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Function of Several Variables

The equation of the surface is :

$$F(x,y,z)=0$$

$$z=f_1(x,y) \qquad y=f_2(x,z) \qquad x=f_3(y,z)$$

$$\frac{\partial z}{\partial x} \quad \frac{\partial z}{\partial y} \qquad \frac{\partial y}{\partial x} \quad \frac{\partial y}{\partial z} \qquad \frac{\partial x}{\partial z} \quad \frac{\partial x}{\partial y}$$

The case where $z=f(x,y)$ it regards

x,y : is independent variable

z : is a function (z is a dependent variable)

① The first partial derivatives are write :

$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}=z_x=f_x=\lim_{\Delta x\to 0}\frac{f(x+\Delta x,y)-f(x,y)}{\Delta x}$$

$$\frac{\partial z}{\partial y}=\frac{\partial f}{\partial y}=z_y=f_y=\lim_{\Delta y\to 0}\frac{f(x,y+\Delta y)-f(x,y)}{\Delta y}$$

② The second partial derivatives

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}(z_x)=z_{xx}=f_{xx}$$

$$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}(z_y)=z_{yy}=f_{yy}$$

$$\frac{\partial^2 z}{\partial x\,\partial y}=\frac{\partial}{\partial y}(z_x)=z_{yx}=f_{yx}$$

$$\frac{\partial^2 z}{\partial y\,\partial x}=\frac{\partial}{\partial x}(z_y)=z_{xy}=f_{xy}$$

Ex: If $Z = f(x,y) = xy^2 + x^2y + 3$ find f_x, f_y, f_{xx}, f_{yy}

$f_x = y^2 + 2xy$, $f_{xx} = 2y$

$f_y = 2yx + x^2$, $f_{yy} = 2x$

Ex: If $Z = e^{2x} \cos y$ find f_x, f_y

$f_x = 2e^{2x} \cos y$

$f_y = -e^{2x} \sin y$

Ex: If $Z = x^3 + y^4 + x\sin y + y\cos x$ find $f_x, f_{xy}, f_y, f_{yx}, f_{xx}, f_{yy}$

$f_x = 3x + \sin y - y\cos x$

$f_{xx} = 3 + y\sin x$

$f_{xy} = \cos y - \sin x$

$f_y = 4y^3 + x\cos y + \cos x$

$f_{yy} = 12y^2 - x\sin y$

$f_{yx} = \cos y - \sin x$

Ex: $Z = \sin(x^2 y)$ find $f_x, f_{xy}, f_y, f_{yx}, f_{xx}, f_{yy}$

$f_x = 2xy \cdot \cos x^2y$

$f_{xx} = -2xy \cdot \sin x^2y \cdot 2xy + 2y \cos x^2y$

$= 2y\,[-x^2y \cdot \sin x^2y + \cos x^2y]$

$f_{xy} = -2xy \cdot \sin x^2y \cdot x^2 + 2x\cos x^2y$

$= 2x\,[-x^2y \sin x^2y + \cos x^2y]$

$f_y = x^2 \cos(x^2y)$

$f_{yy} = -x^2 * \sin x^2y * x^2 = -x^4 \sin x^2y$

$f_{yx} = -x^2 \cdot \sin x^2y \cdot 2xy + 2x \cos x^2y$

$= 2x[-x^2y \sin x^2y + \cos x^2y]$

$\therefore f_{yx} = f_{xy}$