CHAPTER THREE

One-Dimensional, Steady-State Conduction

3.1 The Plane Wall

Consider the plane wall of Figure (3.1) where a direct application of Fourier's law may be made. Integration yields

$$q_x = -kA\frac{dT}{dx} = -\frac{kA}{\Delta x}(T_{s,2} - T_{s,1}) \quad (3.1)$$

$$q_x = \frac{kA}{L}(T_{s1} - T_{s2}) = \frac{T_{s1} - T_{s2}}{R_{t,cond.}} \quad (3.2)$$

Thermal resistance for conduction is

$$R_{t,cond.} = \frac{\Delta x}{kA} = \frac{L}{kA} \quad (3.3)$$

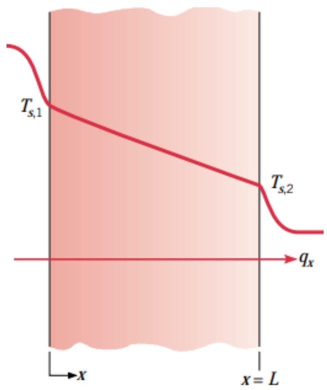


Figure (3.1) Heat Transfer through a Plane Wall.

The equivalent thermal circuit for the plane wall with convection surface conditions is shown in Figure (3.2). The heat transfer rate may be determined from separate consideration of each element in the network. Since (q_x) is constant throughout the network, it follows that

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$

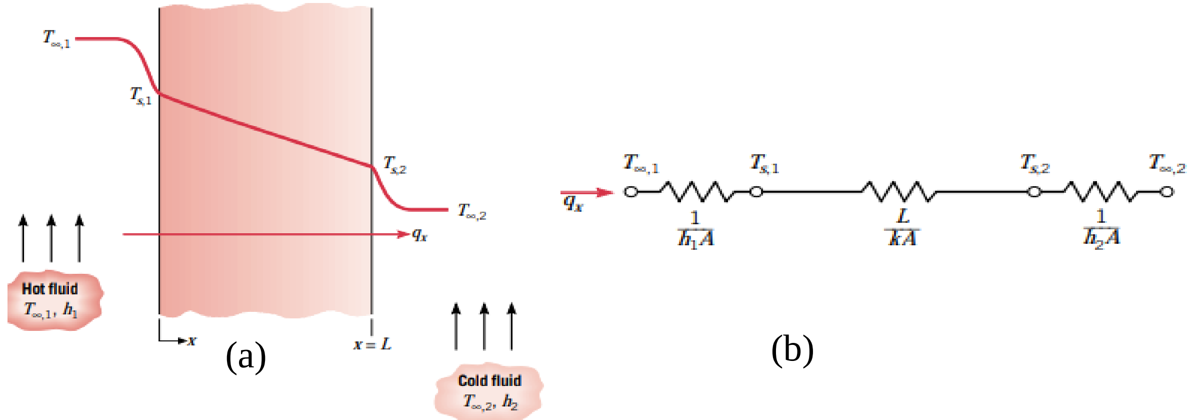


Figure (3.2) Heat Transfer through a Plane Wall. (a) Temperature Distribution. (b) Equivalent Thermal Circuit

The thermal resistance for convection is

$$R_{t,conv.} = \frac{1}{hA} \qquad (3.4)$$

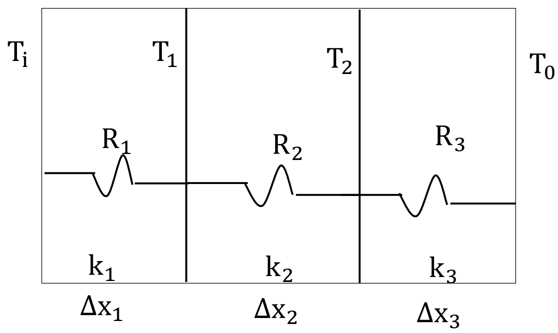
In terms of the overall temperature difference $(T_{\infty,1} - T_{\infty,2})$, and the total thermal resistance (R_{tot}) the heat transfer rate may also be expressed as

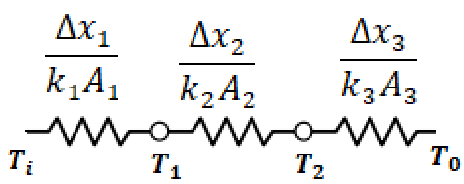
$$q_x = \frac{(T_{\infty,1} - T_{\infty,2})}{R_{tot}}$$

$$R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

3.2 The Composite Wall

As shown in the figure below there are three walls in series with each other a situation which is similar to an electrical circuit consisting of three series resistors and battery across them.





The total resistance through the Composite wall is given by

$$\sum R_{tot} = R_1 + R_2 + R_3 = \frac{\Delta x_1}{k_1 A_1} + \frac{\Delta x_2}{k_2 A_2} + \frac{\Delta x_3}{k_3 A_3}$$

And gives a total heat flow (q) is

$$q = \frac{\Delta T_{overall}}{\sum R_{tot}} = \frac{T_i - T_0}{\Delta x_1/k_1 A_1 + \Delta x_2/k_2 A_2 + \Delta x_3/k_3 A_3} = UA\Delta T$$

Where U is the overall heat transfer coefficient

$$U = \frac{1}{\Delta x_1/k_1 + \Delta x_2/k_2 + \Delta x_3/k_3}$$

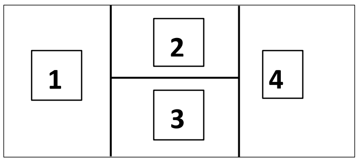
$$UA = \frac{1}{\sum R_{tot}} \qquad (3.5)$$

Noting that the heat flow across the first wall is equal to the second and third walls.

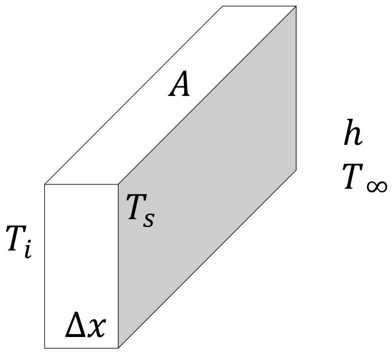
$$q = \frac{T_i - T_1}{\Delta x_1/k_1 A_1} = \frac{T_1 - T_2}{\Delta x_2/k_2 A_2} = \frac{T_2 - T_0}{\Delta x_3/k_3 A_3}$$

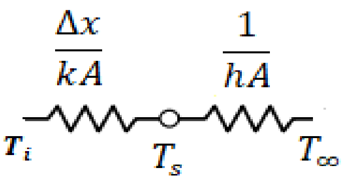
There are many other connect of thermal resistance. For example, the composite wall as shown below the overall resistance is:

$$\sum R_{th} = R_1 + \left[\frac{1}{R_2} + \frac{1}{R_3}\right]^{-1} + R_4$$



And when the case is a combination of conduction and convection as shown in the figure below the electrical analogy will be

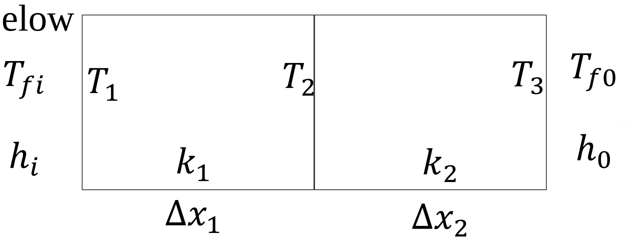


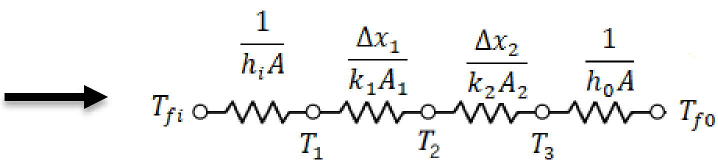


and the heat flow (q) is

$$q = \frac{T_i - T_\infty}{\Delta x/kA + 1/hA}$$

The general case of a combination of conduction and convection as shown in the figure below





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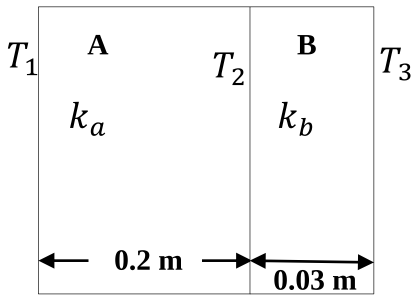
$$q=\frac{\Delta T_{overall}}{\sum R_{tot}}=\frac{T_{fi}-T_{f0}}{\sum R_{tot}}, \qquad \sum R_{th}=\frac{1}{h_iA}+\frac{\Delta x_1}{k_1A_1}+\frac{\Delta x_2}{k_2A_2}+\frac{1}{h_0A}$$

Example (3.1): A laboratory furnace wall is constructed of (0.2 m) thick fireclay with ($k_a = 1\ W/m.K$) this is covered on the outer surface with (0.03 m) thick layer of insulation material having ($k_b = 0.07\ W/m.K$) the furnace inner brick surface is at (1250 K) and the outer surface of the insulation is (310 K). Calculate the steady-state heat transfer rate through the wall in W/m^2, and determined the interfacial temperature (T_2) between the brick and insulation.

Solution:



$$\frac{q}{A}=\frac{T_1-T_3}{\Delta x_a/k_a+\Delta x_b/k_b}$$

$$\frac{q}{A}=\frac{1250-310}{0.2/1+0.03/0.07}=1495\ W/m^2$$

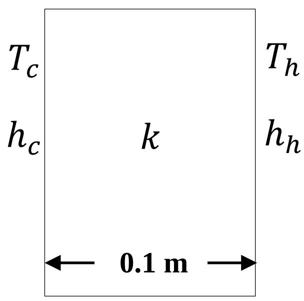
$$\frac{q}{A}=\frac{T_1-T_2}{\Delta x_a/k_a}$$

$$1495=\frac{1250-T_2}{0.2/1}$$

$$T_2=951\ K$$

Example (3.2): A (0.1 m) thick brick wall (k=0.7 W/m. K) is exposed to a cold wind at (270 K) through a convection heat transfer coefficient of (40 $W/m^2.K$) on the other side is air at (330 K) with a natural convection heat transfer coefficient of (10 $W/m^2.K$). Calculate the rate of heat transfer per unit area.

Solution:



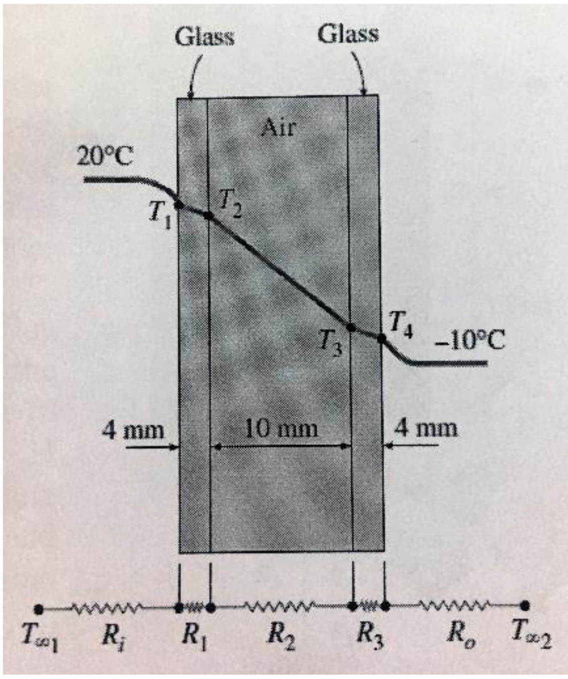
$$\frac{q}{A}=\frac{\Delta T}{1/h_h+\Delta x/k+1/h_c}$$

$$\frac{q}{A}=\frac{330-270}{1/10+0.1/0.7+1/40}$$

$$\frac{q}{A}=223.9\ W/m^2$$

Example (3.3): Consider a (0.8 m) high and (1.5 m) wide double pane window consisting of two (4 mm) thick layer of glass ($k = 0.78\ W/m.°C$) separated by (10 mm) wide stagnant air space ($k = 0.026\ W/m.°C$). Determined the steady rate of heat transfer through this double pane window and temperature of its inner surface for a day during which the room is maintained at (20 °C) while the temperature of outdoor is (-10 °C) take the convection heat transfer coefficient of the inner and outer surface of the window to be ($h_i = 10\ W/m^2.°C$), ($h_0 = 40\ W/m^2.°C$).

Solution:



$$A = 0.8 * 1.5 = 1.2\ m^2$$

$$R_i = \frac{1}{h_i A} = \frac{1}{10 * 1.2} = 0.083\ °C/W$$

$$R_1 = R_3 = R_{glass} = \frac{L_1}{k_1 A} = \frac{0.004}{0.78 * 1.2} = 0.00427\ °C/W$$

$$R_2 = R_{air} = \frac{L_2}{k_2 A} = \frac{0.01}{0.026 * 1.2} = 0.3205\ °C/W$$

$$R_0 = \frac{1}{h_0 A} = \frac{1}{40 * 1.2} = 0.02083\ °C/W$$

$$R_{total} = R_i + R_1 + R_2 + R_3 + R_0$$

$$R_{total} = 0.083 + 0.00427 + 0.3205 + 0.00427 + 0.02083$$

$$R_{total} = 0.4332\ °C/W$$

The steady rate of heat transfer through the double-pane window is:

$$q = \frac{\Delta T_{overall}}{R_{total}} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{20 - (-10)}{0.4332} = 69.2\ W$$

The inner temperature is:

$$q = \frac{T_{\infty 1} - T_1}{R_i}$$

$$69.2 = \frac{20 - T_1}{0.083}$$

$$T_1 = 14.2\ °C$$