Demoiver Theorem

$\boxed{4}$

IF $Z = r[\cos\theta + i\sin\theta]$ and (n) Positive integer number then

① $Z^n = r^n(\cos\theta + i\sin\theta)$ n = Real number ($\mp$)

$Z^n = r^n(\cos(\theta \cdot n) + i\sin(\theta \cdot n))$

EX: Find $\left[\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right]^4$

$= \cos\frac{3\pi}{\cancel{8}_2} * \cancel{4}^1 + i\sin\frac{3\pi}{\cancel{8}_2} * \cancel{4}^1$

$= 0 + i*(-1) = 0 - i = i$

EX: Find $2\left[\cos\frac{7\pi}{12} + i\sin\frac{7\pi}{12}\right]^{-3}$

$= 2\left[\cos\frac{7\pi}{12}*(-3) + i\sin\frac{7\pi}{12}*(-3)\right]$

$= 2[0.707 + i\,0.707]$

$= 1.414 + i\,1.414$

EX: $\sqrt{2}\left[\cos\frac{5\pi}{24} + i\sin\frac{5\pi}{24}\right]^{-6}$

$= \sqrt{2}\left[\cos\frac{5\pi}{24}*(-6) + i\sin\frac{5\pi}{24}*(-6)\right]$

$= \sqrt{2}[-0.707 + i\,0.707]$

$= -1 + i$

Ex: Find $(1+i)^{11}$

$$Z^n = r^n[\cos\theta + i\sin\theta]$$

$$r = \sqrt{x^2+y^2} = \sqrt{1+1} = \sqrt{2}$$

$$\theta = \tan^{-1}\frac{1}{1} = \frac{\pi}{4}$$

$$Z^n = \sqrt{2}^n\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]^n$$

$$= \sqrt{2}^{11}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]^{11}$$

$$= 32\sqrt{2}\left[\cos\frac{\pi}{4}*11 + i\sin\frac{\pi}{4}*11\right]$$

$$= -32 + i32$$

$$\therefore (1+i)^{11} = -32 + 32i$$

Ex: Find $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$

$$Z^n = r^n[\cos\theta + i\sin\theta]^n$$

$$= r^{10}[\cos\theta + i\sin\theta]^{10}$$

$$= r^{10}[\cos\theta*10 + i\sin\theta*10]$$

$$r = \sqrt{x^2+y^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

$$\theta = \tan^{-1}\frac{1/2}{1/2} \Rightarrow \theta = 45^\circ = \frac{\pi}{4}$$

$$= \left(\frac{1}{\sqrt{2}}\right)^{10}\left[\cos\frac{\pi}{4}*10 + i\sin\frac{\pi}{4}*10\right]$$

$$= \frac{1}{\sqrt{2}^{10}}\left[\cos\frac{5\pi}{2} + i\sin\frac{5\pi}{2}\right]$$

$$= \frac{1}{32}[0 + i] = \frac{1}{32}i$$