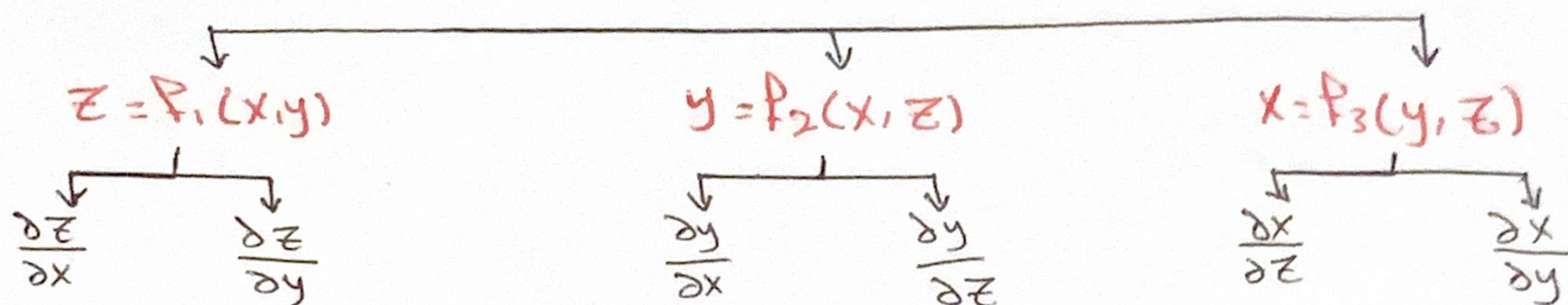


Function of Several Variables

(5)

The equation of the surface is \Rightarrow

$$F(x, y, z) = 0$$



The case where $z = F(x, y)$ it regards

x, y : is independent variable

z : is a function (z is a dependent variable)

① The first partial derivatives are write \Rightarrow

$$\frac{\partial z}{\partial x} = \frac{\partial F}{\partial x} = z_x = F_x = \lim_{\Delta x \rightarrow 0} \frac{F(x + \Delta x, y) - F(x, y)}{\Delta x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial F}{\partial y} = z_y = F_y = \lim_{\Delta y \rightarrow 0} \frac{F(x, y + \Delta y) - F(x, y)}{\Delta y}$$

② The second partial derivatives

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} (z_x) = z_{xx} = F_{xx}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} (z_y) = z_{yy} = F_{yy}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} (z_x) = z_{yx} = F_{yx}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial x} (z_y) = z_{xy} = F_{xy}$$

Ex: If $z = f(x, y) = xy^2 + x^2y + 3$ find f_x, f_y
 f_{xx}, f_{yy}

$$f_x = y^2 + 2xy, \quad f_{xx} = 2y$$

$$f_y = 2yx + x^2, \quad f_{yy} = 2x$$

Ex: If $z = e^{2x} \cos y$ find f_x, f_y

$$f_x = 2e^{2x} \cos y$$

$$f_y = -e^{2x} \sin y$$

Ex: If $z = x^3 + y^4 + x \sin y + y \cos x$ find
 $f_x, f_{xy}, f_y, f_{yx}, f_{xx}, f_{yy}$

$$f_x = 3x + \sin y - y \cos x$$

$$f_{xx} = 3 + y \sin x$$

$$f_{xy} = \cos y - \sin x$$

$$f_y = 4y^3 + x \cos y + \cos x$$

$$f_{yy} = 12y^2 + x \sin y$$

$$f_{yx} = \cos y - \sin x$$

Ex: $z = \sin(x^2y)$ find f_x, f_{xy}, f_y, f_{yx}
 f_{xx}, f_{yy}

$$f_x = 2xy \cdot \cos x^2y$$

$$f_{xx} = -2xy \cdot \sin x^2y \cdot 2xy + 2y \cos x^2y$$
$$= 2y [-x^2y \cdot \sin x^2y + \cos x^2y]$$

$$f_{xy} = -2xy \cdot \sin x^2y \cdot x^2 + 2x \cos x^2y$$
$$= 2x [-x^2y \sin x^2y + \cos x^2y]$$

$$f_y = x^2 \cos(x^2 y)$$

$$f_{yy} = -x^2 \cdot \sin(x^2 y) \cdot x^2 = -x^4 \sin(x^2 y)$$

$$\begin{aligned} f_{yx} &= -x^2 \cdot \sin(x^2 y) \cdot 2xy + 2x \cos(x^2 y) \\ &= 2x [-x^2 y \sin(x^2 y) + \cos(x^2 y)] \end{aligned}$$

$$\therefore f_{yx} = f_{xy}$$