# Equation of the Plane

To find the equation of the plane that passes through the point $P_0(x_0, y_0, z_0)$ and its normal vector is $\vec{N} = a\hat{i} + b\hat{j} + c\hat{k}$

let $P(x, y, z)$ be any point in the plane

$$\vec{P_0P} = (x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)k$$

$$\vec{P_0P} \perp \vec{N} \Rightarrow \vec{P_0P} \cdot \vec{N} = 0$$

$$\Rightarrow a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax - ax_0 + by - by_0 + cz - cz_0 = 0$$

$$ax + by + cz = \boxed{ax_0 + by_0 + cz_0} \Rightarrow d$$

$$ax + by + cz = d \quad \text{Equation of the Plane}$$

**EX** Find the equation of the plane having the points $A(2,3,5)$, $B(7,2,1)$ and $C(1,1,1)$

Sol
$$\vec{AB} = (7-2)\hat{i} + (2-3)\hat{j} + (1-5)k$$
$$= 5\hat{i} - \hat{j} - 4K$$

$$\vec{AC} = (1-2)\hat{i} + (1-3)\hat{j} + (1-5)K$$
$$= -\hat{i} - 2\hat{j} - 4K$$

$$\begin{array}{ccc} \hat{i}^{+} & \hat{j}^{-} & K^{+} \end{array}$$
$$\vec{N} = \vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & K \\ 5 & -1 & -4 \\ -1 & -2 & -4 \end{vmatrix} = \boxed{-4\hat{i} + 24\hat{j} - 11K}$$

$$-4\hat{i} + 24\hat{j} - 11K = d$$

نعوض احد النقاط (A,B,C) في المعادلة لا يجاد قيمة d

نعوض $C(1,1,1)$

$$(-4*1)+(24*1)-(11*1)=d$$

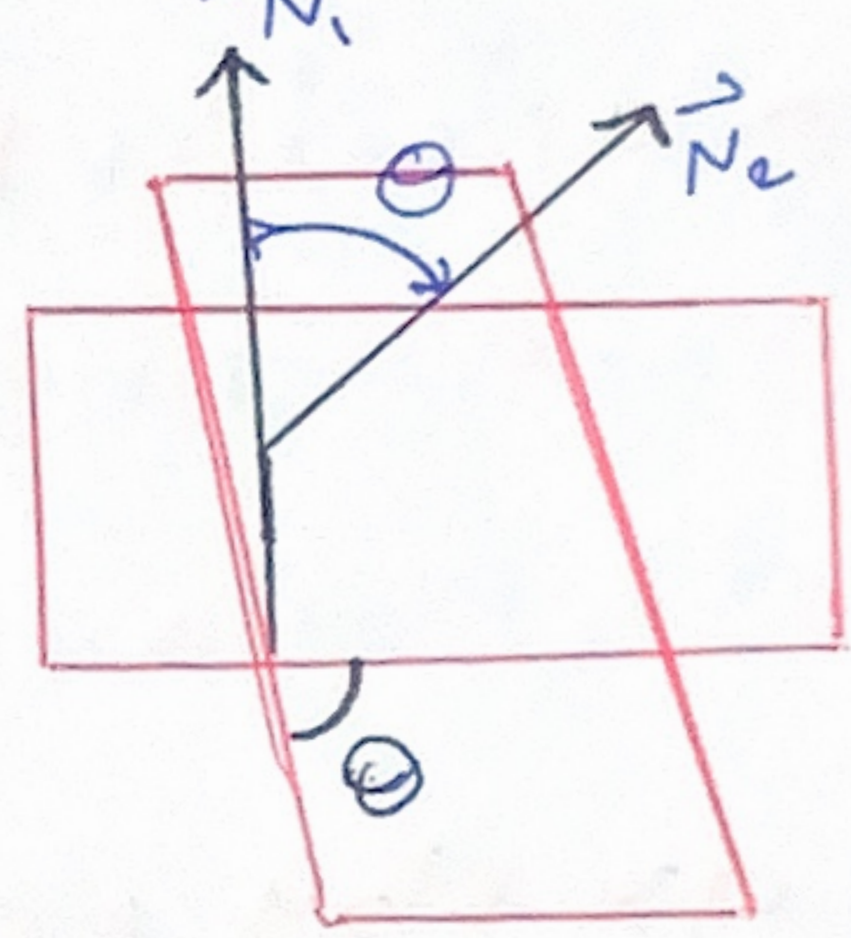
$$\therefore d=-9$$

$\therefore -4x+24y-11z=-9$ is the equation the Plane

## Angles between Planes

The angle between two intersecting Planes is defined to be the angle determined by their normal vectors

$$\theta=\cos^{-1}\left(\frac{\vec{N_1}\cdot\vec{N_2}}{|\vec{N_1}|\cdot|\vec{N_2}|}\right)$$




**Ex** Find the angle between the Planes $3x-6y-2z=15$

$$2x+y-2z=5$$

**Sol** $\vec{N_1}=3i-6j-2K$

$$\vec{N_2}=2i+j-2K$$

$$\theta=\cos^{-1}\left(\frac{\vec{N_1}\cdot\vec{N_2}}{|N_1|\cdot|N_2|}\right)$$

$$=\cos^{-1}\left(\frac{(3*2)+(-6*1)+(-2*-2)}{\sqrt{(3)^2+(-6)^2+(-2)^2}\cdot\sqrt{(2)^2+(1)^2+(-2)^2}}\right)$$

$$\theta=\cos^{-1}\left(\frac{4}{21}\right)$$

$$\therefore \theta=79^\circ$$