3.3 Radial Systems

3.3.1 Cylinders

Consider a long cylinder of the inside radius (r_i), outside radius (r_0), and length (L), such as the one shown in Figure (3.12). We expose this cylinder to a temperature differential $(T_i - T_0)$. For a cylinder with length very large compared to diameter, it may be assumed that the heat flows only in a radial direction so that the only space coordinate needed to specify the system is (r_i). Again, Fourier's law is used by inserting the proper area relation. The area for heat flow in the cylindrical system is

$$A_r = 2\pi rL$$

$$q_r = -kA_r \frac{dT}{dr} = -2\pi Lk \frac{dT}{dr} \qquad (3.6)$$

The solution of the above equation is

$$q_r = 2\pi Lk \frac{(T_i - T_0)}{\ln (r_0/r_i)} = \frac{(T_i - T_0)}{R_{th}} \qquad (3.7)$$

and the thermal resistance in this case is

$$R_{th} = \frac{\ln (r_0/r_i)}{2\pi Lk} \qquad (3.8)$$

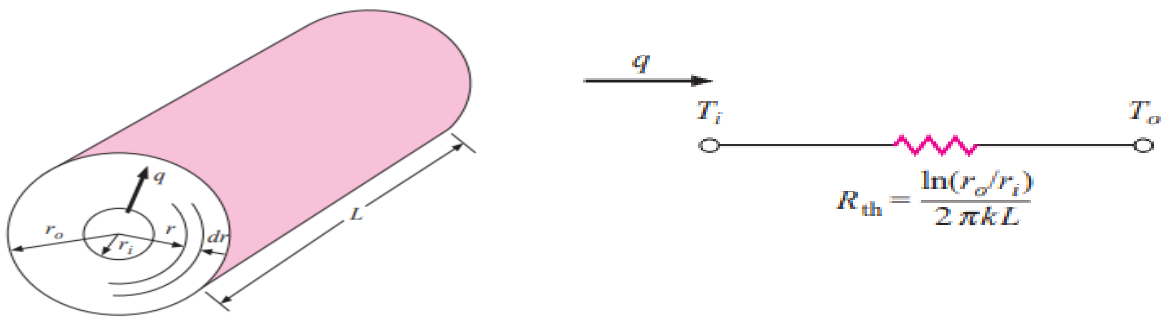


Figure (3.12) One Dimensional Heat Flow through a Hollow Cylinder and Electrical Analog.

The thermal resistance concept may be used for multiple layer cylindrical walls just as it was used for plane walls. For the three-layer system shown in Figure (3.13) the solution is

$$q_r = \frac{(T_i - T_0)}{R_{th}} = \frac{2\pi L(T_i - T_0)}{\ln(r_2/r_1)/k_A + \ln(r_3/r_2)/k_B + \ln(r_4/r_3)/k_C}$$

$$R_{th} = \frac{\ln r_2/r_1}{2\pi L k_A} + \frac{\ln r_3/r_2}{2\pi L k_B} + \frac{\ln r_4/r_3}{2\pi L k_C}$$

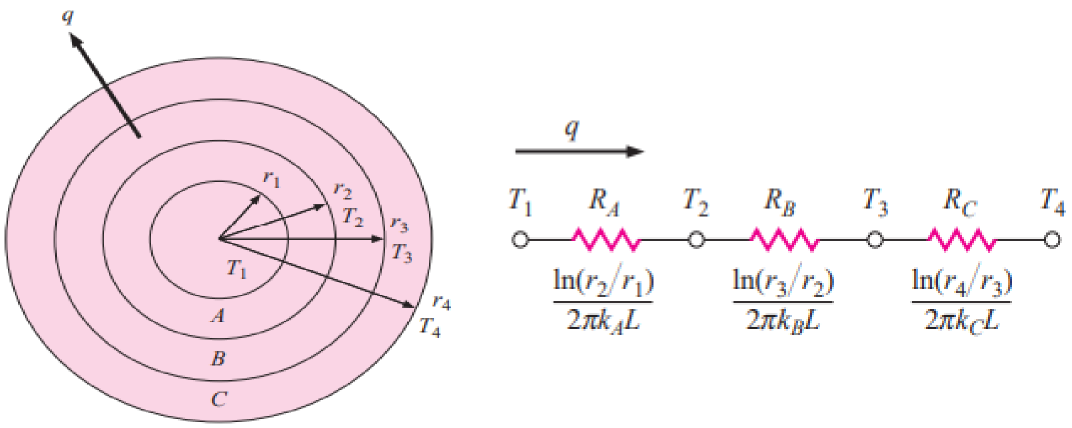


Figure (3.13) Thermal Resistance of Multiple Layer Cylindrical Walls

The hollow cylinder whose inner and outer surfaces are exposed to fluids at different temperatures Figure (3.14). The overall heat transfer would be expressed by

$$q = \frac{(T_{\infty,1} - T_{\infty,2})}{R_{tot}} = \frac{(T_{\infty,1} - T_{\infty,2})}{1/A_i h_i + \ln(r_0/r_i)/2\pi k L + 1/A_0 h_0}$$

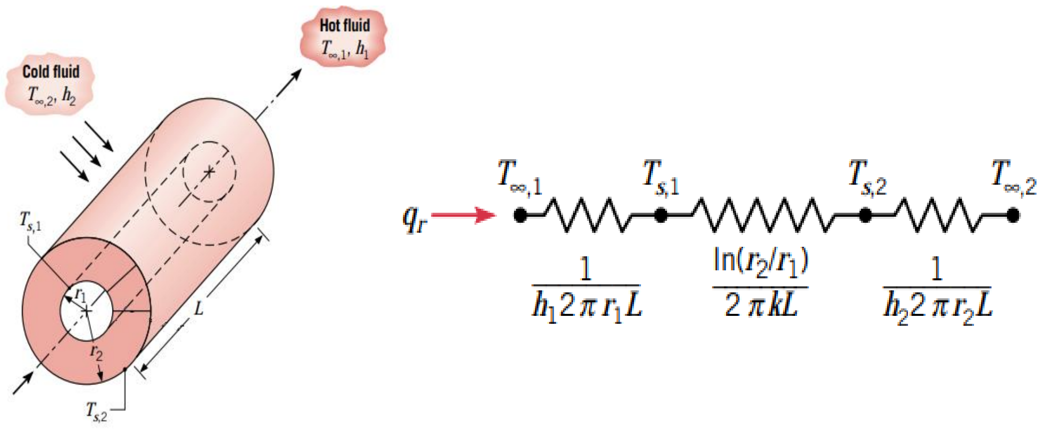
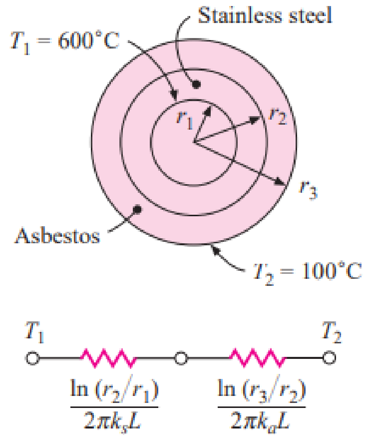


Figure (3.14) Hollow Cylinder with Convective Surface Conditions.

Example (3.4): A thick-walled tube of stainless steel (18% Cr, 8% Ni, k = 19 W/m · °C) with (2 cm) inner diameter and (4 cm) outer diameter is covered with a (3 cm) layer of asbestos insulation (k = 0.2 W/m. °C). If the inside wall temperature of the pipe is maintained at (600 °C) and the outside wall temperature is at (100 °C), calculate the heat loss per meter of length. Also calculate the tube insulation interface temperature.



Solution:

$$q/L = \frac{2\pi(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a}$$

$$q/L = \frac{2\pi(600 - 100)}{\ln(2/1)/19 + \ln(5/2)/0.2}$$

$$q/L = 680\ W/m$$

This heat flow may be used to calculate the interface temperature between the outside tube wall and the insulation. We have

$$q/L = \frac{(T_a - T_2)}{\ln(r_3/r_2)/2\pi k_a}$$

$$680 = \frac{2\pi(T_a - 100)}{\ln(5/2)/0.2}$$

$$T_a = 595.8\ °C$$

Example (3.5): Water flows at (50 °C) inside a (2.5 cm) inside diameter tube such that (h_i = 3500 W/m^2 · °C). The tube has a wall thickness of (0.8 mm) with thermal conductivity of (k=16 W/m. °C). The outside of the tube loses heat by free convection with (h_o = 7.6 W/m^2. °C). Calculate the heat loss per unit length to surrounding air at (20 °C).

Solution:

$$D_i = 0.025\ m \qquad \Rightarrow r_i = 0.0125m$$

$$D_0 = D_i + 2t = 0.025 + 2 * 0.0008 = 0.0266\ m \qquad \Rightarrow\ r_0 = 0.0133\ m$$

for unit length L = 1 m

$$A_i = \pi D_i L = \pi * 0.025 * 1 = 0.0785\ m^2$$

$A_0 = \pi D_0 L = \pi * 0.0266 * 1 = 0.0835\, m^2$

$$q = \frac{(T_i - T_0)}{R_{tot}} = \frac{(T_i - T_0)}{1/A_i h_i + \ln(r_0/r_i)/2\pi k L + 1/A_0 h_0}$$

$$q = \frac{(50 - 20)}{1/0.0785 * 3500 + \ln(0.0133/0.0125)/2\pi * 16 * 1 + 1/0.0835 * 7.6} = 19\, W$$

3.3.2 Spheres

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then

$$q_r = \frac{(T_i - T_0)}{R_{th}} = \frac{4\pi k (T_i - T_0)}{1/r_i - 1/r_0} \qquad (3.9)$$

$$R_{th} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_0}\right) \qquad (3.10)$$

The hollow sphere, whose inner and outer surfaces are exposed to fluids at different temperatures Figure (3.15). The overall heat transfer would be expressed by

$$q = \frac{(T_h - T_c)}{R_{tot}} = \frac{(T_h - T_c)}{1/A_i h_h + (\frac{1}{r_1} - \frac{1}{r_2})/4\pi k_A + (\frac{1}{r_2} - \frac{1}{r_3})/4\pi k_B + 1/A_0 h_c}$$

$where\ A_i = 4\pi r_1^2$

and $A_0 = 4\pi r_3^2$

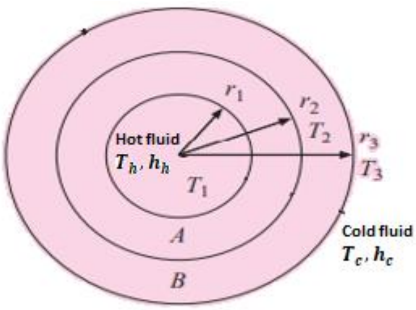


Figure (3.15) Hollow Sphere with Convective Surface Conditions.

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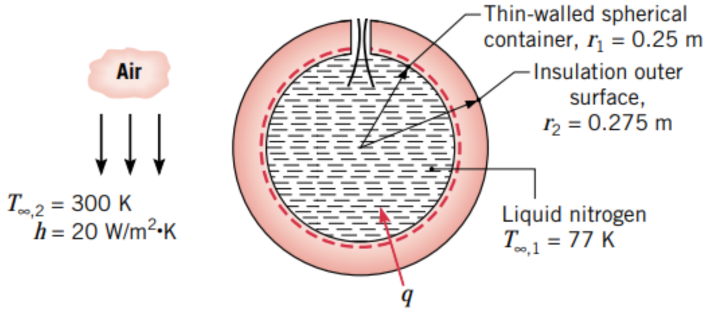
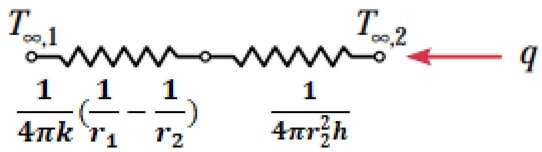
Example (3.6): A spherical, thin-walled metallic container is used to store liquid nitrogen at (77 K). The container has a diameter of (0.5 m) and is covered with an evacuated, reflective insulation composed of silica powder with (k = 0.0017 W/m. K). The insulation is (25 mm) thick, and its outer surface is exposed to ambient air at (300 K). The convection coefficient is known to be (20 W/m^2. K). What is the rate of heat transfer to the liquid nitrogen?

Solution:





$$r_1 = \frac{D_1}{2} = \frac{0.5}{2} = 0.25\ m$$

$$r_2 = r_1 + t = 0.25 + 0.025 = 0.275\ m$$

$$q = \frac{(T_{\infty,2} - T_{\infty,1})}{R_{tot}}$$

$$q = \frac{(T_{\infty,2} - T_{\infty,1})}{(\frac{1}{r_1} - \frac{1}{r_2})/4\pi k + 1/4\pi r_2^2 h}$$

$$q = \frac{(300 - 77)}{\left(\frac{1}{0.25} - \frac{1}{0.275}\right)/4\pi * 0.0017 + 1/4\pi * 0.275^2 * 20}$$

$$q = \frac{223}{17.02 + 0.05}$$

$$q = 13.06\ W$$