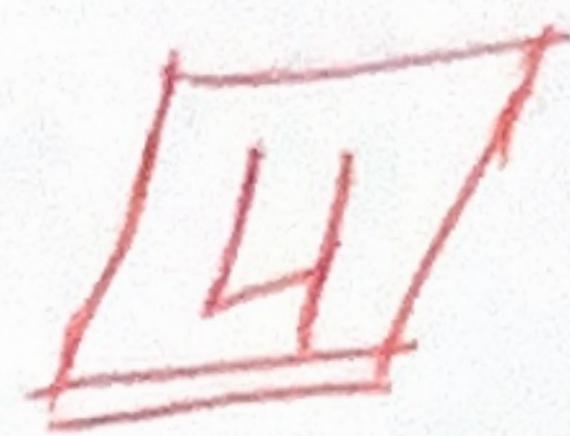


## De Moivre's Theorem



If  $Z = r [\cos \theta + i \sin \theta]$  and  $n$  Positive integer number then

$$\textcircled{1} \quad Z^n = r^n (\cos \theta + i \sin \theta) \quad n = \text{Real number } (\neq)$$

$$Z^n = r^n (\cos (\theta \cdot n) + i \sin (\theta \cdot n))$$

EX: Find  $[\cos \frac{3\pi}{8} + i \sin \frac{3\pi}{8}]^4$

$$= \cos \frac{3\pi}{8} * 4 + i \sin \frac{3\pi}{8} * 4$$

$$= 0 + i * (-1) = 0 - i = -i$$

EX: Find  $2 [\cos \frac{7\pi}{12} + i \sin \frac{7\pi}{12}]^{-3}$

$$= 2 [\cos \frac{7\pi}{12} * (-3) + i \sin \frac{7\pi}{12} * (-3)]$$

$$= 2 [0.707 + i 0.707]$$

$$= 1.414 + i 1.414$$

EX:  $\sqrt{2} [\cos \frac{5\pi}{24} + i \sin \frac{5\pi}{24}]^{-6}$

$$= \sqrt{2} [\cos \frac{5\pi}{24} * (-6) + i \sin \frac{5\pi}{24} * (-6)]$$

$$= \sqrt{2} [-0.707 + i 0.707]$$

$$= -1 + i$$

Ex: Find  $(1+i)^{11}$

$$z^n = r^n [\cos \theta + i \sin \theta]$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$$

$$\theta = \tan^{-1} \frac{1}{1} = \frac{\pi}{4}$$

$$z^n = \sqrt{2}^n \left[ \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]^n$$

$$= \sqrt{2}^{11} \left[ \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right]^{11}$$

$$= 32\sqrt{2} \left[ \cos \frac{\pi}{4} \times 11 + i \sin \frac{\pi}{4} \times 11 \right]$$

$$= -32 + i 32$$

$$\therefore (1+i)^{11} = -32 + 32i$$

Ex: Find  $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$

$$z^n = r^n [\cos \theta + i \sin \theta]^n$$

$$= r^{10} [\cos \theta + i \sin \theta]^{10}$$

$$= r^{10} [\cos \theta \times 10 + i \sin \theta \times 10]$$

$$r = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

$$\theta = \tan^{-1} \frac{1/2}{1/2} \Rightarrow \theta = 45^\circ = \frac{\pi}{4}$$

$$= \left(\frac{1}{\sqrt{2}}\right)^{10} \left[ \cos \frac{\pi}{4} \times 10 + i \sin \frac{\pi}{4} \times 10 \right]$$

$$= \frac{1}{\sqrt{2}^{10}} \left[ \cos \frac{5\pi}{2} + i \sin \frac{5\pi}{2} \right]$$

$$= \frac{1}{32} [0 + i] = \frac{1}{32} i$$