ALMUSTAQBAL UNIVERSITY COLLEGE

DEPARTMENT OF BUILDING & CONSTRUCTION ENGINEERING TECHNOLOGY

ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES II

DIRECT DESIGN METHOD

(DDM)

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DIRECT DESIGN METHOD

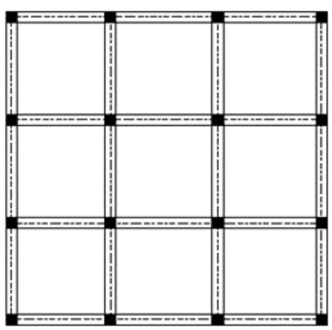
- The direct design method (DDM) ACI 8.10. is an approximate procedure to determine the analysis and design of two-way slabs. The method uses a set of coefficients for determining the design moments at critical sections.

- Two-way slab systems that do not meet the method's limitation of the ACI code 8.10.2 must be analysed by more accurate procedures, such as the ***equivalent frame method***, or ***finite element method***.

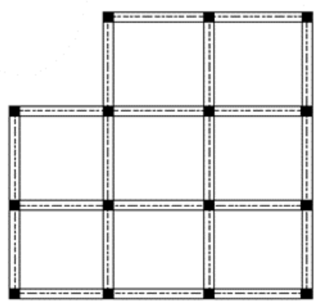
- Direct design method is used with these types of slabs:
  - Slab supported by beams or walls.
  - Flat slabs.
  - Flat plate slabs.
  - Two-way grid slabs (waffle slab).

LIMITATIONS OF THE DIRECT DESIGN METHOD:

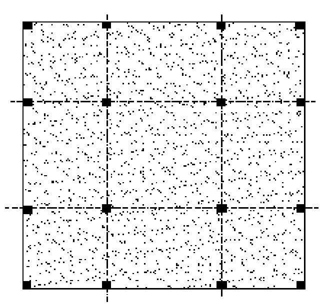
1. There must be **THREE** continuous spans in each direction.

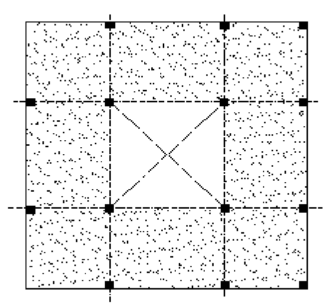


DDM CAN BE USED

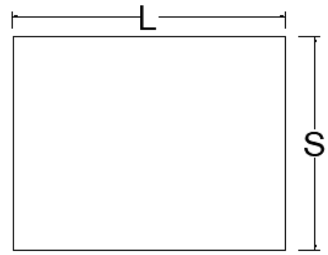


DDM CANNOT BE USED





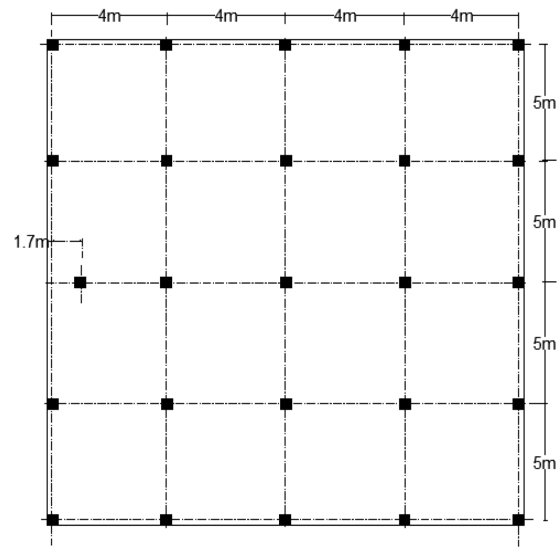
2. Panel should be rectangular, with the ratio of longer to shorter dimensions (centre to centre of columns), is lesser or equal to 2.



$$\frac{L}{S} \leq 2$$

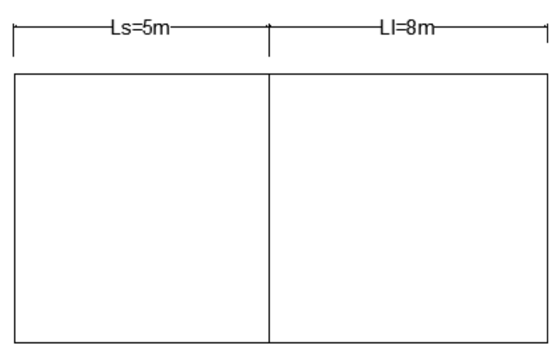
3. Column offset shall not exceed 10% of the span in the direction of offset from either axis between centre lines of successive columns.

$$offset \leq 0.1l$$
$$1.7m \; ? \; 0.1 \times 4m$$
$$1.7m \; ? \; 0.4m$$
$$1.7m > 0.4m \rightarrow not\ okay.$$



4. Successive span lengths measured centre to centre of supports in each direction should not differ by more than 1/3 the longer span.

$$Ll - Ls \leq \frac{1}{3}Ll$$
$$8 - 5 = 3 \; ? \; \frac{1}{3} \times 8$$
$$3m > 2.67m \rightarrow not\ okay.$$



5. All loads shall be due to gravity only (no lateral loads) and uniformly distributed over the entire panel. The direct design method cannot be used for unbraced, laterally loaded frames, foundation mats and prestressed slabs.

6. The service live load shall not exceed twice the service dead load.

$$LL \leq 2 \times DL$$

7. **IF** beams are used between supports on all sides (solid slab), the relative stiffness ratio of the beams in the two directions must be between 0.2 – 5.0.

$$0.2 \leq \frac{\alpha_{f1} {l_2}^2}{\alpha_{f2} {l_1}^2} \leq 5.0$$

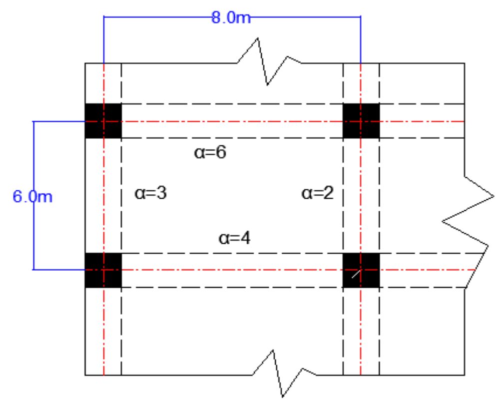
Where:

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \quad , \quad I_b = moment\ of\ inertia\ for\ beam, I_s = moment\ of\ inertia\ for\ slab$$

$$l_1, l_2: length\ perpindicular\ to\ beam\ direction\ which\ has\ (\alpha_f)\ centre\ to\ centre$$

EXAMPLE 1: Find the limit $\frac{\alpha_{f1} {l_2}^2}{\alpha_{f2} {l_1}^2}$ for the slab shown in the figure.

Solution:



$$\alpha_{f1} = \frac{4+6}{2} = 5$$

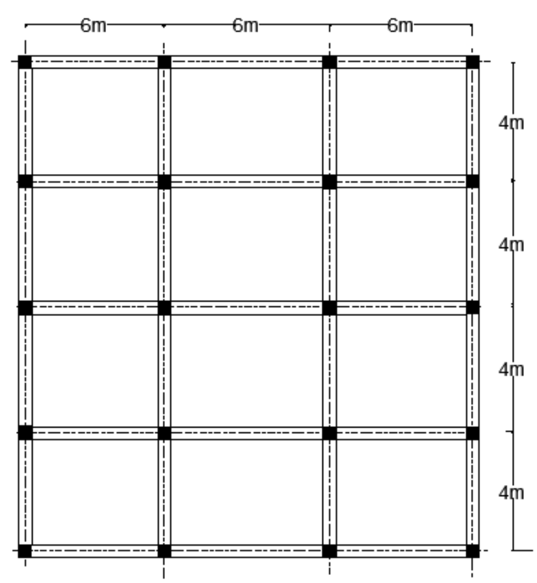
$$\alpha_{f1} = \frac{3+2}{2} = 2.5$$

$$0.2 \leq \frac{\alpha_{f1} {l_2}^2}{\alpha_{f2} {l_1}^2} \leq 5$$

$$\frac{\alpha_{f1} {l_2}^2}{\alpha_{f2} {l_1}^2} = \frac{5 \times 6^2}{2.5 \times 8^2} = 1.125 \qquad \therefore O.K.$$

EXAMPLE 2: for the slab shown below, check the limitations of the direct design method if the live load is 4kN/m², dead load is 3kN/m² (self-weight of slab is included), α_f for all beams in the longitudinal direction is equal to 3.5 and for beams in the transverse direction α_f is equal to 2.

SOLUTION:



1. There must be a minimum of three continuous spans in all directions. ***OK.***

2. The panels must be rectangular and the ration of the longer span to the shorter span within the panel does not exceed 2.

$$\frac{L}{S} = \frac{6}{4} = 1.5 \quad < \quad 2. \quad \textbf{\textit{OK.}}$$

3. Column offset shall not exceed 10% of the span in the Direction of the offset.
   *Since there is no off set and all the columns are on the same centreline This condition is **OK**.*

4. Successive span lengths measured centre to centre of supports in each direction should not differ by more than 1/3 the longer span.
   *Since the successive span lengths are equal in each direction, this condition is **OK.***

5. All loads shall be due to gravity only (no lateral loads) and uniformly distributed over the entire panel. The direct design method cannot be used for unbraced, laterally loaded frames, foundation mats and prestressed slabs.
   *Since there are no lateral loads, this condition is **OK.***

6. The service live load shall not exceed twice the service dead load.

   Live load =4kN/m^2 < 2x Dead Load= 2x3=6kN/m^2…….. ***OK.***

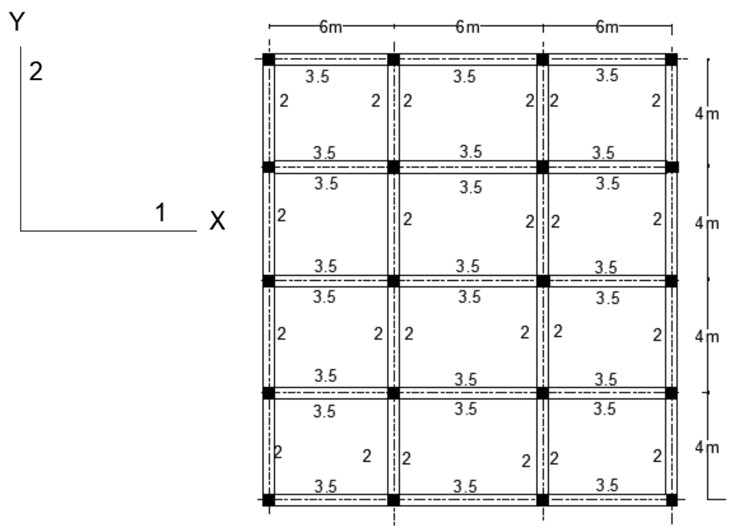
7. **IF** beams are used between supports on all sides (solid slab), the relative stiffness ratio of the beams in the two directions between must be between (0.2 – 5.0).

$$0.2 \leq \frac{\alpha_{f1} l_2^{\,2}}{\alpha_{f2} l_1^{\,2}} \leq 5.0 \quad \text{........} \quad \text{where } \alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s},$$

$$\alpha_{f1} = \frac{3.5+3.5}{2} = 3.5$$

$$\alpha_{f2} = \frac{2+2}{2} = 2.0$$

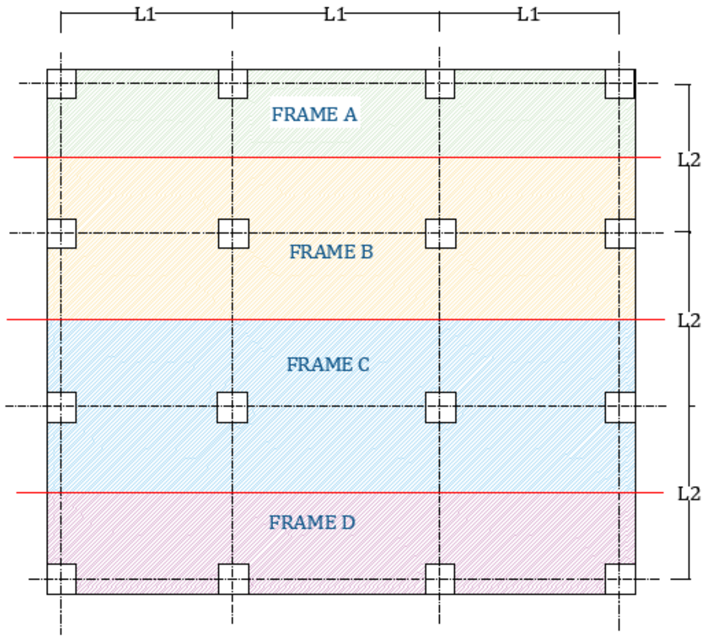
$$\therefore \ \frac{3.5 \times 4^2}{2 \times 6^2} = 0.78 \ \text{..........} \ \textbf{\textit{OK.}}$$



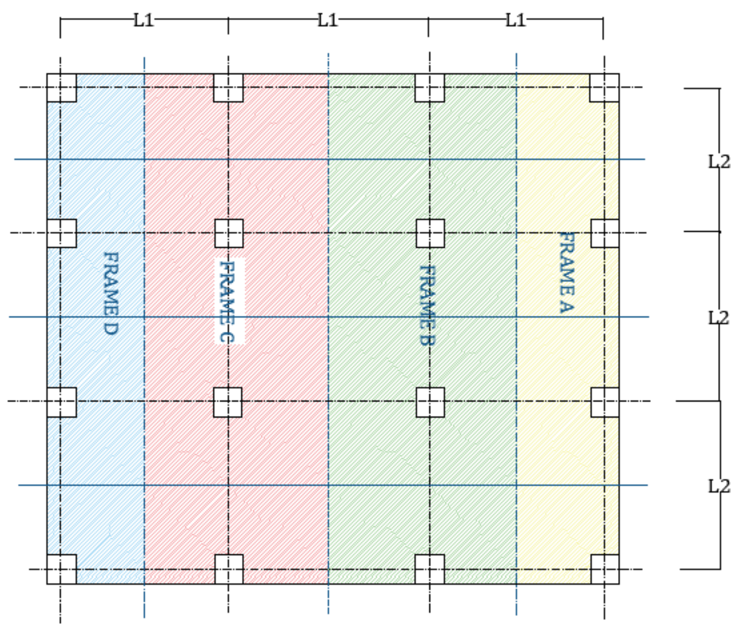
∴ ALL CONDITIONS FOR THE DDM HAVE BEEN SATISFIED.

GENERAL CONCEPT FOR ANALYSING TWOWAY SLABS USING DDM

1. Divide the slab into longitudinal and transverse frames (***usually the frame direction is mentioned in the question***).



Longitudinal (in the X-direction)



Transverse (in the Y-direction)

2. For each frame calculate the total static moment M_o for the exterior and interior spans.

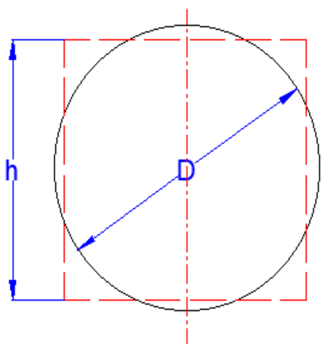
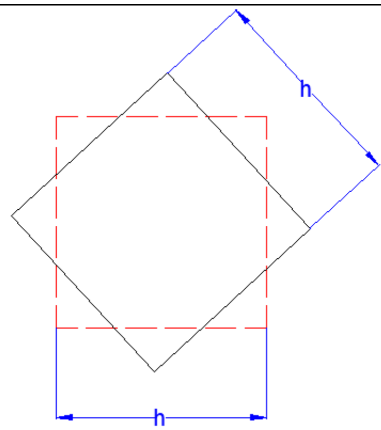
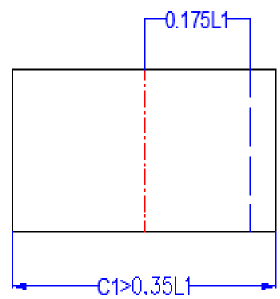
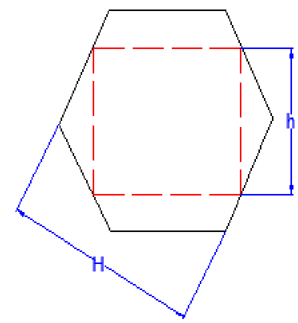
$$\boldsymbol{M_o = \frac{W_u l_2 {l_n}^2}{8}} \text{ ....ACI (8.10.3.2)}$$

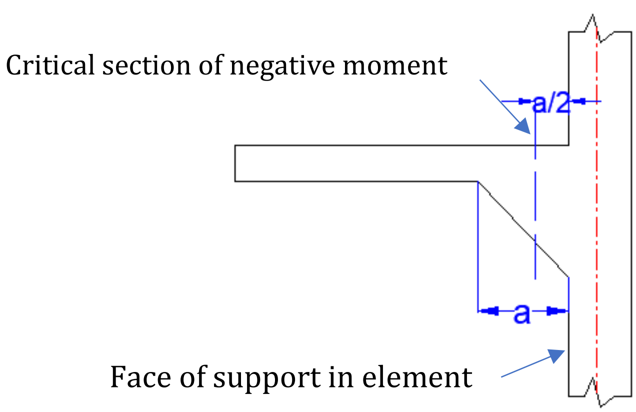
Where:

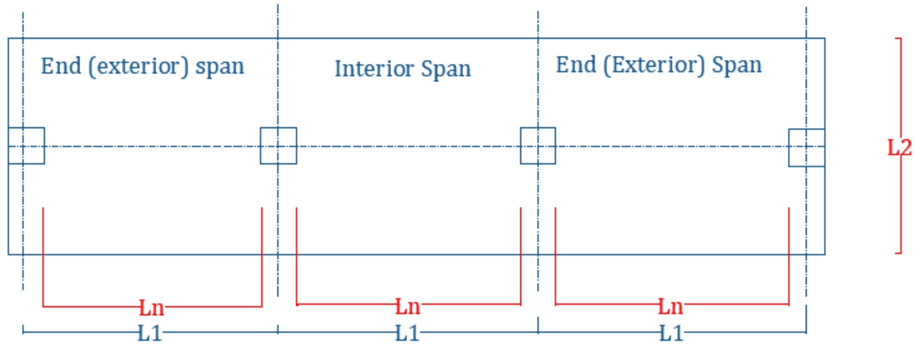
$W_u = 1.2W_D + 1.6W_L$

$l_2 = frame\ width$

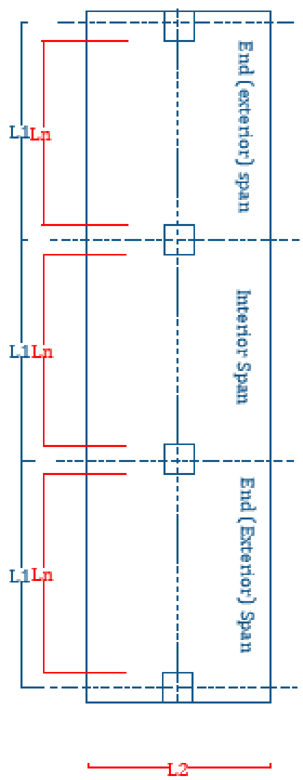
$l_n = clear\ span\ of\ l_1, face\ to\ face\ of\ column, face\ to\ face\ of\ capitals, brackets\ \geq 0.65l_1$

			
$\boldsymbol{h = 0.89D}$	$\boldsymbol{h = h}$	$\boldsymbol{0.175l_1}$	$\boldsymbol{h = 0.93H}$



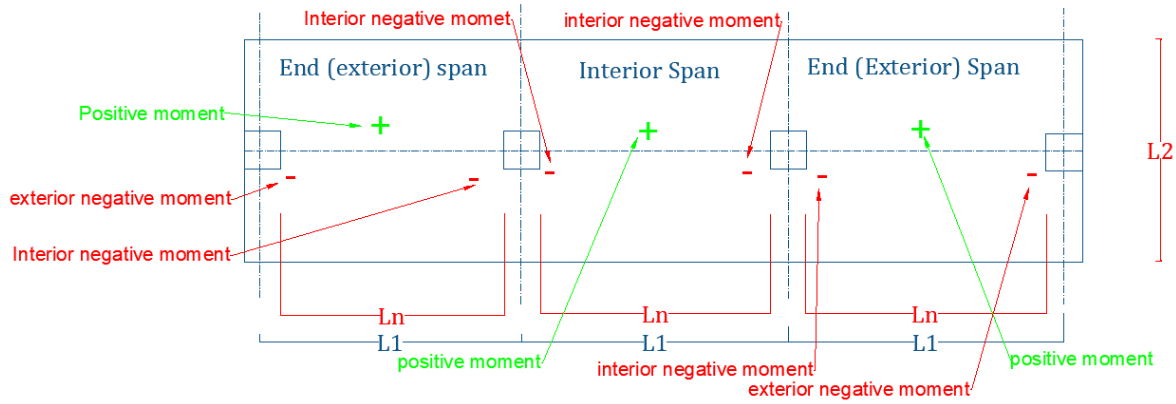


For Longitudinal frame



For Transverse Frame

3. Distribute longitudinally the total static moment M_o into negative and positive moments for each of the exterior and interior spans.
   - Each span has 3 moments (2 negative moments & 1 positive moment).
   - Each frame has a total of 9 moments (6 negative moments & 3 positive moments).



- **For Interior Span:** According to ACI 8.10.4.1, Mo for the interior span shall be distributed as follows:

  (-Ve) Negative moment at the face of support= $0.65M_o$
  (+Ve) Positive moment at the centre of span =$0.35M_o$

- **For exterior (end) span:** in the case of exterior spans, the distribution of the total static moment among the three critical moment sections (interior negative, positive, exterior negative) depend on the flexural restraint provided for the slab by the exterior column or wall, and depends upon the presence or absence of beams on the column lines.

Table 8.10.4.2- distribution coefficients for end span

	(1)	(2)	(3)		(4)
			Slab without beams between interior supports		
	Exterior edge unrestrained	**Slab with beams between all supports**	**Without edge beam**	**With edge beam**	**Exterior edge fully restrained**
Interior negative	0.75	0.70	0.7	0.7	0.65
Positive	0.63	0.57	0.52	0.5	0.35
Exterior Negative	0	0.16	0.26	0.3	0.65

(1) ***Exterior edge unrestrained:*** This means that the exterior edge has no moment restraint, such as in the case where the slab is supported by a brick wall.

(2) ***Slab with beams between all supports:*** this case is a two-way slab that has beams between columns in all directions.

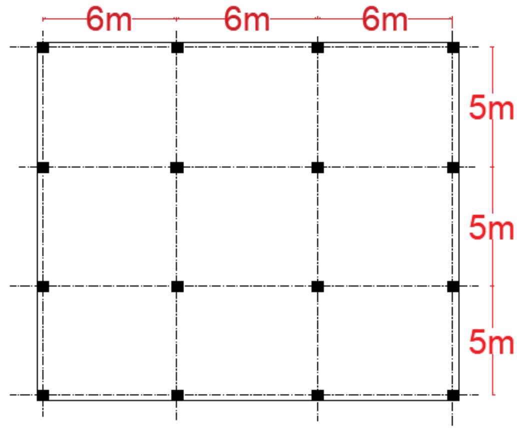
(3) Slab without beams between interior supports: In this case, the slab is without interior beams. However, this case is divided into two parts when the slab has no edge beam around the slab and the second parts is when the slab has an edge beam.

(4) Exterior edge fully restrained: this is the case where the slab is supported by concrete walls and the edges are moment restrained.

EXAMPLE 3: for the flat plate slab shown in the figure below, the slab is subjected to a live load of 3kN/m² and a dead load (including the slab's self-weight) of 2.5kN/m². Columns carrying the slab are square at a size of 400mm. Determine the following:

1. Longitudinal distribution of the total static moment at factored loads for an interior longitudinal frame.
2. Longitudinal distribution of the total static moment at factored loads for an exterior transverse frame.

SOLUTION:



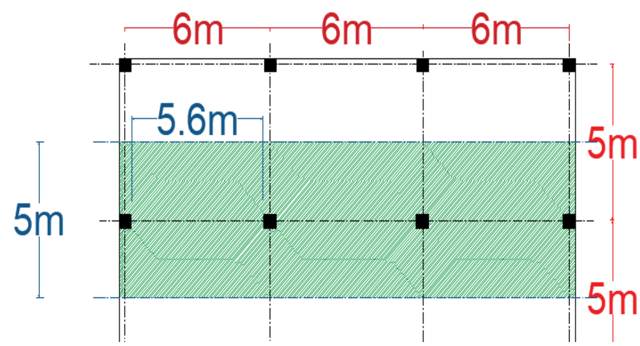
$$W_u = 1.2\,W_D + 1.6\,W_L$$
$$= 1.2\,(2.5) + 1.6\,(3) = 7.8kN/m^2$$

1. Longitudinal distribution of the total static moment at factored loads for an interior longitudinal frame.

$$l_n = 6 - 0.4 = 5.6m \quad or\ 0.65l_1 = 0.65 \times 6 = 3.9$$

Use $l_n = 5.6m$.

$$l_2 = \left(\frac{5}{2} + \frac{5}{2}\right) = 5.0m$$



$$M_o = \frac{W_u {l_n}^2 l_2}{8} = \frac{7.8\ \times 5.6^2 \times 5}{8} = 152.88kN.m$$

For Interior span:

$$(+Ve) Moment\ at\ centre\ of\ span = 0.35(152.88)$$
$$= 53.508kN.m$$
$$(-Ve) Moment\ at\ the\ face\ of\ supports = 0.65(152.88)$$
$$= 99.37kN.m$$

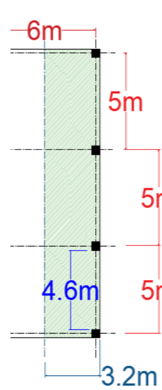
For exterior span:

Table 8.10.4.2- distribution coefficients for end span

	(1)	(2)	(3)		(4)
			Slab without beams between interior supports		
	Exterior edge unrestrained	Slab with beams between all supports	Without edge beam	With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.7	0.7	0.65
Positive	0.63	0.57	0.52	0.5	0.35
Exterior Negative	0	0.16	0.26	0.3	0.65

$$Interior\ (-Ve) moment = 0.7(152.88) = 107.88kN.m$$
$$(+Ve) Moment = 0.52(152.88) = 79.497kN.m$$
$$Exterior\ (-Ve) moment = 0.26(152.88) = 39.7488kN.m$$

2. Longitudinal distribution of the total static moment at factored loads for an exterior transverse frame.



$$l_1 = 5m, l_2 = 3.0m + 0.5(0.4) = 3.2m$$

$$l_n = 5 - 0.4 = 4.6m\ \ or\ 0.65(5) = 3.25m$$

$$\therefore\ l_n = 4.6m.$$

$$M_o = \frac{W_u {l_n}^2 l_2}{8} = \frac{7.8\ \times 4.6^2 \times 3.2}{8} = 66.0192\ kN.m$$

For Interior span:

$$(+Ve)Moment\ at\ centre\ of\ span = 0.35(66.0192)$$
$$= 23.12kN.m$$
$$(-Ve)Moment\ at\ the\ face\ of\ supports = 0.65(66.0192)$$
$$= 42.913kN.m$$

For exterior span:

$$Interior\ (-Ve)moment = 0.7(66.0192) = 46.213kN.m$$
$$(+Ve)Moment = 0.52(66.0192) = 34.33kN.m$$
$$Exterior\ (-Ve)moment = 0.26(66.0192) = 17.165kN.m$$

Table 8.10.4.2- distribution coefficients for end span

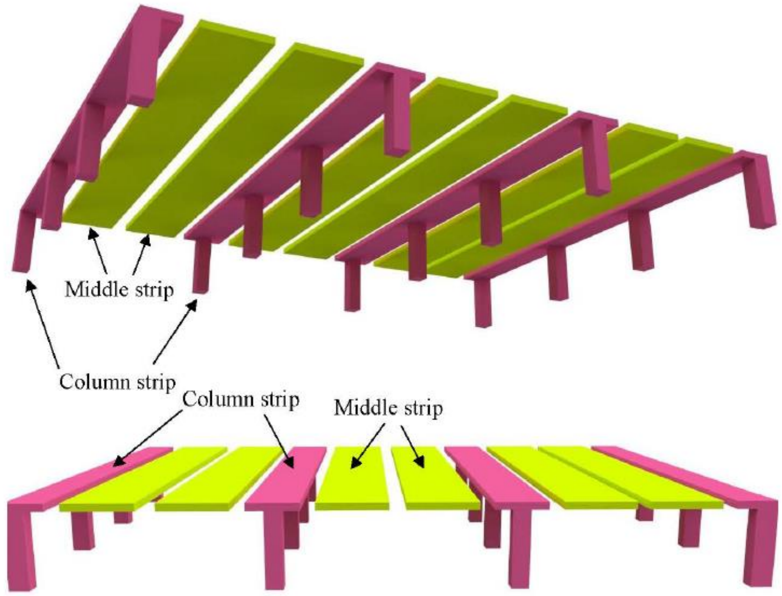
	(1)	(2)	(3)		(4)
			Slab without beams between interior supports		
	Exterior edge unrestrained	Slab with beams between all supports	Without edge beam	With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.7	0.7	0.65
Positive	0.63	0.57	0.52	0.5	0.35
Exterior Negative	0	0.16	0.26	0.3	0.65

4. LATERAL DISTRIBUTION OF LONGITUDINALMOMENT:

- Having distributed the total factored static moment (Mo) to the positive and negative moments as described previously, the designer still must distribute each of the positive and negative moment into **column strip** and **middle strip** for each design frame.
- The longitudinal moment values mentioned in the previous section are for the entire width of the equivalent building frame. This frame width is the sum of the widths of two half column strips and two half middle strips of two adjacent panels.
- The width of a column strip is taken as follows:

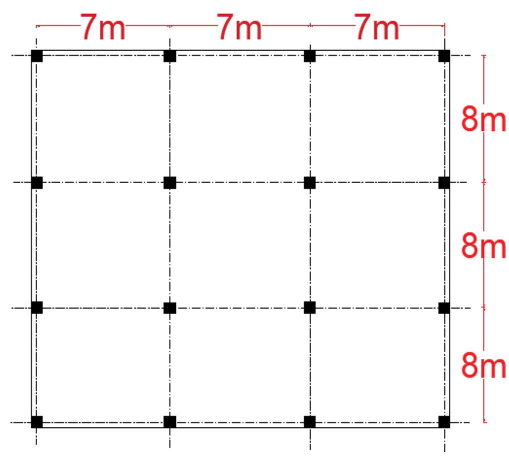
$$\boldsymbol{0.25\ l_1\ or\ 0.25\ l_2\ which\ ever\ is\ smaller.}$$

  Look at the example below.
- The lateral distribution on moments depends on the:
  - Relative stiffness of parallel beam (α_{f1})
  - The ratio of l_2/l_1
  - Torsional stiffness of the torsional member (C).
  - Relative stiffness for torsional member (β_t).



Example 4: Define the column and middle strips' width for the slab shown below for both interior and exterior longitudinal frames. The columns are square shaped having a dimension of 900mm.

Solution:



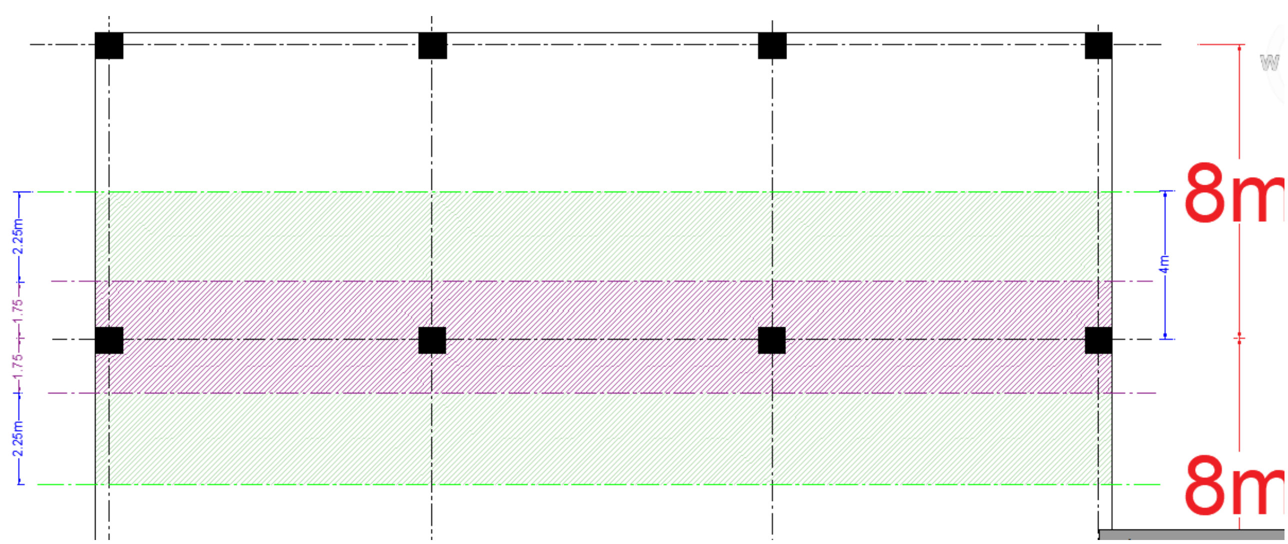
For longitudinal interior frame:

$width\ of\ column\ strip = 0.25\ l_1\ or\ 0.25\ l_2$

$= 0.25(7m) = 1.75m\ or\ 0.25(8m) = 2m.$

Use W.C. S=**1.75m** on **both sides**.

$width\ of\ half\ middle\ strip = \frac{l_2}{2} - 1.75 = 2.25m.$

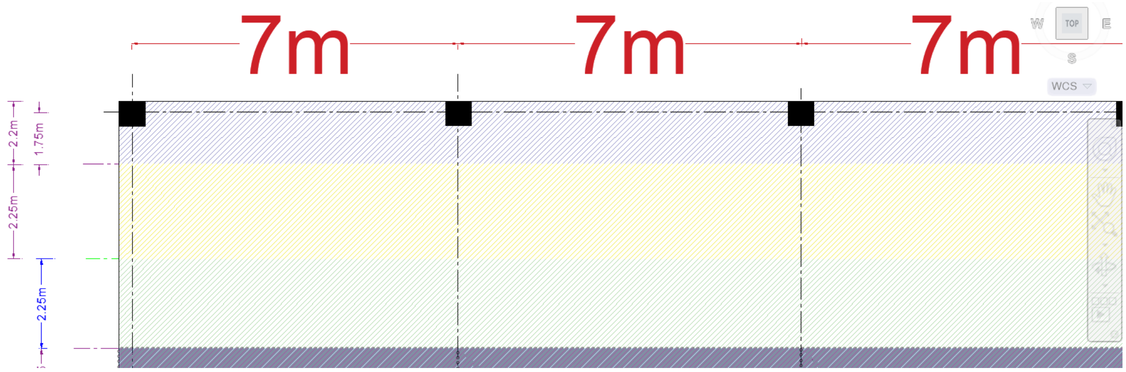


For external Longitudinal frame:

$$width\ of\ column\ strip = 0.25(7) or\ 0.25(8) + \frac{column}{2}$$

$$= 1.75 + \frac{0.9}{2} = 2.2m.$$

$$width\ of\ half\ middle\ strip = 4 + 0.45 - 2.2 = 2.25m.$$



4.1. Lateral Distribution for Negative Internal Moment:

Column strip shall be proportioned to resist the following portions in percent of interior negative factored moments ACI 8.10.5.1.

Table 8.10.5.1- Portion of Interior Negative Mu, in column strip			
$\alpha_{f1}\frac{l_2}{l_1}$	l_2/l_1		
	0.5	**1.0**	**2.0**
0	0.75	0.75	0.75
≥ 1.0	0.9	0.75	0.45

Note: the linear interpolation for transverse distribution of interior negative moment between the column strips can be based on the following equation:

$$Interior\ C.S\ coeff.\% = 75 + 30(\alpha_{f1}\frac{l_2}{l_1}) \times (1 - \frac{l_2}{l_1})$$

Where:

- $\alpha_{f1}\frac{l_2}{l_1}$: value of $\alpha_{f1} = \frac{I_b}{I_s}$ in the direction of the moment.
- $\frac{l_2}{l_1}$: centre to centre of support.
- When there is no beam in the direction of moment α_{f1} =0.
- When $\alpha_{f1}\frac{l_2}{l_1} > 1$, use 1 in the above equation.
- The common equation used to calculate negative moment at column strip at interior support is:

$$Negative\ Moment_{CS} = CS_{coefficiant} \times (-ve)\ Moment_{\ interior}$$

4.2. Lateral Distribution of Positive Moment:

Column strips shall be proportioned to resist the following portions in percent of positive factored moments

Table 8.10.5.5- portion of positive Mu, in column strip			
$\alpha_{f1}\frac{l_2}{l_1}$	l_2/l_1		
	0.5	**1.0**	**2.0**
0	0.6	0.6	0.6
≥ 1.0	0.9	0.75	0.45

Note: the linear interpolation for transverse distribution of positive moment between the column strips can be based on the following equation:

$$+ve\ C.S\ coefficiant\ \% = 60 + 30(\alpha_{f1}\frac{l_2}{l_1}) \times (1 - \frac{l_2}{l_1})$$

Where:

- $\alpha_{f1}\frac{l_2}{l_1}$: value of $\alpha_{f1} = \frac{I_b}{I_s}$ in the direction of the moment
- $\frac{l_2}{l_1}$: centre to centre of support.
- When there is no beam in the direction of moment α_{f1} =0.
- When $\alpha_{f1}\frac{l_2}{l_1} > 1$, use 1 in the above equation.
- The common equation used to calculate negative moment at column strip at interior support is:

$$(+ve)Moment\ C.S. = C.S\ coefficiant \times (+ve)\ Moment$$

$$(+ve)Moment\ M.S. = (+ve)Moment - (+ve)Moment\ C.S.$$

4.3. Lateral Distribution of Exterior Negative Moment:

Column strip shall be proportioned to resist the following portions in percent of exterior negative factored moment.

Table 8.10.5.2- portion of exterior negative Mu in column strip				
$\alpha_{f1}\frac{l_2}{l_1}$	β_t	l_2/l_1		
		0.5	**1.0**	**2.0**
0	0	1.0	1.0	1.0
	≥ 2.5	0.75	0.75	0.75
≥ 1.0	0	1.0	1.0	1.0
	≥ 2.5	0.9	0.75	0.45

Note: The linear interpolation for transverse distribution of exterior negative moment between the column strips and middle strip can be based on the following equation:

$$\boldsymbol{exterior\ C.S = 100 - 10\beta_t + 12\beta_t(\alpha_{f1}\frac{l_2}{l_1}) \times (1 - \frac{l_2}{l_1})}$$

Where:

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s}$$

$$I_s = \frac{l_2 {h_f}^3}{12}$$

$$C = \sum(1 - 0.63\frac{x}{y})\frac{x^3 y}{3}$$

- The common equation used to calculate negative moment at column strip at interior support:

$$\boldsymbol{Negative\ Moment\ Column\ strip\ exterior = CS\ coeff. \times (-ve)Moment}$$

$$\boldsymbol{(-ve)Moment\ M.S. = Moment\ exterior - (-ve)Moment\ C.S.}$$

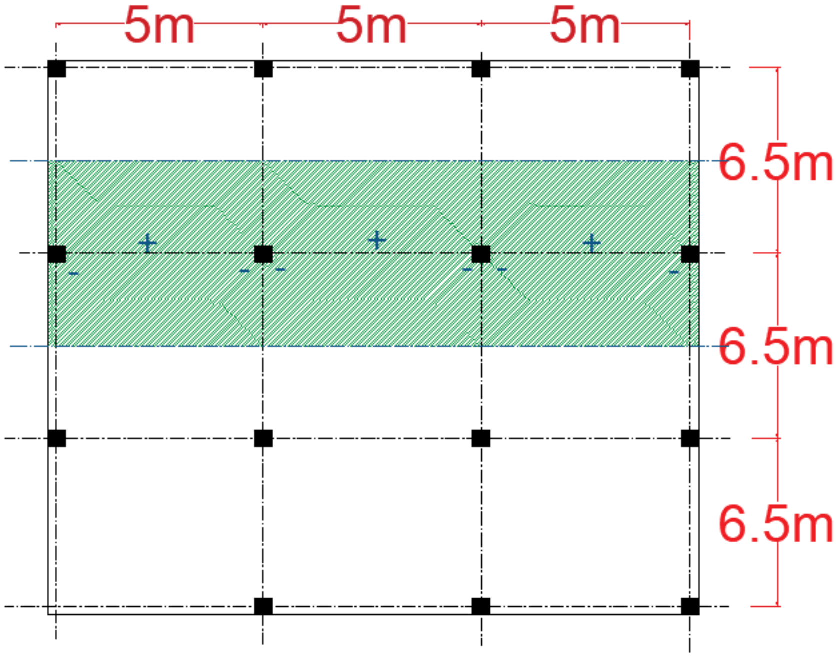
- The effect of the torsional stiffness parameter β_t is to assign all of the exterior negative factored moments to the column strip, and none to middle strip unless the beam torsional stiffness is high relative to the flexural stiffness of the supports slab.

Notes: ACI code R (8.10.5.2).

1. β_t=0 (for the flat plate without edge beam and brick wall), $\beta_t = 2.5$ (reinforced concrete wall).
2. Where walls are used as supports along column lines, they can be regarded as very stiff beams with $\boldsymbol{\alpha_{f1}\frac{l_2}{l_1} > 1}$.

EXAMPLE 5: for the longitudinal frame A of the flat plate slab shown below, by using the direct design method, find:

- Longitudinal distribution of the total static moment at factored loads.
- Lateral distribution of moment at interior and exterior panels.
  Slab thickness=200mm
  d= 180mm
  $Wu = 15.2kN/m^2$
  All columns are squared shaped with the dimension of 400mm.
  $f_c' = 25MPa, fy = 400MPa.$



SOLUTION:

1. Longitudinal distribution of the total static moment at factored loads.

$$M_o = \frac{W_u {l_n}^2 l_2}{8}$$
$$l_n = 5 - 0.4 = 4.6m \ldots or \ldots 0.65(5) = 3.25m$$
$$\therefore l_n = 4.6m$$
$$l_2 = \left(\frac{6.5}{2} + \frac{6.5}{2}\right) = 6.5m$$
$$\therefore M_o = \frac{W_u {l_n}^2 l_2}{8} = \frac{15.2 \times 4.6^2 \times 6.5}{8} = \mathbf{261.33kN/m^2}$$

For interior span:

$$(-ve)Moment = 0.65 \times 261.33 = 169.8kN.m$$
$$(+ve)\ Moment = 0.35 \times 261.33 = 91.46kN.m$$

For exterior span:

$$interior\ (-ve)M = 0.7 \times 261.33 = 183kN.m$$
$$(+ve)M = 0.35 \times 261.33 = 135.88kN.m$$
$$exterior\ (-ve)M = 0.26 \times 261.33 = 68kN.m$$

Table 8.10.4.2- distribution coefficients for end span

	(1)	(2)	(3)		(4)
			Slab without beams between interior supports		
	Exterior edge unrestrained	Slab with beams between all supports	Without edge beam	With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.7	0.7	0.65
Positive	0.63	0.57	0.52	0.5	0.35
Exterior Negative	0	0.16	0.26	0.3	0.65

2. Lateral Distribution of Moment at Interior and Exterior Panels.

For interior Panels:

Negative Moment:

$Total\ negative\ moment = 169.8kN.m$

$\alpha_{f1} = 0\ (No\ beam\ in\ the\ direction\ of\ moment)$

$C.S\%\ coeff. = 75\%$

$(-ve)M\ C.S = 0.75 \times 169.8 = 127.35kN.m$

$(-ve)M\ M.S = 169.8 - 127.35 = 42.45kN.m$

Positive Moment:

$Total\ positive\ moment = 91.46kN.m$

$\alpha_{f1} = 0\ (No\ beam\ in\ the\ direction\ of\ moment)$

$(+ve)M\ C.S = 0.6 \times 91.46 = 54.88kN.m$

$(+ve)M\ M.S = 91.46 - 54.88 = 36.58kN.m$

For end (exterior) Panel:

- Total negative interior moment= 183kN.m
  $(-ve)at\ \ column\ strip = 0.75 \times 183 = 137.25kN.m$
  $(-ve)Moment\ at\ Middle\ strip = 183 - 137.25 = 45.75kN.m$

- Total positive moment= 135.88kN.m
  $(+ve)Moment\ at\ Column\ strip = 0.6 \times 135.88 = 81.53kN.m$
  $(+ve)Moment\ at\ Middle\ Strip = 135.88 - 81.53 = 54.35kN.m$

- Total negative exterior moment= 68kN.m
  $\beta_t = 0\ (No\ edge\ beam)$
  $\alpha_{f1} = 0\ (No\ beam\ in\ the\ direction\ of\ moment)$
  $Column\ Strip\ coeff. = 100\%$
  $(-ve)\ Moment\ at\ Column\ Strip = 1 \times 68 = 68kN.m$
  $(-ve)Moment\ at\ Middle\ Strip = 68 - 68 = 0kN.m$