* Scalar product (Dot_product) :-

Let $\vec{A} = a_1 i + a_2 j + a_3 k$ and $\vec{B} = b_1 i + b_2 j + b_3 k$

* $\boxed{\vec{A} \cdot \vec{B} = |A| \cdot |B| \cdot \cos\theta}$

where θ is the angle between $\vec{A}$ and $\vec{B}$

Properties :-

1- $\vec{A} \cdot \vec{A} = |A|^2$

2- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = a_1 b_1 + a_2 b_2 + a_3 b_3$

3- $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A| \cdot |B|} = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \cdot \sqrt{b_1^2 + b_2^2 + b_3^2}}$

4- $\vec{A} \perp \vec{B} \Leftrightarrow \vec{A} \cdot \vec{B} = 0$ [Orthogonal vectors]

5- $ai + bj \perp bi - aj$

6- projection :-

a- Scalar projection :-

$$|C| = \text{Proj}_{\vec{B}} \vec{A} = \frac{\vec{A} \cdot \vec{B}}{|B|}$$

b- Vector projection :-

$$\vec{C} = \vec{\text{Proj}}_{\vec{B}} \vec{A} = \left(\frac{A \cdot B}{B \cdot B}\right) \vec{B}$$

EX(1):- Find the angle θ between $\vec{A} = i - 2j - 2K$ and $\vec{B} = 6i + 3j + 2K$?

Answer:-

$$A \cdot B = (1)(6) + (-2)(3) + (-2)(2) = 6 - 6 - 4 = \boxed{-4}$$

$$|A| = \sqrt{(1)^2 + (-2)^2 + (-2)^2} = \sqrt{9} = \boxed{3}$$

$$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2} = \sqrt{49} = \boxed{7}$$

$$|A| \cdot |B| = 3 * 7 = \boxed{21}$$

$$\therefore \cos\theta = \frac{A \cdot B}{|A| \cdot |B|} = \frac{-4}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

$$\therefore \boxed{\theta \approx 101^\circ}$$

EX(2):- Show that the two vectors $\vec{A} = 3i - 2j + K$ and $\vec{B} = 2j + 4K$ are orthogonal ?

Answer:-

$$A \cdot B = (3)(0) + (-2)(2) + (1)(4)$$
$$= 0 - 4 + 4 = 0$$

$$\because A \cdot B = 0$$

$\therefore$ The two vectors $\vec{A}$ and $\vec{B}$ are orthogonal

EX(3):- Find the vector projection of $(\vec{B} = 6i + 3j + 2k)$ onto $(\vec{A} = i - 2j - 2k)$ and the scalar component of $\vec{B}$ in the direction of $\vec{A}$?

Answer:- The vector projection is

$$\text{Proj}_{\vec{A}} \vec{B} = \left(\frac{A \cdot B}{A \cdot A}\right) \vec{A}$$

$$= \left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{(1)^2 + (-2)^2 + (-2)^2}\right)(i - 2j - 2k)$$

$$= \frac{6 - 6 - 4}{1 + 4 + 4}(i - 2j - 2k)$$

$$= \frac{-4}{9}(i - 2j - 2k)$$

$$= \boxed{\frac{-4}{9} i + \frac{8}{9} j + \frac{8}{9} k}$$

and the scalar component is

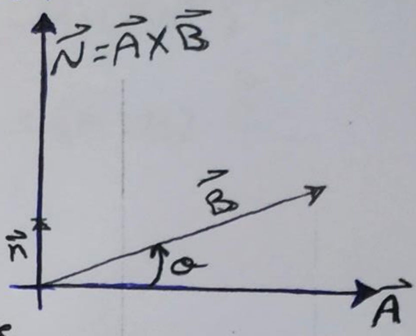
$$\text{Proj}_{\vec{A}} \vec{B} = \frac{\vec{A} \cdot \vec{B}}{|A|}$$

$$= \frac{(1)(6) + (-2)(3) + (-2)(2)}{\sqrt{(1)^2 + (-2)^2 + (-2)^2}}$$

$$= \frac{-4}{\sqrt{9}} = \boxed{\frac{-4}{3}}$$

Vector Product (Cross Product):-

$$* \ \boxed{\vec{N} = \vec{A} \times \vec{B} = \vec{n}\,|A||B| \sin\theta}$$



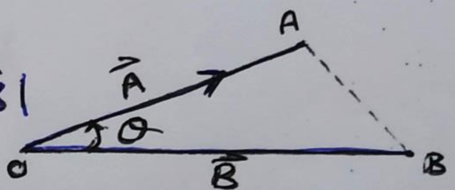
where $\vec{n}$ is the normal unit vector normal to $\vec{A}$ and $\vec{B}$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

where,
$\vec{A} = a_1 i + a_2 j + a_3 k$
$\vec{B} = b_1 i + b_2 j + b_3 k$

Properties:-

1. $\vec{A} \times \vec{A} = 0$
2. $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
3. $\vec{A} \parallel \vec{B} \Leftrightarrow \vec{A} \times \vec{B} = 0$
4. Area of $\triangle OAB = \frac{1}{2} |\vec{A} \times \vec{B}|$



Ex(1):- Find $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$ if $(\vec{A} = 2i + j + k)$ and $(\vec{B} = -4i + 3j + k)$?

Answer:-

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 1 \\ -4 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ -4 & 3 \end{vmatrix} k$$

$$= [(1*1) - (3*1)]\,i - [(2*1) - (-4*1)]\,j + [(2*3) - (-4*1)]\,k$$

$$\boxed{\vec{A} \times \vec{B} = -2i - 6j + 10k} \text{ but } \boxed{\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} = 2i + 6j - 10k}$$

$\boxed{9}$

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Triple Product:-

(1) Scalar triple product $(\vec{A},\vec{B},\vec{C}) = \vec{A}\cdot(\vec{B}\times\vec{C})$

$$= (\vec{A}\times\vec{B})\cdot\vec{C}$$

Note:- Volume of box is

$$\boxed{V = |\vec{A}\cdot\vec{B}\times\vec{C}|}$$

Note: Volume of pyramid (Tetrahedron)

$$\boxed{V = \frac{1}{6}|\vec{A}\cdot\vec{B}\times\vec{C}|}$$

(2) Vector triple product

$$\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$$

Note:- $i\cdot i = j\cdot j = k\cdot k = 1$

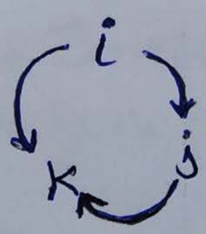
$i\times i = j\times j = k\times k = 0$

$i\cdot j = j\cdot k = k\cdot i = 0$

$i\times j = k$

$j\times k = i$

$k\times i = j$



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H.W No.1

(1)- Find the length and direction of each vector and the angle it makes with the positive X-axis

(a)/ $i + j$ (b)/ $\sqrt{3}\, i + j$ (c)/ $5i + 12j$

(2)- Find a unit vector in the direction of the vector from $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

(3)- Find a vector 6 units long in the direction of $\vec{A} = 2i + 2j - K$.

(4)- Find the length and direction of $A \times B$ and $B \times A$

(a) $\vec{A} = 2i - 2j - K$ $\vec{B} = i + j + K$

(b) $\vec{A} = 2i$ $\vec{B} = -3j$

(5)- Find the area of the tringle whose vertices are $A(1, -1, 0)$; $B(2, 1, -1)$ and $C(-1, 1, 2)$

(6)- If $\vec{A} = 2i - j$ and $\vec{B} = i + 3j - 2K$, find $A \times B$ then calculate $(A \times B) \cdot A$

(7)- Let $\vec{A} = 5i - j + k$; $B = j - 5K$ and $\vec{C} = -15i + 3j - 3K$ which pairs of vector are (a) Perpendicular (b) Parallel ?