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Equation of line in Space :-

If L is aline in Space that Passes through apoint $P_0(x_0, y_0, z_0)$ and it Parallel to avector $v = Ai + Bj + Ck$. Then $P(x, y, z)$ is any Point lies on L only if

$$\overrightarrow{P_0P} = tv \quad \text{----} \; (1)$$

where t is some number

So eq(1) can be write:

$$\therefore (x - x_0)i + (y - y_0)j + (z - z_0)k = t(Ai + Bj + Ck)$$

$$x - x_0 = At \;, \quad y - y_0 = Bt \;, \quad z - z_0 = Ck$$

$$\Rightarrow x = At + x_0 \;, \quad y = Bt + y_0 \;, \quad z = Ck + z_0$$

EX Find Parametric equations for the line through the Point $(-2, 0, 4)$ Parallel to the vector $v = 2i + 4j - 2k$

Sol $P_0(x_0, y_0, z_0) = (-2, 0, 4)$

$Ai + Bj + Ck = 2i + 4j - 2k$

$x = 2t - 2$

$y = 4t$

$z = -2t + 4 = 4 - 2t$

EX Find Parametric equations for line through the Point $P(-3, 2, -3)$ and $Q(1, -1, 4)$?

Sol

$\overrightarrow{PQ} = (1-(-3))i + ((-1)-2)j + (4-(-3))K$

$= 4i - 3j + 7K$

With $(x_0, y_0, z_0) = (-3, 2, -3)$

$x = 4t - 3 \quad , y = -3t + 2 \quad , z = 7t - 3$

and for $(x_0, y_0, z_0) = (1, -1, 4)$

$x = 4t + 1 \quad , \quad y = -3t - 1 \quad , z = 7t + 4$

The distance from a point to a line :-

① First method

a- Find the Point $Q(x, y, z)$

b- Calculate the distance from P to Q

ex Find the distance from Point $P(1,1,5)$ to the line

$x = 1 + t \quad , \quad y = 3 - t \quad , \quad z = 2t$

Sol

$Q(x, y, z) = (1+t, 3-t, 2t)$

$\overrightarrow{PQ} = ((1+t)-1)i + ((3-t)-1)j + (2t-5)K = ti + (2-t)j + (2t-5)K$

$|\overrightarrow{PQ}| = \sqrt{t^2 + (2-t)^2 + (2t-5)^2}$

$= \sqrt{t^2 + 4 - 4t + t^2 + 4t^2 - 20t + 25}$

$= \sqrt{6t^2 - 24t + 29}$ بالتربيع

let $(|\overrightarrow{PQ}|)^2 = f(t) = d^2$

$f(t) = 6t^2 - 24t + 29$

$$\frac{df}{dt} = 0 = 12t - 24 \Rightarrow t = 2$$

نعوض قيمة (t=2) في النقطة Q

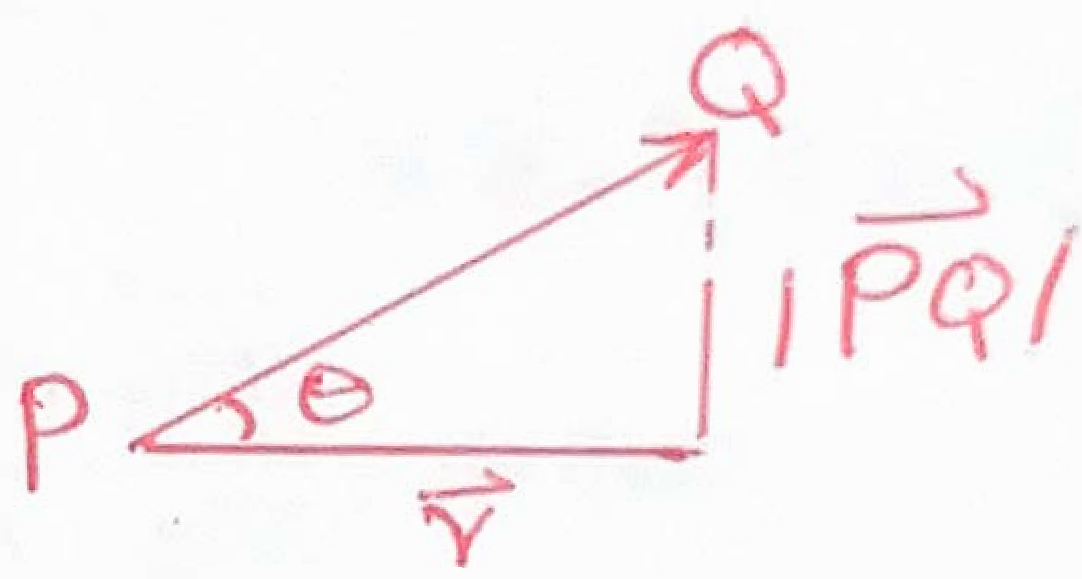
$$Q(1+2\ ,\ 3-2\ ,\ 2*2)$$

$$\therefore\ Q = (3,1,4)$$

(2) Second method

The distance from a point Q to a line that passes through a point P parallel to a vector

$$d = \frac{|\overrightarrow{PQ} \times \vec{v}|}{|\vec{v}|}$$



Ex $P(1,1;5)$ and line $x = 1+t$, $y = 3-t$, $z = 2t$

Sol $\therefore\ x = \overset{x_0}{1} + \overset{A}{t}\ ,\ y = \overset{y_0}{3} - \overset{B}{t}\ ,\ z = \overset{C}{2}t + \overset{z_0}{0}$

$$\therefore\ \vec{v} = i - j + 2k$$

$$Q = (1,3,0)$$

$$\overrightarrow{PQ} = (1-1)i + (3-1)j + (0-5)k$$

$$\therefore\ \overrightarrow{PQ} = 2j - 5k$$

$$\overrightarrow{PQ} \times \vec{v} = \begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ 0 & 2 & -5 \\ 1 & -1 & 2 \end{vmatrix}$$

$$= (2\times2 - (-1\times-5))i - (0\times2 - (1*-5))j + ((0\times-1) - (1\times2))k$$

$$= -i - 5j - 2k$$

$$d = \frac{|\overrightarrow{PQ} \times \vec{v}|}{|\vec{v}|} = \frac{\sqrt{(-1)^2 + (-5)^2 + (-2)^2}}{\sqrt{(1)^2 + (-1)^2 + (2)^2}} = \frac{\sqrt{30}}{\sqrt{6}} = \boxed{\sqrt{5}}$$

(3) Third method

$$d = |\overrightarrow{PQ}| * \sin\theta$$

Ex // $P = (1, 1, 5)$

line $x = 1 + t$, $y = 3 - t$, $z = 2t$

Sol //

$\vec{V} = i - j + 2k$, $Q(1, 3, 0)$

$\overrightarrow{PQ} = (1-1)i + (3-1)j + (0-5)K$

$\overrightarrow{PQ} = 2j - 5K$

$$\cos\theta = \frac{\overrightarrow{PQ} \cdot \vec{v}}{|PQ| \cdot |\vec{v}|}$$

$$\cos\theta = \frac{(0*1) + (2*-1) + (-5*2)}{\sqrt{(2)^2 + (-5)^2} \cdot \sqrt{1^2 + (-1)^2 + (2)^2}} = \frac{-12}{\sqrt{29} * \sqrt{6}}$$

$\cos\theta = -0.9097 \Rightarrow \theta = \cos^{-1}(-0.9097)$

$\therefore \theta = 155.5$

$d = |\overrightarrow{PQ}| * \sin\theta$

$d = \sqrt{2^2 + (-5)^2} * \sin\theta$

$= \sqrt{29} * \sin(155.5)$

$\boxed{= 2.233 = \sqrt{5}} = d$