المادة: الرياضيات

المرحلة: الاولى

2023-2022

Lecture (5)

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Laws of Derivatives:

1- $\frac{d}{dx}c = 0$ where c is constant.

- $\frac{d}{dx}4 = 0$
- $\frac{d}{dx}\sqrt{2} = 0$

2- $\frac{d}{dx}x^n = nx^{n-1}$

- $\frac{d}{dx}x^4 = 4x^3$
- $\frac{d}{dx}x^{-3} = -3x^{-4}$
- $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$

3- If U and V are two functions of x then:

a) $\frac{d}{dx}(c * U) = c\frac{d}{dx}(U)$

- $\frac{d}{dx}3x^3 = 3\frac{d}{dx}x^3 = 3(3x^2) = 9x^2$

b) $\frac{d}{dx}(U \pm V) = \frac{d}{dx}(U) \pm \frac{d}{dx}(V)$

- $\frac{d}{dx}(x^2 + x^3) = \frac{d}{dx}(x^2) + \frac{d}{dx}(x^3) = 2x + 3x^2$

c) $\frac{d}{dx}(U * V) = U\frac{d}{dx}(V) + V\frac{d}{dx}(U)$

- $\frac{d}{dx}(x^2 * x^3) = x^2\frac{d}{dx}(x^3) + x^3\frac{d}{dx}(x^2) = x^2 * (3x^2) + x^3 * (2x)$

d) $\frac{d}{dx}U^n = nU^{n-1} * \frac{d}{dx}U$

- $\frac{d}{dx}(x^5)^3 = 3(x^5)^2 * \frac{d}{dx}x^5 = 3(x^5)^2 * 5x^4$

e) $\frac{d}{dx}\left(\frac{U}{V}\right) = \frac{V\frac{dU}{dx} - U\frac{dV}{dx}}{V^2}$

- $\frac{d}{dx}\left(\frac{x^2}{x^3}\right) = \frac{x^3 * (2x) - x^2 * 3x^2}{(x^3)^2}$

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Example: If $y = x^3 + 4x^2 - 5x + 4$ find $f'(2)$.

Sol: $f'(x) = 3x^2 + 8x - 5$

$$f'(2) = 3 * (2)^2 + 8 * (2) - 5$$

$$f'(2) = 23$$

Example: If $y_1 = 3x^2 + 2x + 1$, $y_2 = 5x + 8$ find $f'(y_1 + y_2)_{(2)}$

Sol: $f'(y_1 + y_2) = f'(y_1) + f'(y_2)$

$$= f'(3x^2 + 2x + 1\ ) + f'(5x + 8\ \ )$$

$$= (6x + 2\ ) + (5\ \ )$$

$$= 6x + 7$$

$$f'(y_1 + y_2)_{(2)} = 6 * (2) + 7$$

$f'(y_1 + y_2)_{(2)} = 19.$

H.W:

1- If $y = 3x^4 + 4x^3 - 2x + 6$ find $f'(3)$.

2- If $y_1 = 5x^4 + 2x^3 - 1$, $y_2 = 5x^3 - x + 2$ find $f'(y_1 + y_2)_{(3)}$

3- If $y = \frac{2x^2 - 6x}{4x}$ find $f'(1)$.

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(Second and Higher Order Derivative) المشتقة الثانية

$$f''(x) = \frac{d}{dx}\left(\frac{d}{dx}y\right)$$

Example: If $y = x^4 + 2x^3 - 3x + 1$ find $f''(1)$.

Sol: $f'(x) = 4x^3 + 2 * 3x^2 - 3 = 4x^3 + 6x^2 - 3$

$$f''(x) = 4 * 3x^2 + 6 * 2x = 12x^2 + 12x$$

$$f''(x) = 12(1)^2 + 12 = 24.$$

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Example: If $y = 2x^4 + 5x^3 - 3x^2 + 6$ find $f''(-1)$.

Sol: $f'(x) = 8x^3 + 15x^2 - 6x$

$$f''(x) = 24x^2 + 30x - 6$$

$$f''(x) = 24(-1)^2 + 30(-1) - 6$$

$$= 24 - 30 - 6 = -12$$

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H.W:

1- If $y = 3x^4 + 4x^3 - 2x + 6$ find $f''(4)$.

2- If $y_1 = 5x^4 + 2x^3 - 1$, $y_2 = 5x^3 - x + 2$ find $f''(y_1 - y_2)_{(1)}$

3- If $y = 5x^3 + 4x - 7$ find $f''(2)$.

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(Implicit Differentiation) المشتقة الضمنية

In some cases, it is difficult to solve **y=f(x),** so to find $\frac{d}{dx}$ for such cases, implicit differentiation will be use.

Example: Find $\frac{d}{dx}$ of the following

1- $y^2 + x^2 = 1$

Sol: $2y\frac{d}{dx} + 2x = 0$

$$2y\frac{d}{dx} = -2x$$

$$\frac{d}{dx} = \frac{-2x}{2y}$$

2- Find the Implicit Differentiation of the function

$2y = x^2 + 3xy^2$ **at** **x=1, y=2**

Sol: $2\frac{d}{dx} = 2x + \left(3x * 2y * \frac{d}{dx}\right) + (y^2 * 3)$

$$2\frac{d}{dx} = 2x + \left(6xy\frac{d}{dx}\right) + (3y^2)$$

$$2\frac{d}{dx} - 6xy\frac{d}{dx} = 2x + (3y^2)$$

$$(2 - 6xy)\frac{d}{dx} = 2x + (3y^2)$$

$$\frac{d}{dx} = \frac{2x + 3y^2}{2 - 6xy} = \frac{2 * (1) + 3 * (2)^2}{2 - (6 * (1) * (2))} = \frac{14}{-10}$$