Heat generation & Thermal Resistance

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Review

In order to start this lecture, it is important to recall the general heat equations for a plane wall, cylinder, and sphere, respectively.

Plane wall, Cartesian

$$\frac{\partial^2 T}{\partial x^2}k + \frac{\partial^2 T}{\partial y^2}k + \frac{\partial^2 T}{\partial z^2}k + \dot{q} = \rho c \frac{\partial T}{\partial t}$$

Cylindrical

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(kr\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

Spherical

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k \sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$$

Considering: One dimensional heat flow (x-direction),
steady-state (no change with time),
With heat generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{g}}{k} = 0$$

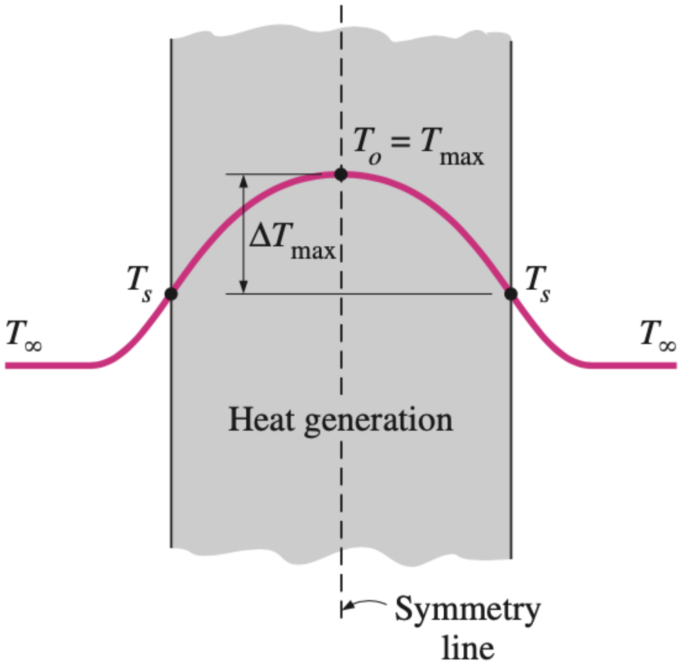
$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

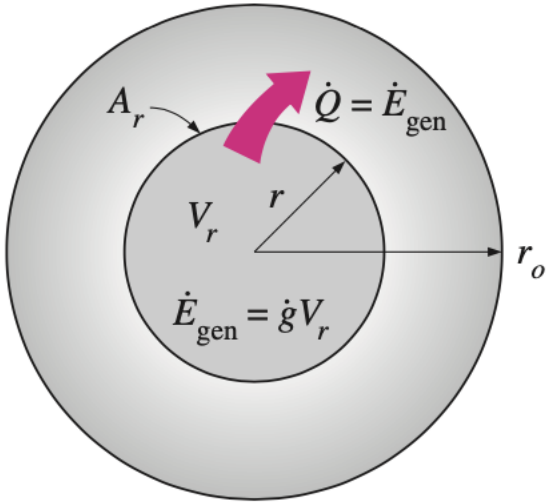
$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) + \frac{\dot{g}}{k} = 0$$

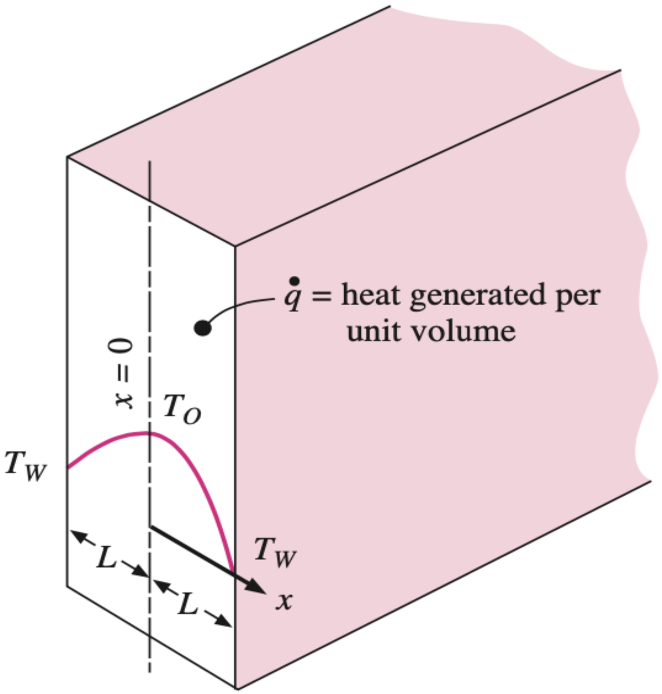
HEAT GENERATION IN A SOLID

Heat generation is usually expressed *per unit volume* of the medium and is denoted by $\dot{Q}$ *or* $\dot{q}$, whose unit is W/m^3. Some examples of heat generation are *resistance heating* in wires, exothermic *chemical reactions* in a solid, and *nuclear reactions* in nuclear fuel rods where electrical, chemical, and nuclear energies are converted to heat, respectively

$$q_{gen} = \dot{q} * Volume. \quad \text{W}$$





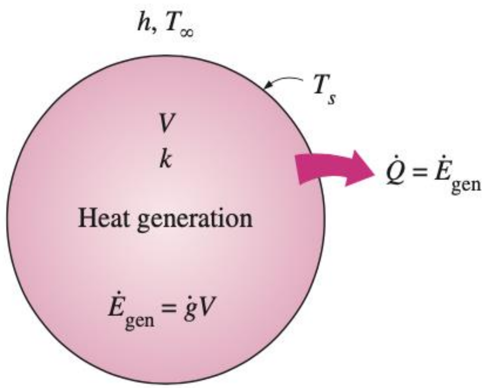


HEAT GENERATION IN A SOLID

Consider a solid medium of surface area A_s, volume V, and constant thermal conductivity k, where heat is generated at a constant rate. Heat is transferred from the solid to the surrounding medium at T_∞ , with a constant heat transfer coefficient of h. All the surfaces of the solid are maintained at a common temperature T_s. Under *steady* conditions, the energy balance for this solid can be expressed as

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat transfer}\\ \text{from the solid}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{energy generation}\\ \text{within the solid}\end{pmatrix}$$

$$\dot{Q} = \dot{g}V \qquad \text{(W)}$$

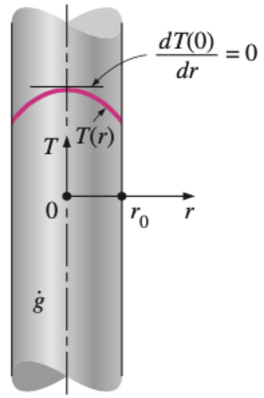


Disregarding radiation, the heat transfer rate can also be expressed from Newton's law of cooling as

$$\dot{Q} = hA_s\,(T_s - T_\infty) \qquad \text{(W)}$$

$$T_s = T_\infty + \frac{\dot{g}V}{hA_s}$$

For a large *plane wall* of thickness $2L$ ($A_s = 2A_{wall}$ and $V = 2LA_{wall}$), a long solid *cylinder* of radius r_o ($A_s = 2\pi r_o L$ and $V = \pi r_o^2 L$), and a solid *sphere* of radius r_0 ($A_s = 4\pi r_o^2$ and $V = 4/3\,\pi r_o^3$), Equation reduces to

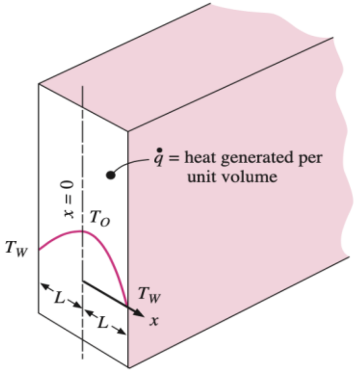


$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{g}L}{h}$$

$$T_{s,\text{ cylinder}} = T_\infty + \frac{\dot{g}r_o}{2h}$$

$$T_{s,\text{ sphere}} = T_\infty + \frac{\dot{g}r_o}{3h}$$

Only In case of:
Heat generation = Convection



HEAT GENERATION IN A SOLID

- Plane Wall with Heat generation:

The general differential equation that governs the heat flow is: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho c}{k}\frac{\partial T}{\partial t}$

Applying double integration:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

One dimensional, steady-state, with heat generation

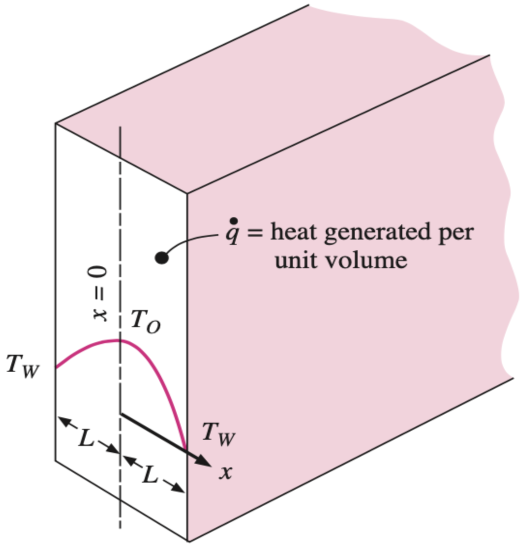
$$\int \frac{d^2T}{dx^2} = -\int \frac{q^{\cdot}}{k}$$

Boundary conditions:

At center , maximum curve point

$X = 0$, $\frac{dT}{dx} = 0$ ⟶ $\frac{dT}{dx} = -\frac{q^{\cdot}}{k}x + C_1$ ⟶ $C_1 = 0$

$X = 0$, $T = T_o$ ⟶ $T = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2$ ⟶ $C_2 = To$



At wall edge,

$X = L$, $T = T_w$ ⟶ $T = -\frac{q^{\cdot}x^2}{2k} + T_o$

Plane wall with heat generation Temperature distribution equation

$$T_o = \frac{q^{\cdot}L^2}{2k} + T_w$$

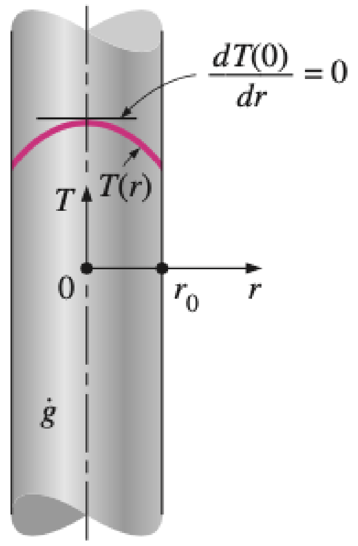
Plane wall heat generation center point Temperature equation

HEAT GENERATION IN A SOLID

CYLINDER WITH HEAT SOURCES: $\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(kr\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{g} = \rho C\frac{\partial T}{\partial t}$ general differential equation

One dimensional, steady-state, with heat generation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{q^{\cdot}}{k} = 0$$



$$\int \frac{d}{dr}\left(r\frac{dT}{dr}\right) = \int -\frac{q^{\cdot}}{k}r$$

<u>Boundary Conditions:</u>

At center , maximum curve point

r = 0 , $\frac{dT}{dr} = 0$

$$r\frac{dT}{dr} = -\frac{q^{\cdot}r^2}{2k} + C_1 \longrightarrow C_1 = 0$$

$$\int \frac{dT}{dr} = \int -\frac{q^{\cdot}r}{2k}$$

At wall edge,

r= r_o, T = T_s

$$T = -\frac{q^{\cdot}r^2}{4k} + C_2 \longrightarrow C_2 = T_s + \frac{q^{\cdot}r^2}{4k}$$

$$T = T_s + \frac{q^{\cdot}}{4k}(r_o^2 - r^2) \longrightarrow$$ Cylinder with heat generation Temperature distribution equation

At center point r = 0, $T(o) = T_s + \frac{q^{\cdot}}{4k}r_o^2$ Cylinder heat generation center point Temperature equation

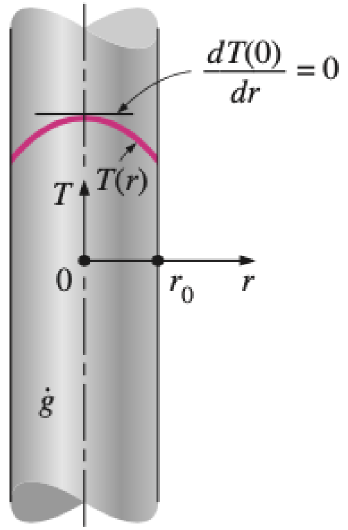
Example:

A long homogeneous resistance wire of radius r_0 = 0.005 m. and thermal conductivity k = 15 w/m.ºc is being used to boil water at atmospheric pressure by the passage of electric current. Heat is generated in the wire uniformly as a result of resistance heating at a rate of $g = 0.318 \times 10^9$ w/m^3 · in .Determine the temperature at the centerline of the wire when steady operating conditions are reached.

Solution:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{q^{\cdot}}{k}=0$$

$$T = T_s + \frac{q^{\cdot}}{4k}(r_o^2 - r^2)$$



The temperature at the centerline ($r = 0$) is obtained by replacing r in Eq. by zero and substituting the known quantities,

$$T(o) = T_s + \frac{q^{\cdot}}{4k}r_o^2$$

$$T(0) = T_s + \frac{\dot{g}}{4k}(r_o^2) = 105 + \frac{0.318x10^9}{4x15}(0.005)^2 \longrightarrow T(0) = 237.5\ ºC$$

Thermal Resistance

- **Thermal resistance:** refers to the ability of a system of particular configuration to resist heat transfer in any mode.

❖ Thermal resistance concept extracted from the concept of Ohm's law in electricity, $\boldsymbol{I = \frac{V}{R}}$.

Where,

$I = q$

$V = \Delta T$

$$q = \frac{\Delta T}{R_{th}}$$

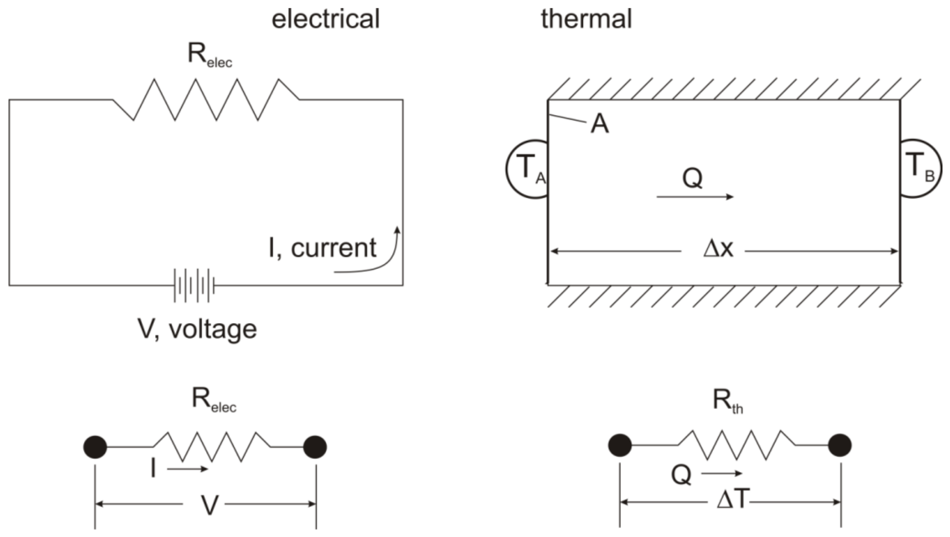
For conduction, $\boldsymbol{q = -kA\frac{\Delta T}{L}}$, $\boldsymbol{R_{th} = \frac{L}{kA}}$

For convection, $\boldsymbol{q = hA(T_s - T_\infty)}$, $\boldsymbol{R_{th} = \frac{1}{hA}}$

For radiation, $\boldsymbol{q = h_r A(T_s - T_{sur})}$, $\boldsymbol{R_{th} = \frac{1}{h_r A}}$

Where,

$h_r = \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$



Plane Wall Thermal Resistance Network

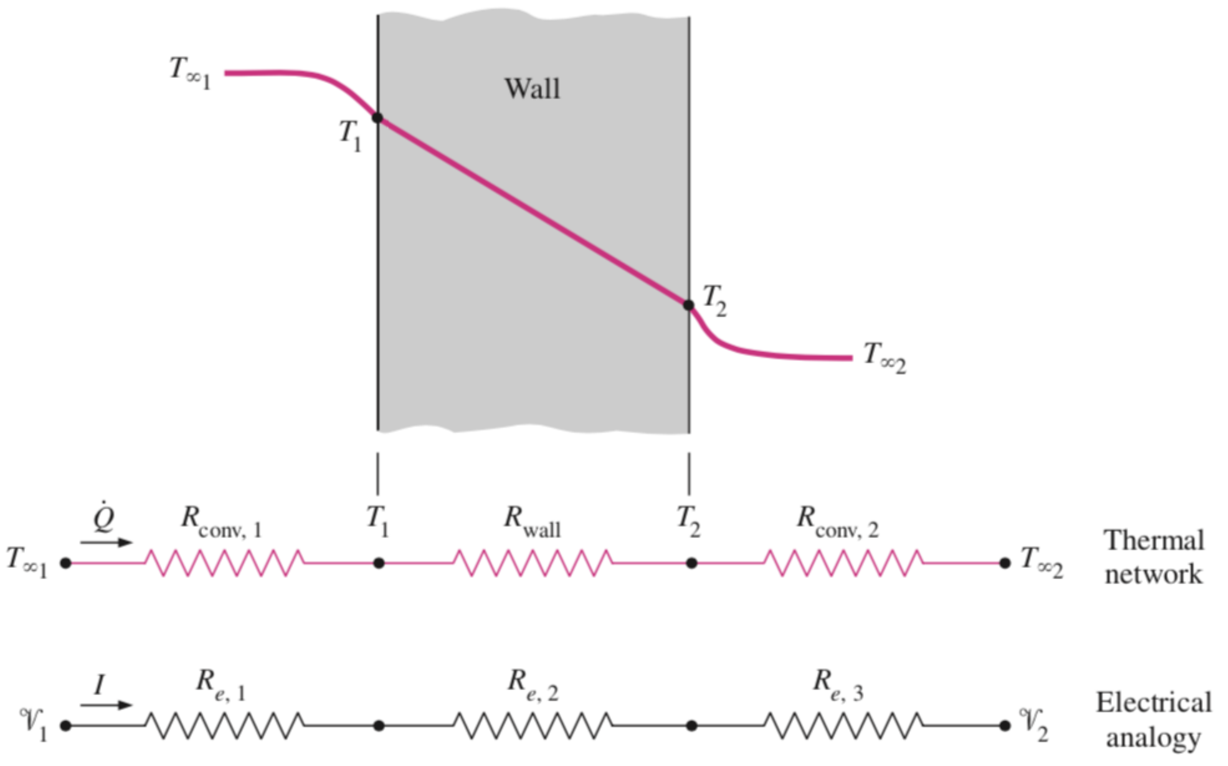
Conduction thermal resistance

$$\dot{Q} = \frac{T_1 - T_2}{R}$$

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad R_{\text{wall}} = \frac{L}{kA}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}}}$$

$$I = \frac{V_1 - V_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$



(Rate of *heat convection* into the wall) = (Rate of *heat conduction* through the wall) = (Rate of *heat convection* from the wall)

Convection thermal resistance

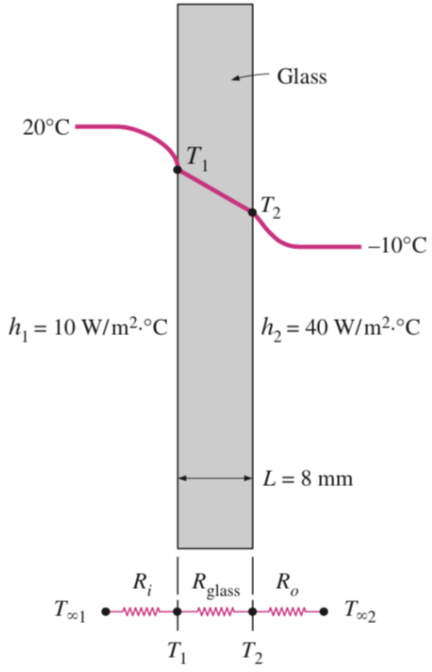
$$\dot{Q}_{\text{conv}} = \frac{T_s - T_\infty}{R_{\text{conv}}}$$

$$R_{\text{conv}} = \frac{1}{hA_s}$$

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{wall}} + R_{\text{conv, 2}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°\text{C/W})$$

Heat Loss Thought a Window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is = 10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h1$ = 10W/m2 ·°C and $h2$ = 40W/m2 ·°C, which includes the effects of radiation.



Solution:

This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown. Noting that the area of the window is A =0.8 m X 1.5 m = 1.2 m2, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m} \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.00855^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass}} + R_{\text{conv, 2}} = 0.08333 + 0.00855 + 0.02083 \\ &= 0.1127^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{[20 - (-10)]°C}{0.1127°C/W} = \mathbf{266\ W}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{conv,\ 1}} \longrightarrow T_1 = T_{\infty 1} - \dot{Q}R_{conv,\ 1}$$
$$= 20°C - (266\ W)(0.08333°C/W)$$
$$= \mathbf{-2.2°C}$$

GENERALIZED THERMAL RESISTANCE NETWORKS

- Parallel resistances

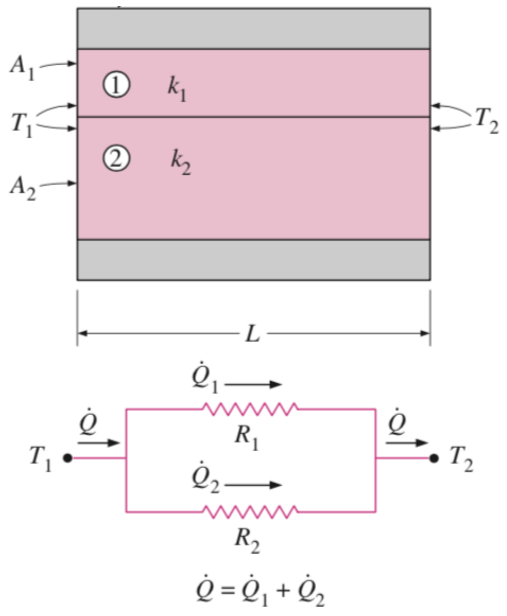
The total heat transfer is the sum of the heat transfers through each layer

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Utilizing electrical analogy, we get

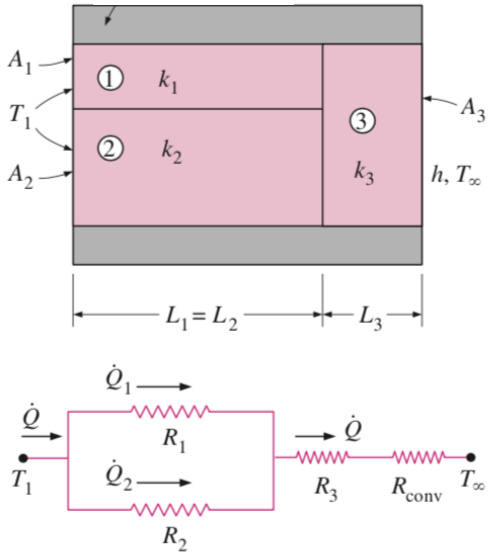
$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}}$$

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2}$$



- Combined series-parallel arrangement

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}}$$



Conductive Heat Through a Composite Wall

Example: For the composite wall shown in the figure, the area of sections B and C is equal. Use the information fixed on the figure to calculate the conductive heat transfer (q_x)

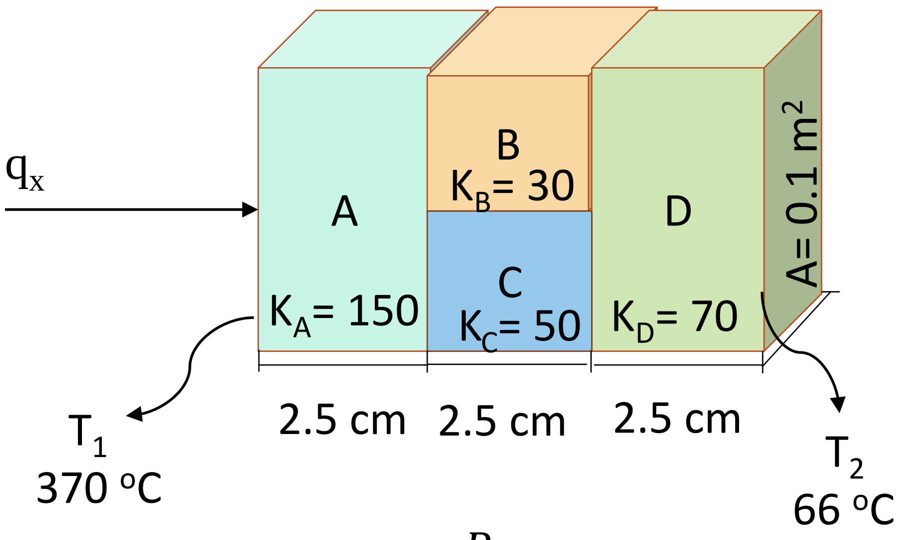
$$R_A = \frac{\Delta x}{KA} = \frac{0.025}{150\ x\ 0.1} = 1.66X10^{-3}\ {}^{o}C/W$$

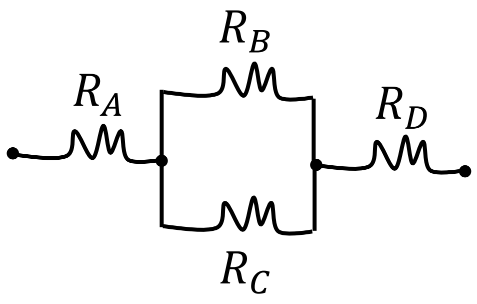
$$R_B = \frac{\Delta x}{KA} = \frac{0.025}{30\ x\ (\frac{0.1}{2})} = 0.0166\ {}^{o}C/W$$

$$R_C = \frac{\Delta x}{KA} = \frac{0.025}{50\ x\ (\frac{0.1}{2})} = 0.01\ {}^{o}C/W$$

$$R_D = \frac{\Delta x}{KA} = \frac{0.025}{70x\ 0.1} = 0.0875\ {}^{o}C/W$$

$$R_{B+C} = \frac{R_B . R_C}{R_B + R_C} = \frac{1.66x10^{-4}}{0.0266} = 6.24X10^{-3}$$





Conductive Heat Through a Composite Wall

$$\sum R_{total} = R_A + R_{B+C} + R_D$$

$$\sum R_{total} = (1.66X10^{-3})+(6.24X10^{-3})+(0.0875)=0.0939$$

$$q_x = \frac{\Delta T}{R_{total}} = \frac{(370 - 66)}{0.0939} = 3237.25\ W$$

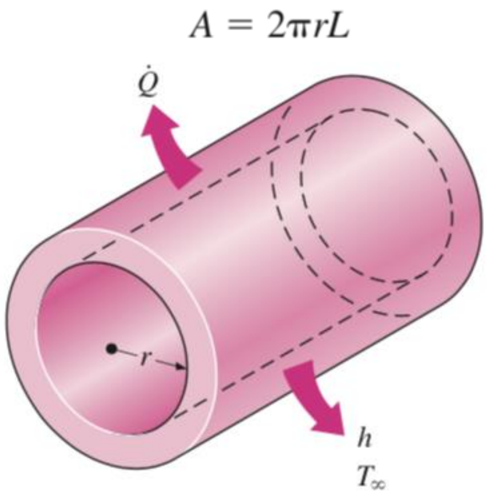
Cylinders & Spheres Thermal Resistance

Heat is lost from a hot water pipe to the air outside in the radial direction, and thus heat transfer from a long pipe is one-dimensional.

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr}$$

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)}$$

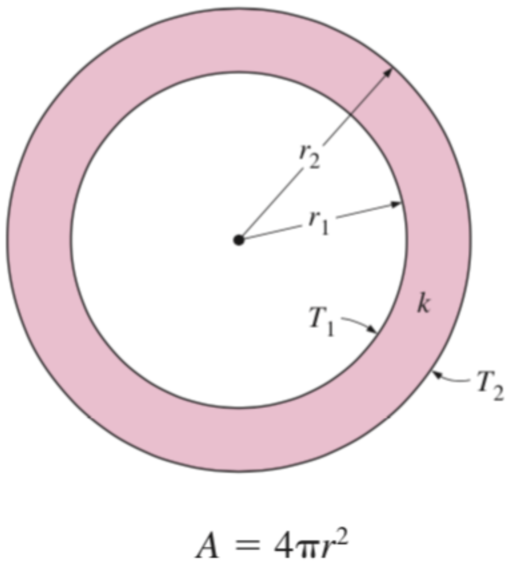
$$R_{\text{cyl}} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})}$$



Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o}$$

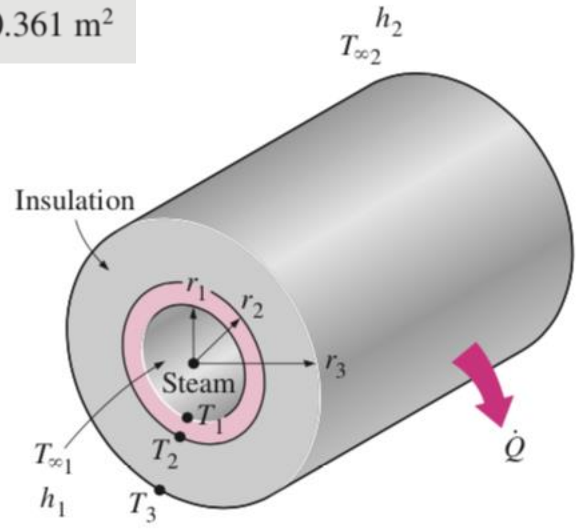
$$R_{\text{sph}} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})}$$



Cylinders & Spheres Thermal Resistance Network

Steam at $T_{\infty 1}$ = 320 °C flows in a cast iron pipe (k = 80 W/m · °C) whose inner and outer diameters are $D1$ = 5 cm and $D2$ = 5.5 cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with k = 0.05 W/m · °C. Heat is lost to the surroundings at $T_{\infty 2}$ = 5 °C by natural convection and radiation, with a combined heat transfer coefficient of $h2$ = 18 W/m2 · °C. Taking the heat transfer coefficient inside the pipe to be $h1$ = 60 W/m2 · °C, determine the rate of heat loss from the steam per unit length of the pipe.

$$A_1 = 2\pi r_1 L = 2\pi(0.025\text{ m})(1\text{ m}) = 0.157\text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575\text{ m})(1\text{ m}) = 0.361\text{ m}^2$$



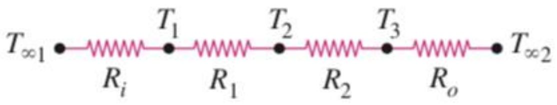
Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(60\text{ W/m}^2\cdot{}^\circ\text{C})(0.157\text{ m}^2)} = 0.106^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80\text{ W/m}\cdot{}^\circ\text{C})(1\text{ m})} = 0.0002^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05\text{ W/m}\cdot{}^\circ\text{C})(1\text{ m})} = 2.35^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18\text{ W/m}^2\cdot{}^\circ\text{C})(0.361\text{ m}^2)} = 0.154^\circ\text{C/W}$$



Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = \mathbf{121\text{ W}} \qquad \text{(per m pipe length)}$$