CHAPTER FIVE

TRANSIENT CONDUCTION

Our objective in this chapter is to develop procedures for determining the time dependence of the temperature distribution within a solid during a transient process, as well as for determining heat transfer between the solid and its surroundings. The nature of the procedure depends on assumptions that may be made for the process. For example, temperature gradients within the solid may be neglected, a comparatively simple approach, termed the lumped capacitance method, may be used to determine the variation of temperature with time.

5.1 The Lumped Capacitance Method

Consider a hot metal forging that is initially at a uniform temperature T_i and is quenched by immersing it in a liquid of lower temperature $T_\infty < T_i$ Figure (5.1). If the quenching is said to begin at time $t = 0$, the temperature of the solid will decrease for time $t > 0$, until it eventually reaches T_∞. This reduction is due to convection heat transfer at the solid-liquid interface. The essence of the lumped capacitance method is the assumption that the temperature of the solid is spatially uniform at any instant during the transient process. This assumption implies that temperature gradients within the solid are negligible.

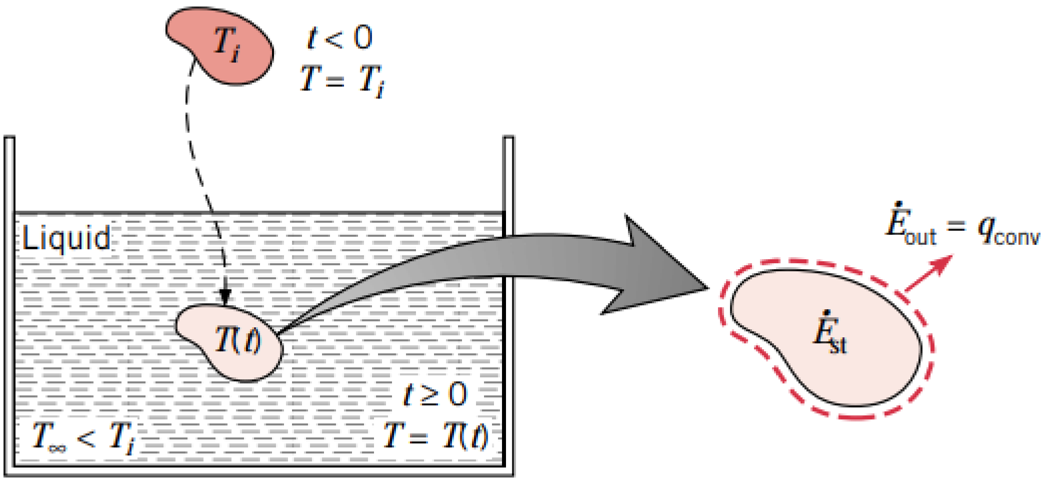


Figure (5.1) Cooling of a Hot Metal Forging

From Fourier's law, heat conduction in the absence of a temperature gradient implies the existence of infinite thermal conductivity. Such a condition is impossible. However, the condition is closely approximated if the resistance to conduction within the solid is small compared with the resistance to heat transfer between the solid and its surroundings. Applying conservation energy to the control volume of Figure (5.1) this requirement takes the form

$$-\dot{E}_{out} = \dot{E}_{st} \qquad (5.1)$$

$$-hA_s(T - T_\infty) = \rho V C_p \frac{dT}{dt} \qquad (5.2)$$

Introducing the temperature difference

$$\theta = T - T_\infty \qquad (5.3)$$

And recognizing that $(d\theta/dt) = (dT/dt)$ if T_∞ is constant, it follows that

$$\frac{\rho V C_p}{hA_s}\frac{d\theta}{dt} = -\theta \qquad (5.4)$$

Separating variables and integrating from the initial condition, for which $t = 0$ and $T(0) = T_i$, we then obtain

$$\frac{\rho V C_p}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

$$\theta_i = T_i - T_\infty$$

Evaluating the integrals, it follows that

$$\frac{\rho V C_p}{hA_s}\ln\frac{\theta_i}{\theta} = t \qquad (5.5)$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V C_p}\right)t\right] \qquad (5.6)$$

Where

T_i: initial temperature (°C)

T_∞: surrounding temperature (°C)

t: time (sec)

C_p: specific heat capacity (J/kg. °C)

V: volume (m^3)

ρ: density (kg/m^3)

A_s: surface area (m^2)

h: convection heat transfer coefficient $(W/m^2.K)$

Eq. (5.5) maybe used to determine the time required for the solid to reach some temperature (T), or, conversely, Eq. (5.6) maybe used to compute the temperature reached by the solid at some time (t).

The foregoing results indicate that the difference between the solid and fluid temperatures must decay exponentially to zero as t approaches infinity. This behavior is shown in Figure (5.2). From Eq. (5.6) it is also evident that the quantity (V_c/hA_s) may be interpreted as a thermal time constant expressed as

$$\tau_t = \left(\frac{1}{hA_s}\right)(\rho V C_p) = R_t C_t \qquad (5.7)$$

5.2 Validity of the Lumped Capacitance Method

To develop a suitable criterion consider steady-state conduction through the plane wall of area (A) Figure (5.2). One surface is maintained at a temperature $(T_{s,1})$ and the other surface is exposed to a fluid of temperature $(T_\infty < T_{s,1})$. The temperature of this surface will be some intermediate value, $(T_{s,2})$, for which $(T_\infty < T_{s,2} < T_{s,1})$.

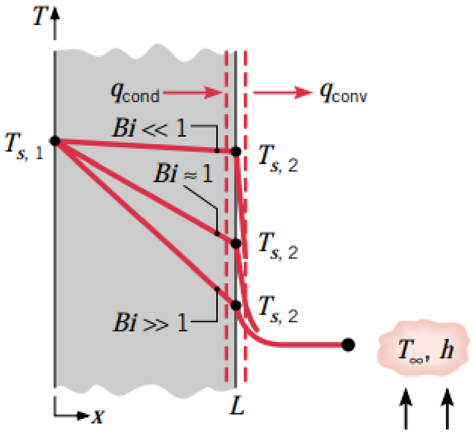


Figure (5.2) Effect of Biot Number on Steady State Temperature Distribution in a Plane Wall with Surface Convection.

Hence under steady-state conditions, the surface energy balance reduces to

$$\frac{kA}{L}\left(T_{s,1}-T_{s,2}\right)=hA\left(T_{s,2}-T_{\infty}\right)$$

where k is the thermal conductivity of the solid. Rearranging, we then obtain

$$\frac{T_{s1}-T_{s2}}{T_{s2}-T_{\infty}}=\frac{L/kA}{1/hA}=\frac{hL}{k}=\frac{R_{cond.}}{R_{conv.}}$$

$$Bi=\frac{hL}{k} \qquad (5.8)$$

The quantity (hL/k) appearing in Eq. (5.8) is a dimensionless parameter. It is termed the Biot number (Bi), and it plays a fundamental role in conduction problems that involve surface effects. Biot number provides a measure of the temperature drop in the solid relative to the temperature difference between the surface and the fluid. If $Bi \ll 1$, the resistance to conduction within the solid is much less than the resistance to convection across the fluid boundary layer. Hence the assumption of uniform temperature distribution is reasonable.

If the following condition is satisfied

$$Bi=\frac{hL_c}{k}<0.1 \qquad (5.9)$$

The error associated with using the lumped capacitance method is small. For convenience, it is customary to define the characteristic length of Eq. (5.9) as the ratio of the solid's volume to surface area ($L_c=V/A_s$).

Geometry	**The characteristic length (L_c)**
Flat plate	$L/2$
Long cylinder	$r_0/2$
Sphere	$r_0/3$
Cube	$L/6$

Note: Volume and Area

1- For a cylinder

$$V=\pi r^2 L=\frac{\pi}{4}D^2 L$$

and $A_s = 2\pi r L=\pi DL$

2- For a sphere

$$V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi D^3$$

and $\quad A_s = 4\pi r^2 = \pi D^2$

Finally, we note that, with $(L_c = V/A_s)$, the exponent of Eq. (5.6) maybe expressed as

$$\frac{hA_s t}{\rho V C_p} = Bi.Fo$$

Where Fourier number (F_0) is

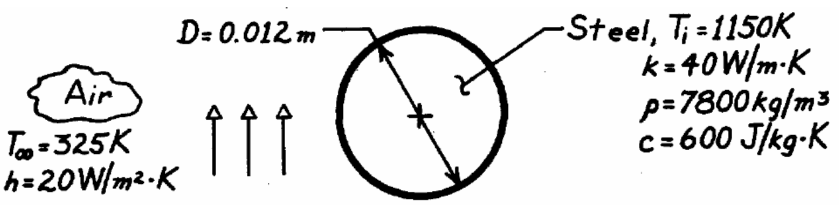
$$Fo = \frac{\alpha t}{L_c^2} \qquad (5.10)$$

Substituting Eq. (5.10) into (5.6), we obtain

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi\,.Fo) \qquad (5.11)$$

Example (5.1): Steel balls $(12\ mm)$ in diameter are annealed by heating to $(1150\ K)$ and then slowly cooling to $(400\ K)$ in an air environment for which $(T_\infty = 325\ K)$ and $(h = 20\ W/m^2.\ K)$. Assuming the properties of the steel to be $(k =\ 40\ W/m.K), (\rho = 7800\ kg/m^3)\ , and\ \left(C_p = 600\ J/kg.K\right)$, estimate the time required for the cooling process.

Solution:



$$Bi = \frac{hL_c}{k} = \frac{h(r_0/3)}{k} = \frac{20 * (0.006/3)}{40} = 0.001 < 0.1$$

The lumped capacitance method may be used

$$t = \frac{\rho V C_p}{h A_s} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

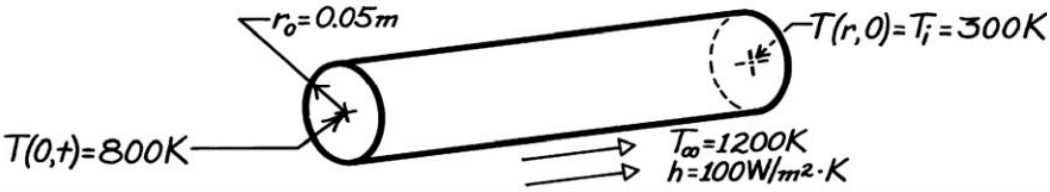
$$t = \frac{\rho (\pi\, D^3/6\,) C_p}{h \pi D^2} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7800 * (\pi\,(0.012)^3/6\,) * 600}{20 * \pi (0.012)^2} \ln \frac{1150 - 325}{400 - 325}$$

$$t = 1122\, s = 0.312\, hr$$

Example (5.2): Carbon steel shafts of $(0.1\, m)$ diameter are heat treated in a gas-fired furnace whose gases are at $(1200\, K)$ and provide a convection coefficient of $(100\, W/m^2.K)$. If the shafts enter the furnace at $(300\, K)$, how long must they remain in the furnace to achieve a centerline temperature of (800 K)? Take $(\rho = 7832\, kg/m^3)$, $(k = 51.2\, W/m.K)$, $(C_p = 541\, J/Kg.K)$.

Solution:



$$Bi = \frac{h L_c}{k} = \frac{h(r_0/2)}{k} = \frac{100 * (0.05/2)}{51.2} = 0.0488 < 0.1$$

The lumped capacitance method may be used

$$t = \frac{\rho V C_p}{h A_s} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{\rho (\pi\, r^2 L\,) C_p}{h * 2\pi r L} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

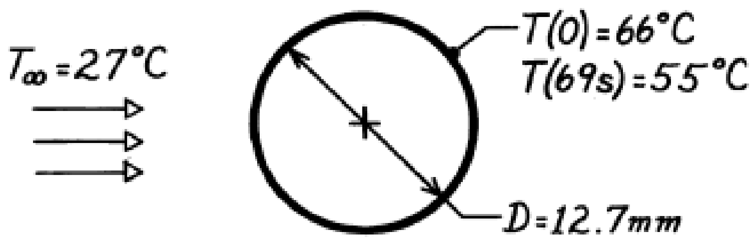
$$t = \frac{\rho r C_p}{2h} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{7832 * 0.05 * 541}{2 * 100} \ln \frac{300 - 1200}{800 - 1200}$$

$$t = 859\, s$$

Example (5.3): The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature-time history of a sphere fabricated from pure copper. The sphere, which is $(12.7\ mm)$ in diameter, is at $(66\ °C)$ before it is inserted into an airstream having a temperature of $(27\ °C)$. A thermocouple on the outer surface of the sphere indicates $(55\ °C)$, after $(69\ s)$ the sphere is inserted in the airstream. Assume, and then justify, that the sphere behaves as a space-wise isothermal object and calculate the heat transfer coefficient. Take $(\rho = 8933\, kg/m^3), (k = 398\, W/m.K), (C_p = 389\, J/Kg.K)$.

Solution:



$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V C_p}\right)t\right]$$

Where $A_s = \pi D^2$ $\qquad V = \dfrac{\pi D^3}{6}$

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{h\pi D^2}{\rho(\pi D^3/6)C_p}\right)t\right] = \exp\left[-\left(\frac{6h}{\rho D C_p}\right)t\right]$$

$$\frac{55 - 27}{66 - 27} = \exp\left[-\left(\frac{6h}{8933 * 389 * 12.7 * 10^{-3}}\right) * 69\right]$$

$$h = 35.3\, W/m^2.K$$

$$L_c = D_0/6$$

$$Bi = \frac{hL_c}{k} = \frac{h(D_0/6)}{k} = \frac{35.3 * (0.0127/6)}{398} = 1.88 \times 10^{-4}$$

Hence, Bi < 0.1 and the spatially isothermal assumption is reasonable.

Home Work (5):

1- A (20 by 20 cm) slab of copper (5 cm) thick at a uniform temperature of (260 $°C$) suddenly has its surface temperature lowered to (35 $°C$). Find the time required for the plate to reach the temperature of (90 $°C$) ; take ($\rho = 8900\, kg/m^3$), ($cp = 0.38\, kJ/kg.°C$), ($k = 370\, W/m \cdot °C$) and ($h = 90\, W/m^2.°C$).

2- A solid copper sphere of (10 cm) diameter initially at a temperature of (250 $°C$) is suddenly immersed in a fluid at (50 $°C$). The convection heat transfer coefficient is (200 $W/m2.°C$). Estimate the temperature of the copper block at ($t = 5\, min$) after the immersion. Take($\rho = 8954\, kg/m^3$), ($cp = 383\, J/kg.°C$), and ($k = 386\, W/m \cdot °C$).

3- A (15 mm) diameter mild steel sphere ($k = 42\, W/m.°C$) is exposed to cooling air flow at (20 $°C$) resulting in the convection coefficient ($h = 120\, W/m^2.°C$). Determine the time required to cool the sphere from (550 $°C$) to (90 °C). Take ($\rho = 7850\, kg/m^3$) , ($cp = 475\, J/kg\, .°C$), and($\alpha = 0.045\, m^2/h$).

4- A piece of aluminum weighing (6 kg) and initially at a temperature of (300 $°C$) is suddenly immersed in a fluid at (20 $°C$). The convection heat transfer coefficient is (58 $W/m^2.°C$). Taking the aluminum as a sphere having the same weight as that given, estimate the time required to cool the aluminum to (90 $°C$), using the lumped capacity method of analysis. Take ($\rho = 2707\, kg/m^3$) and $(C_p = 896\, J/kg.°C)$.

5- A stainless steel rod (6.4 mm) in diameter is initially at a uniform temperature of (25 °C) and is suddenly immersed in a liquid at (150 $°C$) with ($h = 120\, W/m2.°C$). Using the lumped capacity method of analysis, calculate the time necessary for the rod temperature to reach (120 $°C$). Take ($\rho = 7817\, kg/m^3$) and $(C_p = 460\, J/kg.°C)$.