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Vectors

vectors in plane

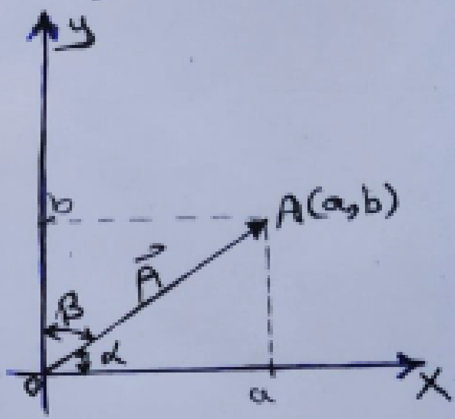
$$* \ \vec{A} = \vec{OA} = ai + bj$$

where

i, j :- are the fundamental unit vectors

$$* \ \text{Length of } \vec{A} = |A| = \sqrt{a^2 + b^2}$$

$$* \ \text{Unit vector} = \vec{U_A} = \frac{\vec{A}}{|A|} = \frac{ai + bj}{|A|}$$



$$\vec{U_A} = \frac{a}{|A|} i + \frac{b}{|A|} j \quad \text{—— (1)}$$

but, $\cos\alpha = \sin\beta = \dfrac{a}{|A|}$ —— (2)

$\cos\beta = \sin\alpha = \dfrac{b}{|A|}$ —— (3)

Sub. eq (2) & eq (3) in eq (1) give:-

$$* \ \vec{U_A} = \cos\alpha \, i + \cos\beta \, j$$

$$\text{or } \ \vec{U_A} = \cos\alpha \, i + \sin\alpha \, j$$

$$* \ \vec{A} = \vec{U_A} \cdot |A| = (\cos\alpha \, i + \sin\alpha \, j) \cdot |A|$$

where

a, b :- Direction numbers

α, β :- Direction angles

$\cos\alpha, \cos\beta$:- Direction cosines

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EX.(1):- Find avector in plane (R^2) of length (7 units) which makes angle (35°) with X- axis ?

Answer:-

Since $|A| = 7$ & $\alpha = 35^\circ$

$$\vec{A} = |A|(\cos\alpha\, i + \sin\alpha\, j)$$

$$\therefore \vec{A} = 7 * (\cos 35^\circ\, i + \sin 35^\circ\, j)$$

$$= \boxed{5.7\, i + 4 j}$$

EX(2):- Find the angle between the Vector $\vec{A} = 2i + 3j$ and the x- axis ?

Answer:-

$$|A| = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$\cos\alpha = \frac{a}{|A|} = \frac{2}{\sqrt{13}} \Rightarrow \alpha = \cos^{-1}\left(\frac{2}{\sqrt{13}}\right)$$

$$\boxed{\alpha = 56.3^\circ}$$

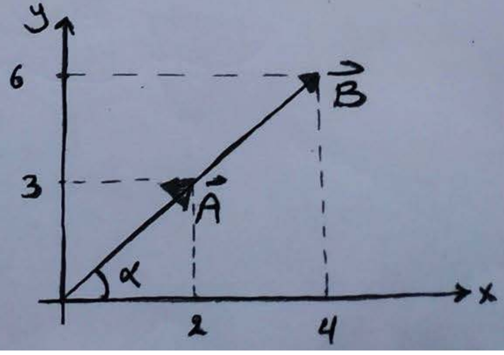
Note:- Two vectors are parallel if either is proportional to an other

i.e $\vec{A} \parallel \vec{B} \Leftrightarrow \vec{B} = t\vec{A}$ $(\vec{U}_A = \vec{U}_B)$

where t is a scalar quantity

EX(3):- The two vectors $\vec{A} = 2i + 3j$ and $\vec{B} = 4i + 6j$ are parallel, for the reason that

$$\vec{B} = 2(2i + 3j) = 2\vec{A}$$



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Vectors in aspace :-

$\vec{A} = \vec{OA} = a\vec{i} + b\vec{j} + c\vec{k}$

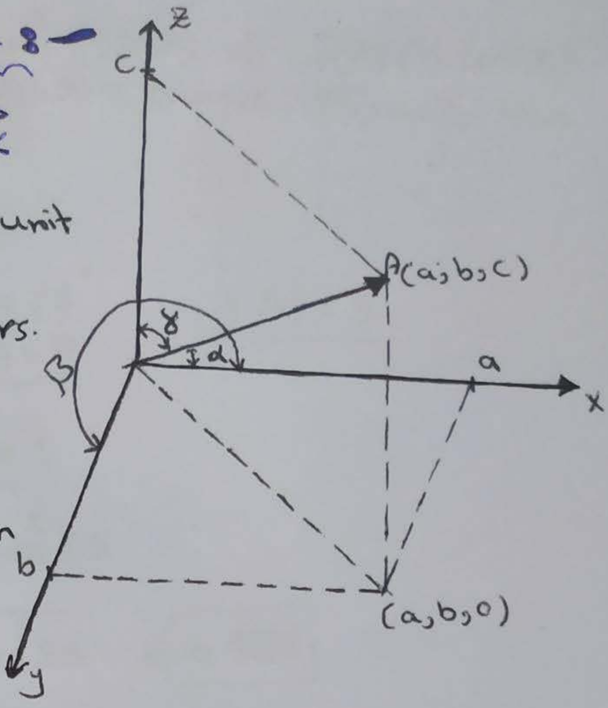
where :

i, j, k :- are the fundamental unit vectors.

a, b, c :- Direction numbers.

α, β, γ :- Direction angle

$\cos\alpha, \cos\beta, \cos\gamma$:- Direction Cosines.



* $\vec{U_A} = \dfrac{\vec{A}}{|A|}$

* Length of $\vec{A} = |A| = \sqrt{a^2 + b^2 + c^2}$

$$\vec{U_A} = \frac{ai + bj + ck}{|A|}$$

$$\vec{U_A} = \frac{a}{|A|} i + \frac{b}{|A|} j + \frac{c}{|A|} k$$

$$\cos\alpha = \frac{a}{|A|} \; ; \; \cos\beta = \frac{b}{|A|} \; ; \; \cos\gamma = \frac{c}{|A|}$$

$$\therefore \vec{U_A} = \cos\alpha \, i + \cos\beta \, j + \cos\gamma \, k$$

where

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

and $\vec{A} = |A| \cdot \vec{U_A}$

EX(1):- Find a vector in R^3 (space) of length (5 units) that makes angles (70°) with the x-axis, (85°) with the y-axis

Answer:-

$\alpha = 70^\circ$ $\quad$ $\beta = 85^\circ$ $\quad$ $|A| = 5$

$\gamma = ?$ $\quad$ Vector $\vec{A} = ?$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2\gamma = 1 - \cos^2 70 - \cos^2 85$$

$$\cos\gamma = \sqrt{1 - \cos^2 70 - \cos^2 85} = \boxed{0.935}$$

$$\vec{A} = |A| \cdot (\cos\alpha\, i + \cos\beta\, j + \cos\gamma\, K)$$

$$= 5 * (\cos 70^\circ\, i + \cos 85^\circ\, j + 0.935\, K)$$

$$= \boxed{5 * (1.7\, i + 0.435\, j + 0.935\, K)}$$

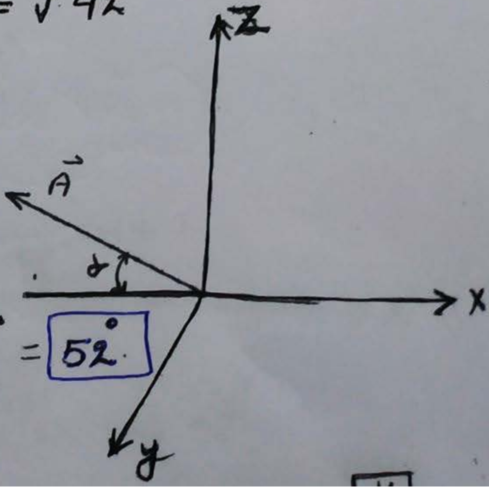
EX(2):- Find the acute angle between the x-axis and the vector $\vec{A} = -4i + 5j + K$?

Answer:-

$$|A| = \sqrt{(-4)^2 + (5)^2 + (1)^2} = \sqrt{42}$$

$$\cos\alpha = \frac{a}{|A|} = \frac{-4}{\sqrt{42}} = 0.617$$

$$\alpha = \cos^{-1}(0.617) \cong 128^\circ$$



$\therefore$ The required angle is $180^\circ - 128^\circ = \boxed{52^\circ}$

Definition

(1) Algebrac addition :-

If $\vec{A} = a_1 i + b_1 j$ and $\vec{B} = a_2 i + b_2 j$

Then $\vec{A}+\vec{B} = (a_1+a_2) i + (b_1+b_2) j$

(2) Subtraction :-

$\vec{A} - \vec{B} = (a_1-a_2) i + (b_1-b_2) j$

EX(1) :- If $\vec{A} = 2i + 3j$ and $\vec{B} = 4i + j$, then find $\vec{A}+\vec{B}$ and $\vec{B}-\vec{A}$ and sketch ?

Answer :-

$$\vec{A}+\vec{B} = (2i+3j) + (4i+j)$$
$$= (2+4) i + (3+1) j$$
$$= \boxed{6i + 4j}$$

$$\vec{B}-\vec{A} = (4i+j) - (2i+3j)$$
$$= (4-2) i + (1-3) j$$
$$= \boxed{2i - 2j}$$