



Ministry of Higher Education and Scientific Research
Al-Mustaqbal University College
Department of Chemical Engineering and petroleum
Industrials

Mathematics II
2nd Stage
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1. Double integral

The definite integral can be extended to functions of more than one variable. Consider, for example, a function of two variables $z = f(x, y)$. The double integral of function $f(x, y)$ is denoted by

$$\iint F(x,y)$$

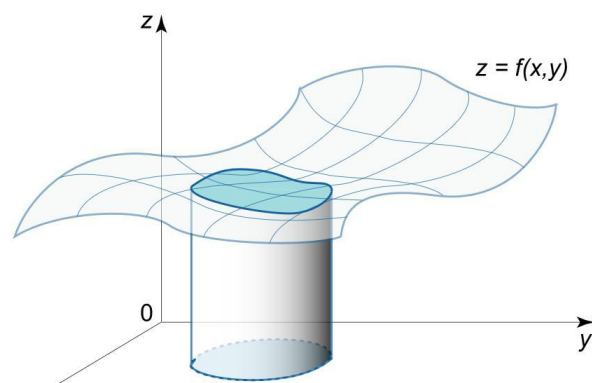


Figure 1

Where R is the region of integration in the xy -plane.

The definite integral $\int_a^b f(x)dx$ of a function of one variable $f(x) \geq 0$ is the area under the curve $f(x)$ from $x=a$ to $x=b$, then the double integral is equal to the volume under the surface $z=f(x, y)$ and above the xy -plane in the region of integration R (Figure 1).

a- Properties of double integral

If $f(x, y)$ and $g(x, y)$ are continuous on the bounded region R , then the following properties hold.

1. *Constant Multiple:*
$$\iint_R cf(x, y) dA = c \iint_R f(x, y) dA \quad (\text{any number } c)$$

2. *Sum and Difference:*

$$\iint_R (f(x, y) \pm g(x, y)) dA = \iint_R f(x, y) dA \pm \iint_R g(x, y) dA$$

3. *Domination:*

(a)
$$\iint_R f(x, y) dA \geq 0 \quad \text{if} \quad f(x, y) \geq 0 \text{ on } R$$

$$(b) \iint_R f(x, y) dA \geq \iint_R g(x, y) dA \quad \text{if} \quad f(x, y) \geq g(x, y) \text{ on } R$$

$$4. \text{ Additivity: } \iint_R f(x, y) dA = \iint_{R_1} f(x, y) dA + \iint_{R_2} f(x, y) dA$$

if R is the union of two nonoverlapping regions R_1 and R_2

b- Cartesian form

Double integral of $f(x, y)$ over the region R is denoted by:

$$\iint_R F(x, y) dA = \iint_R F(x, y) dx dy = \int_c^d \int_{x_1}^{x_2} F(x, y) dx dy$$

or

$$\iint_R F(x, y) dA = \iint_R F(x, y) dy dx = \int_a^b \int_{y_1}^{y_2} F(x, y) dy dx$$

c- Finding Limits of Integration in Cartesian form

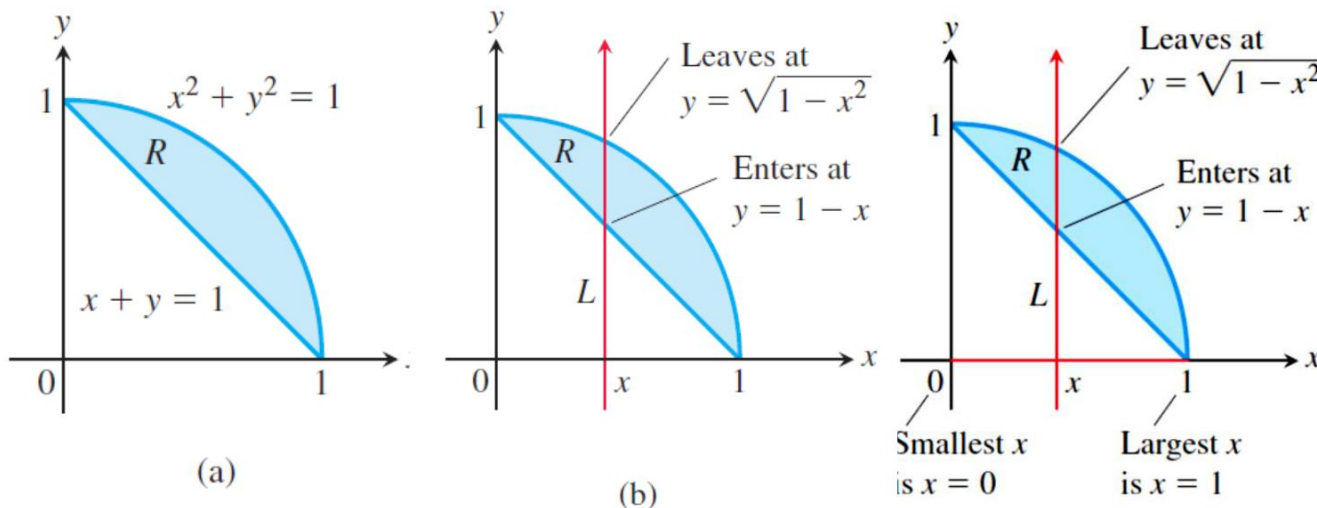
- **Using Vertical Cross-Sections**

When faced with evaluating $\iint ()$, integrating first with respect to y and then with respect to x , do the following three steps:

- 1- Sketch. Sketch the region of integration and label the bounding curves. (Figure 3 a).
- 2- Find the y -limits of integration. Imagine a vertical line L cutting through R in the direction of increasing y . Mark the y -values where L enters and leaves. These are the y -limits of integration and are usually functions of x (instead of constants) (Figure 3 b).

3- Find the x-limits of integration. Choose x-limits that include all the vertical lines through R. The integral shown here (see Figure 3 c) is

$$\iint_R f(x, y) dA = \int_{x=0}^{x=1} \int_{y=1-x}^{y=\sqrt{1-x^2}} f(x, y) dy dx.$$



• **Using Horizontal Cross-Sections**

To evaluate the same double integral as an iterated integral with the order of integration reversed, use horizontal lines instead of vertical lines in Steps 2 and 3 (see Figure 4). The integra

$$\iint_R f(x, y) dA = \int_0^1 \int_{1-y}^{\sqrt{1-y^2}} f(x, y) dx dy.$$

