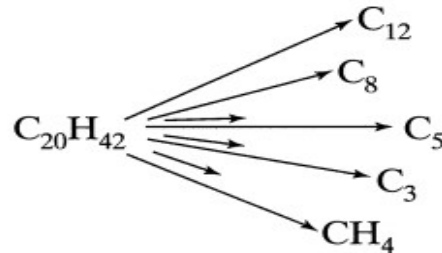
$$O2_{\text{totalmoles}} := C_{\text{moles}} + \frac{H2_{\text{moles}}}{2} = (6.667 \cdot 10^3) \frac{\text{mol}}{\text{s}}$$

Only 50% of coal feed are burned in the beds

$$\text{rate}_{\text{batteries}} := \frac{0.5 \cdot O2_{\text{totalmoles}}}{V_{\text{batteries}}} = 4.167 \frac{\text{mol}}{\text{m}^3 \cdot \text{s}}$$

**1.3. Fluid cracking crackers (FCC).** FCC reactors are among the largest processing units used in the petroleum industry. Figure P1.3 shows an example of such units. A typical unit is 4-10 m ID and 10-20 m high and contains about 50 tons of  $\rho = 800 \text{ kg/m}^3$  porous catalyst. It is fed about 38 000 barrels of crude oil per day ( $6000 \text{ m}^3/\text{day}$  at a density  $\rho \cong 900 \text{ kg/m}^3$ ), and it cracks these long chain hydrocarbons into shorter molecules.

To get an idea of the rate of reaction in these giant units, let us simplify and suppose that the feed consists of just  $\text{C}_{20}$  hydrocarbon, or



If 60% of the vaporized feed is cracked in the unit, what is the rate of reaction, expressed as  $-r'$  (mols reacted/kg cat · s) and as  $r'''$  (mols reacted/ $\text{m}^3$  cat · s)?

**Sol.**

$$W_{cat} := 50000 \text{ kg} \quad \rho_{cat} := 800 \frac{\text{kg}}{\text{m}^3} \quad V_{cat} := \frac{W_{cat}}{\rho_{cat}} = (6.25 \cdot 10^4) \text{ L}$$

$$Feedrate := 6000 \frac{\text{m}^3}{\text{day}} \quad \rho_{feed} := 900 \frac{\text{kg}}{\text{m}^3} \quad Feed := Feedrate \cdot \rho_{feed} = 62.5 \frac{\text{kg}}{\text{s}}$$

$$M.Wt.C_{20}H_{42} := (20) \cdot 12 \frac{\text{gm}}{\text{mol}} \frac{\text{kg}}{1000 \text{ gm}} + (42) \cdot 1 \frac{\text{gm}}{\text{mol}} \frac{\text{kg}}{1000 \text{ gm}} = 0.282 \frac{\text{kg}}{\text{mol}}$$

$$dN_{perdt} := \frac{Feed}{M.Wt.C_{20}H_{42}} \cdot 0.6 = 132.979 \frac{\text{mol}}{\text{s}} \quad \text{The 0.6 is the 60\% of the vaporized feed cracked (given)}$$

$$r' := \frac{1}{W_{cat}} dN_{perdt} = 0.003 \frac{\text{mol}}{\text{kg} \cdot \text{s}}$$

$$r''' := \frac{1}{V_{cat}} dN_{perdt} = 2.128 \frac{\text{mol}}{\text{m}^3 \cdot \text{s}}$$

Q/ The reactor chamber burns  $1000 \text{ kg/hr}$  of coal (consisting of 80% carbon and remain is hydrogen), The chamber is cylindrical, 4m long, and 40 cm in diameter. find the rate of reaction if used 60% of feed?