ALMUSTAQBAL UNIVERSITY COLLEGE

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

EXAMPLES IN PUNCHING SHEAR

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EXAMPLE ONE: the flat slab of 200mm total thickness and 160mm effective depth is carried by 300mm square columns with a span of 4.5m from centre to centre of columns in each direction. Determine if shear reinforcement is required for the slab for the interior column. Wu = 29.94 kPa, fy=414MPa and fc'=30MPa.

SOLUTION:

$b_o = (300 + 160) \times 4 = 1840mm$

$V_u = 29.94 \times (4.5 \times 4.5 - 0.46 \times 0.46) = 580kN$

$$v_u = \frac{V_u}{b_o d} = \frac{580 \times 10^3}{1840 \times 160} = 1.97MPa$$

$$v_c = min. \begin{Bmatrix} 0.33 \times 1 \times \sqrt{f_c'} = 0.33\sqrt{30} = \mathbf{1.807MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{30} = 2.973MPa \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 160}{1840}\right) \times \sqrt{30} = 2.49MPa \end{Bmatrix}$$

$\beta = \frac{300}{300} = 1$

$\therefore \ \boldsymbol{v_c = 1.807MPa}$

$\emptyset v_c = 0.75 \times 1.807 = 1.355MPa \quad < \ v_u = 1.97\ MPa \quad not\ o.k.$

$\therefore$ shear reinforcement is required.

$v_u = 1.97 < \emptyset 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{30}$

$v_u = 1.97 < 2.054\ MPa \qquad ok$

Find $\emptyset v_s$:

$v_s = \frac{v_u}{\emptyset} - v_c, where\ v_c = 0.17\sqrt{f_c'} = 0.17\sqrt{30} = 0.931MPa.$

$v_s = \frac{v_u}{\emptyset} - v_c = \frac{1.97}{0.75} - 0.931 = 1.696$ MPa.

$v_s = \frac{A_v f_y}{b_o s} \rightarrow A_v = \frac{v_s b_o s}{f_y} = \frac{1.696 \times 1840 \times 80}{414} \qquad s = \frac{d}{2} = 80mm$

$\therefore A_v = 603mm^2$

The required area of vertical shear reinforcement = 603mm^2

EXAMPLE TWO: Check the two-way shear (punching shear) only around an interior column (400x500) mm in a flat plate floor of a span (5.6x5.6) m. Find the area of vertical shear reinforcement if required. Assume d=170mm. Total Wu=18kPa (including slab weight). fc'= 30MPa, fy=420MPa.

SOLUTION:

$b_o = (500 + 170 + 400 + 170) \times 2 = 2480mm$

$V_u = 18 \times (5.6 \times 5.6 - 0.67 \times 0.57) = 557.6kN.$

$$v_u = \frac{V_u}{b_o d} = \frac{557.6 \times 10^3}{2480 \times 170} = 1.322MPa.$$

$$v_c = \begin{Bmatrix} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = \mathbf{1.807}\boldsymbol{MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1.25}\right)\sqrt{30} = 2.42MPa \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 170}{2480}\right)\sqrt{30} = 2.155MPa \end{Bmatrix}$$

$\therefore v_c = 1.807MPa., \beta = \frac{500}{400} = 1.25$

$\emptyset v_c = 0.75 \times 1.807 = 1.355MPa > v_u = 1.322MPa$ ok

$\therefore$ No shear reinforcement is required.

EXAMPLE THREE: Check the two-way shear (punching shear) only around the edge column (400x400) mm in a flat plate floor of a span (6.0x 6.0) m. Find the area of vertical shear reinforcement if required. Assume d=158mm. Total Wu=16kPa (including slab weight). fc'= 25MPa, fy=400MPa.

SOLUTION:

$b_o = (400 + 79) \times 2 + (400 + 158) = 1516mm.$

$V_u = 16 \times (6 \times 3.2 - 0.558 \times 0.479) = 302.923kN.$

$$v_u = \frac{V_u}{b_o d} = \frac{302.923 \times 10^3}{1516 \times 158} = 1.265MPa.$$

$$\beta = \frac{400}{400} = 1$$

$$v_c = \begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.65MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\sqrt{25} = 2.55MPa \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{30\times 158}{1516}\right)\sqrt{25} = 2.128MPa \end{cases}$$

$\therefore v_c = 1.65MPa$

$\emptyset v_c = 0.75 \times 1.65 = 1.238MPa < v_u = 1.265MPa$ ..... Not ok

$v_u \leq 0.5\sqrt{f_c'} \rightarrow 1.265 < 1.875$ .... Ok.

$$v_c = 0.17\sqrt{f_c'} = 0.17\sqrt{25} = 0.85MPa$$

$$v_s = \frac{v_u}{\emptyset} - v_c = \frac{1.265}{0.75} - 0.85 = 0.837\ MPa$$

$$s = \frac{d}{2} = \frac{158}{2} = 79mm.$$

$$v_s = \frac{A_v f_y}{b_o s} \rightarrow A_v = \frac{0.837 \times 1516 \times 79}{400} = 250.6mm^2$$

$\therefore$ The required area of vertical shear reinforcement is $\mathbf{250.6mm^2}$.

EXAMPLE FOUR: Check the two-way shear (punching shear) only around the corner column (400x400) mm in a flat plate floor of a span (6.0x 6.0) m. Find the area of vertical shear reinforcement if required. Assume d=158mm. Total Wu=19kPa (including slab weight). fc'= 25MPa, fy=400MPa.

SOLUTION:

$$b_o = (400 + 79) \times 2 = 958mm.$$

$$V_u = 19 \times (3.2 \times 3.2 - 0.479 \times 0.479) = 190.201\ kN.$$

$$v_u = \frac{V_u}{b_o d} = \frac{190.201 \times 10^3}{958 \times 158} = 1.257MPa.$$

$$\beta = \frac{400}{400} = 1$$

$$v_c = \left\{ \begin{matrix} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.65MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\sqrt{25} = 2.55MPa \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{20\times158}{958}\right)\sqrt{25} = 2.199MPa \end{matrix} \right\}$$

$\therefore v_c = 1.65MPa$

$\emptyset v_c = 0.75 \times 1.65 = 1.238MPa < v_u = 1.257MPa$ ..... Not ok

$\therefore$ Shear reinforcement is required.

$v_u \leq 0.5\sqrt{f_c'} \rightarrow 1.257 < 1.875$ .... Ok.

$$v_c = 0.17\sqrt{f_c'} = 0.17\sqrt{25} = 0.85MPa$$

$$v_s = \frac{v_u}{\emptyset} - v_c = \frac{1.257}{0.75} - 0.85 = 0.826\ MPa$$

$$s = \frac{d}{2} = \frac{158}{2} = 79mm.$$

$$v_s = \frac{A_v f_y}{b_o s} \rightarrow A_v = \frac{0.826 \times 957 \times 79}{400} = 156.12mm^2$$

$\therefore$ The required area of vertical shear reinforcement is $\mathbf{156.12mm^2}$.

EXAMPLE FIVE: Check the two-way shear (punching shear) only around the interior column (450x450) mm in a flat plate floor of a span (5.8x 5.8) m. Find the area of vertical shear reinforcement if required. Assume d=150mm. Total Wu=17.5kPa (including slab weight). fc'= 25MPa, fy=420MPa.

SOLUTION:

$$b_o = (450 + 150) \times 4 = 2400mm.$$

$$V_u = 17.5 \times (5.8 \times 5.8 - 0.6 \times 0.6) = 562.1\ kN.$$

$$v_u = \frac{V_u}{b_o d} = \frac{562.1 \times 10^3}{2400 \times 150} = 1.561MPa.$$

$$\beta = \frac{450}{450} = 1$$

$$v_c = \begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = \mathbf{1.867MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\sqrt{25} = 2.885MPa \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{30\times150}{2400}\right)\sqrt{25} = 2.133MPa \end{cases}$$

$\therefore v_c = 1.867MPa$

$\emptyset v_c = 0.75 \times 1.867 = 1.4MPa < v_u = 1.561MPa$ ….. Not ok

∴ Shear reinforcement is required.

$v_u \le 0.5\sqrt{f_c'} \to 1.561 < 2.12\ MPa$ ……. Ok.

$$v_c = 0.17\sqrt{f_c'} = 0.17\sqrt{32} = 0.962MPa$$

$$v_s = \frac{v_u}{\emptyset} - v_c = \frac{1.561}{0.75} - 0.962 = 1.119\ MPa$$

$$s = \frac{d}{2} = \frac{150}{2} = 75mm.$$

$$v_s = \frac{A_v f_y}{b_o s} \to A_v = \frac{1.119 \times 2400 \times 75}{420} = 479.6\ mm^2$$

∴ The required area of vertical shear reinforcement is $\mathbf{479.6\ mm^2}$.