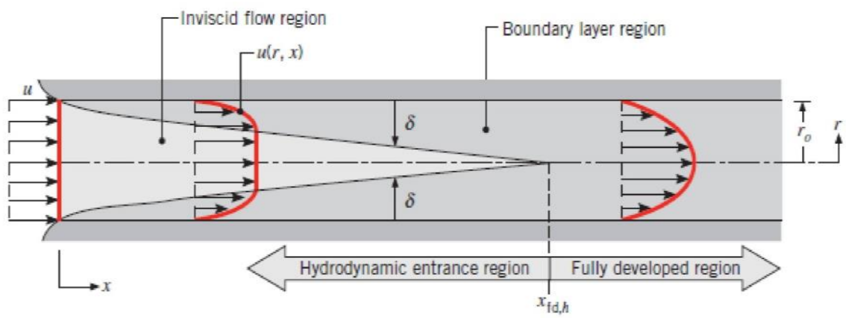
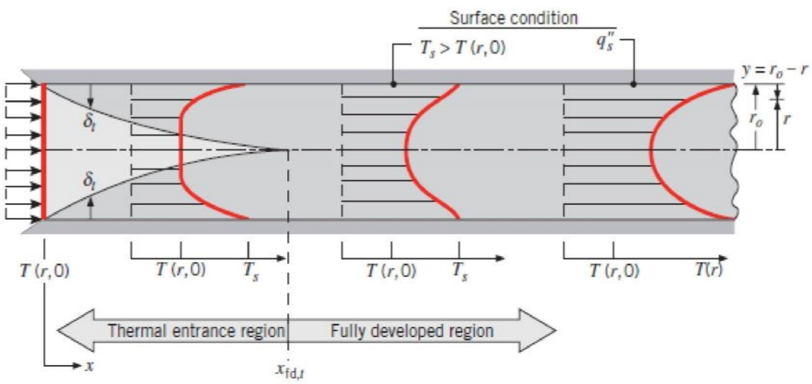
Chapter six

Empirical and practical Relation for Forced-Convection heat transfer

Convection heat transfer for internal flow [flow in conduits]

Conduits “pipe-tube-duct”





For pipes: [circular cross section area]

$$R_e = \frac{\rho v D}{\mu} = \frac{vD}{\nu} = \frac{4\dot{m}}{\pi D\mu} = \frac{\dot{m}}{A\mu} = \frac{4\rho\dot{V}}{\pi D\mu} = \frac{4\dot{V}}{\pi D\nu}$$

D: internal diameter for pipe , m

v: Fluid velocity m/s

$\dot{V}$: Volume flow rate m^3/s

$\dot{m}$: mass flow rate kg/s

ν: kinematic viscosity (m^2/s)

μ: dynamic viscosity of fluid (N s/m^2)

ρ:fluid density kg/ m^3

For pipes :

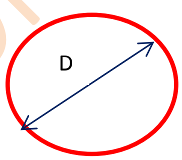
Re < 2300 , **Laminar Flow**

Re > 2300 , **Turbulent Flow**

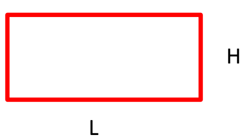
For non-circular cross sectional area,

We use what is called hydraulic diameter (D_H) in calculation, (Re and Nu … etc.).

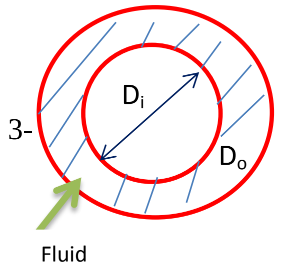
$$D_H = \frac{4A}{P} = \frac{4*\text{cross sectional area}}{\text{wet perameter}}$$

1- 

$$D_H = \frac{4*\frac{\pi}{4}D^2}{\pi D} = D$$

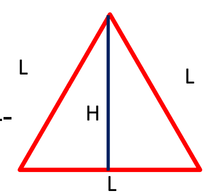
2- 

$$D_H = \frac{4*LH}{2L*2H} = = \frac{2*LH}{L*H}$$

3- 

$$D_H = \frac{4*\frac{\pi}{4}(D_o{}^2 - Di^2)}{\pi Di * \pi Do} = \frac{D_o{}^2 - Di^2}{Di*Do} = \frac{(Do-Di)(Do+Di)}{Do+Di}$$

$$D_H = Do - Di$$

4- 

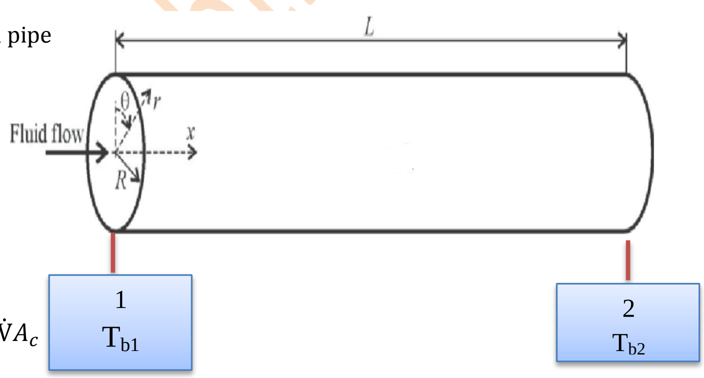
$$H = \sqrt{L^2 - (\frac{1}{2}L)^2} = \frac{\sqrt{3}}{2}L$$

$$D_H = \frac{4*\frac{1}{2}*L*H}{3*L} = \frac{4*\frac{1}{2}*L*\frac{\sqrt{3}}{2}L}{3*L} = \frac{L}{\sqrt{3}}$$

Sec 6-2 : Empirical Relations for pipe and tube flow

flow in pipe

$\dot{m}$, Cp,



$\dot{m} = \rho \dot{V} A_c$

$A_c = \frac{\pi}{4} D^2$

- 1 درجة حرارة عند النقطه= Tb1
- 2 درجة حرارة عند النقطه = Tb2
- Bulk temperature represents energy average or " mixing cup" condition
- (The bulk temperature is defined as the energy-average fluid temperature across the tube)

$q = \dot{m}cp(T_{b_2} - T_{b_1})$

also

$q = hA(T_w - \overline{T}_b)$ $\overline{T}_b = \frac{T_{b_1} + T_{b_2}}{2}$

$A = \pi dL$

$\overline{T}_b$: ونقصد بها درجة حرارة المائع ولكن هنا توجد درجة حرارة عند المدخل والمخرج لذلك استخدمنا average لهما

$Nu_d = \frac{h * D_H}{k}$

D = hydraulic dimeter of pipe

1- Fully developed turbulent flow in smooth tube

$Nu_d = 0.023 Re_d^{0.8} Pr^n$

$0.6 < p_r < 100$

$2500 < Re_d < 1.25 * 10^5$

All properties @ $\overline{T}_b$ $n = \begin{cases} 0.4 & \text{for heating of the fluid.} \\ 0.3 & \text{for cooling of the fluid.} \end{cases}$

2- Turbulent flow in smooth tube suggest better results

$Nu_d = 0.0214(Re_d^{0.8} - 100)Pr^{0.4}$ $0.5 < Pr < 1.5$, $10^4 < Re_d < 5 \times 10^6$

$Nu_d = 0.012(Re_d^{0.87} - 280)Pr^{0.4}$ $1.5 < Pr < 500$, $3000 < Re_d < 10^6$

3- Fully developed Turbulent flow but there is large temperature difference (the Temp. effect is taken into consideration)

$Nu_d = 0.027 Re_d^{0.8} Pr^{1/3} (\frac{\mu}{\mu_w})^{0.14}$

All properties @ $\overline{T}_b$ except μ_w @ T_w

4 − *more accurate* Relation for fully developed flow in smooth or Rough tube (there is friction involves)

$$Nu_d = \frac{(f/8)Re_d P_r}{1.07+12.7(f/8)^{\frac{1}{2}}(P_r^{\frac{2}{3}}-1)}\left(\frac{\mu_b}{\mu_w}\right)^n$$

$$0.5 < P_r < 2000$$
$$10^4 < Re_d < 5 \times 10^6$$
$$0 < \frac{\mu_b}{\mu_w} < 40$$

All properties @ $T_f = \frac{T_w - \overline{T}_b}{2}$ except

μ_b @ $\overline{T}_b$ and μ_w @ T_w

$n = 0.11$ If $T_w > \overline{T}_b$

$n = 0.25$ If $T_w < \overline{T}_b$

$n = 0$ For constant heat flux

5- Turbulent in entrance region (the flow is not developed)

$$Nu_d = 0.036\, Re_d^{0.8} Pr_r^{1/3} \left(\frac{d}{L}\right)^{0.55} \qquad 10 < \frac{L}{d} < 400$$

L = length of tube

d= diameter of tube

All properties are evaluated @ $\overline{T}_b$

6- Fully developed **Laminer** flow in tube @ constant wall temperature

$$Nu_d = 366 + \frac{0.668\left(\frac{d}{l}\right)Re_d P_r}{1+0.04\left[\left(\frac{d}{l}\right)Re_d P_r\right]^{2/3}} \qquad Tw = \text{constant}$$

Nu_d = 3.66 If tube is sufficiently long ,

7 − for **Laminer** flow in tube

$Nu_d = 1.86\,(Re_d P_r)^{1/3}(\frac{d}{L})^{1/3}(\frac{\mu}{\mu_w})^{9.4}$ Tw = constan , $Re_d P_r \frac{d}{l} > 10$

All properties @ $\overline{T}_b$ except μ_w @ T_w

This equation can't be used for L >>> d (entirely long tube)

Problem 6-6: water at the rate of 3kg/s is heated from 5 to 15 °C by passing it through a 5 cm ID copper tube. The tube wall temperature is maintained at 90°C. What is the length of the tube?

Solution

$\dot{m} = \frac{3\text{kg}}{\text{s}}$, $T_{b1} = 5°C, T_{b2} = 15°C, D = 0.05\text{m}$

$T_k = 90oc$, find tube length L?

$q = hA(T_v - \overline{T}_b) = h(\pi dL)(T_v - \overline{T}_b)$

$l = \frac{q}{h\pi d h(T_w - \overline{T}_b)}$

1- $\overline{T}_b = \frac{T_{b1}+T_{b2}}{2} = \frac{5+15}{2} = 10°C$

$T_b = 10^o$

From table A – 9

$C_p = 4.195\ \frac{Kj}{Kg.C}$

2- $\rho = 999.2\ \text{Kg/m}^3$

$\mu = 1.31 * 10^{-3}\text{Kg/m.S}$

$K = 0.585\ \text{w/m} \cdot \text{C}$

$\text{Pr} = 9.4$

3- $q = \dot{m}\,c\,p(T_{b2} - T_{b1}) = 3 \times 4195 \times (15 - 5) = 125850\ \text{w}$

4- $Re_d = \frac{\rho VD}{\mu} = \frac{4\dot{m}}{\pi D\mu} = \frac{4*3}{\pi*0.05\times1.31\times10^{-3}} \Rightarrow Re_d = 58316\ Turbulent$

5- Fully developed turbulent flow in smooth tube , table 6.8

6 – $Nu_d = 0.023\ Re_d^{0.8}\ Pr^n$ $n = 0.4 \Rightarrow$ heating

$Nu_d = 0.023\ (58316)^{0.8}(9.4)^{0.4} = 366.1$

$$Nu_d = \frac{hD}{k} \Rightarrow h = \frac{Nu \cdot k}{D} = \frac{366 \cdot 1 \times 0.535}{0.05} = 4283.4$$

$$7 - \qquad L = \frac{q}{h\pi d(T_w - \overline{T}_b)} = \frac{125850}{4283.4 \times \pi \times 0.05(90-10)}$$

L =2.338 m OR

∗ using another relation هذه المعادلة تعطي دقة اكثر بالحل

$$Nu_d = 0.012\left({Re_d}^{0.87} - 280\right)\mathrm{Pr}^{0.4} \qquad 1.5 < P_r < 500, \quad 3000 < \mathrm{Re} < 10^6$$

$$Nu_d = 0.012((58316)^{0.87} - 280)(9.4)^{0.4}$$

$$Nu_d = 403.5$$

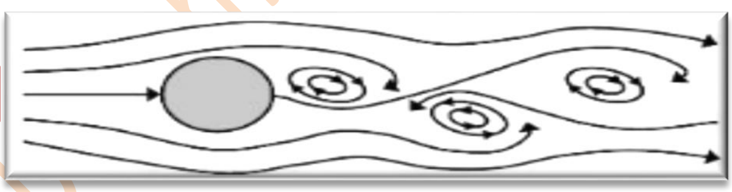
$$Nu_d = \frac{\mathrm{hD}}{k} \Rightarrow h = \frac{Nu_d \cdot k}{D} = \frac{403.5 \times 0.585}{0.05} = 4721.5$$

$$l = \frac{q}{h \cdot \pi\mathrm{d}(T_w - \overline{T}_b)} = \frac{125850}{4721.5 \times \pi \times 0.05(90-10)}$$

$$l = 2.121\ m$$

Sec 5-3 Flow across cylinders and spheres

Flow



For cross flow over circular cylinders :

$$Nu_{df} = c\ Re_d^n\ Pr_f^{1/3} = \frac{\mathrm{hd}}{k_f} \qquad P_r \geq 0.7\ ,\ 0.4 < Re_d < 400000,$$

[M1]Comment: الخواص تحسب بدلاله T_f

f : (film)هذه ترمز الى استخدام الخواص بدلاله درجه حراره

$$T_f = \frac{T_w + T_\infty}{2}$$

$$\mathrm{Nu}_{df} = \frac{hd}{k_f} = C\left(\frac{u_\infty d}{\nu_f}\right)^n \mathrm{Pr}_f^{1/3} \quad \textbf{[6-17]}$$

C, n from table 6-2 $Re_d = \frac{u_f\, d}{v_f}$

Table 6-2 | Constants for use with Equation (6-17), based on References 8 and 9.

Re_{df}	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

Other formula

$$Nu_d = 0.3 + \frac{0.62\, Re_d^{1/2} p_r^{1/3}}{[1+(0.4/p_r)^{2/3}]^{1/4}}[1 + (\frac{Re_d}{282000})^{5/8}]^{4/5} \qquad P_r > 0.2$$

$10^2 < Re_d < 10^7$

properites @ T_f

Example 6-7 : Air at 1 atm and 35oC flows across 0.5 cm diameter cylinder @ a velocity of 50 m/s . The cylinder surface is maintained at a Temperature of 50 °C. Calculate the heat loss per unit length of the cylinder .

Sol:

$q = \mathrm{hA}(T_w - T_\infty) = h(\pi d L)(T_w - T_\infty)$

P = 1 atm , air , T_∞ = 35°C , d= 5 cm , u = 50 m/s , T_w = 150°C find $\frac{\mathrm{q}}{\mathrm{L}}$?

$$1 - T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 35}{2} = 92.5°\mathrm{C} = 365.5\mathrm{k}$$

$$(2)\rho = \frac{P}{RT} = \frac{1.0132 \times 0^5}{287(365.5)} = 0.996\frac{\mathrm{kg}}{\mathrm{m}^3}$$

(3) from table $A - 5 \Rightarrow @T_f = 365 \cdot 5\ \mathrm{K}$

$$\mu_f = 2 \cdot 14 \times 10^{-3}\frac{\mathrm{Kg}}{\mathrm{m \cdot c}}\ , k_f = 0.0312\frac{\mathrm{W}}{\mathrm{m \cdot C}}, \mathrm{Prf} = 0.695$$

$$(5)Re_d = \frac{\rho_f u_\infty \cdot d}{\mu_f} = \frac{0.446 \times 50 * 0.05}{2.14 * 10^{-5}} = 1.129 \times 10^5$$

By using the following relation, (eq 6.17)

$Nu_{d_f} = c\ Re_d^n\ Pr_f^{1/3}$ From Table (6-2) $C = 0.0266, n = 0.805$

$$Nu_{d_f} = 0.0266 \times (1.129 \times 10^5)^{0.805}(0.695)^{\frac{1}{3}} = 275.1$$

$$Nu_{d_f} = \frac{\mathrm{hd}}{k_f}\ , \mathrm{h} = \frac{275.1 * 0.0312}{0.05} = 171.7\mathrm{w}/m^2.°\mathrm{C}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = 171.7 \times \pi \times 0.05(150 - 35)$$

$\frac{q}{L} = 3100\ \mathrm{W/m}$

Or

By using Relation (2)(b)

$$Nu_d = 0.3 + \frac{0.62\ Re_d^{1/2} Pr^{1/3}}{[1+(0.4/P_r)^{2/3}]^{1/4}}\left[1+\left(\frac{Re_d}{282000}\right]^{1/2}\right]$$ $2\times10^4 < Re < 4\times10^5$, $Pr > 0.2$

$$Nu_d = 0.3 + \frac{0\cdot 62(1.129+10^5)^{\frac{1}{2}}(0.695)^{\frac{1}{3}}}{[1+(0.4/0\cdot 695)^{\frac{2}{3}}]^{\frac{1}{4}}}(1+(\frac{1.129+10^5}{282000})^{1/2})$$

$$Nu_d = 264.47$$

$$Nu_{df} = \frac{hd}{k_f}\ ,\ h = \frac{264.47 * 0.0312}{0.05} = 165.03\ \mathrm{W}/m^2.\mathrm{°C}$$

$$\frac{q}{L} = h\pi d(T_w - T_\infty) = 165.03\times\pi\times0.05(150-35)$$

$$\frac{q}{L} = 2981.15\ \mathrm{W}$$