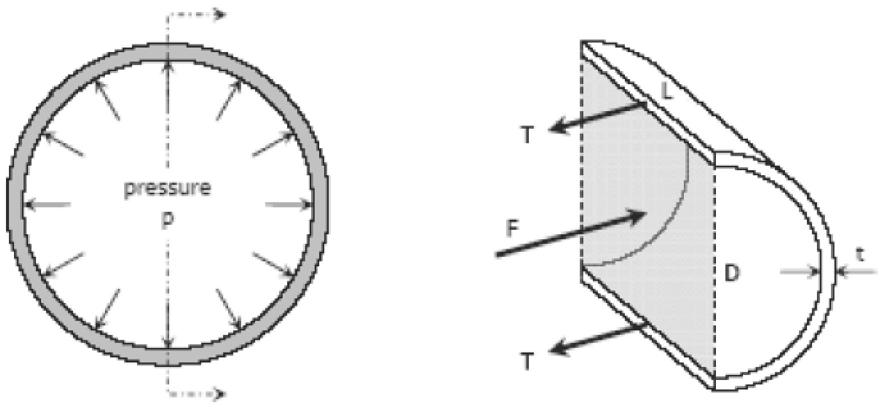
Thin-Walled Cylindrical Pressure Vessels

A tank or pipe carrying a fluid or gas under a pressure is subjected to tensile forces, which resist bursting, developed across longitudinal and transverse sections.

1- Tangential Stress (Circumferential Stress):

Consider the tank shown being subjected to an internal pressure p. The length of the tank is L and the wall thickness is t. Isolating the right half of the tank:



$F = P_o * A = P_o * D * L$

$T = \sigma_t * A_{wall} = \sigma_t * t * L$

$\sum F_H = 0$

$F = 2 * T$

$P_o * D * L = 2 * \sigma_t * t * L$

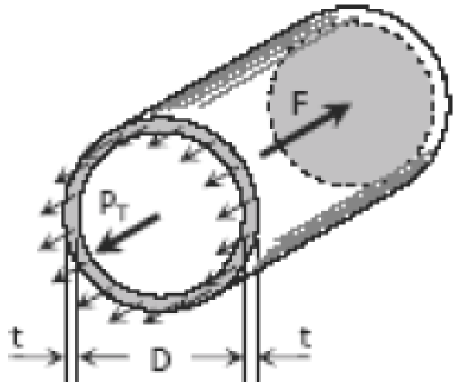
$\sigma_t = \dfrac{P_o * D}{2 * t}$

$$\boldsymbol{\sigma_t = \frac{P_o * \text{مساحة الفراغ}}{t * \text{محيط القص}}}$$

2- Longitudinal Stress:

Consider the free body diagram in the transverse section of the tank:

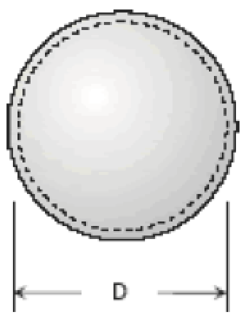
The total force acting at the rear of the tank (F) must equal to the total longitudinal stress on the wall ($P_T = \sigma_L * A_{wall}$) . Since (t) is so small compared to (D), the area of the wall is close to (π*D*t)



$$\sigma_L = \frac{\mathbf{P_o} * \text{مساحة الفراغ}}{t * \text{محيط القص}}$$

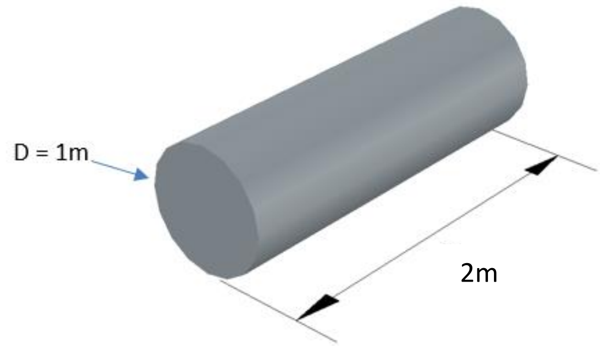
Spherical Shell:

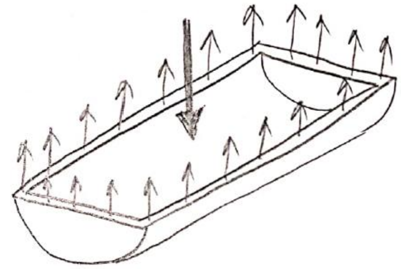
If a spherical tank of diameter (D) and thickness (t) contains gas under a pressure of (p), the stress at the wall can be expressed as:

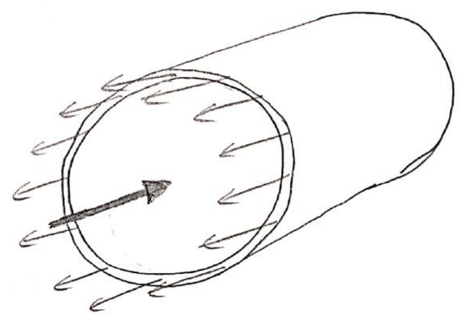


$$\sigma = \frac{\mathbf{P_o} * \text{مساحة الفراغ}}{t * \text{محيط القص}}$$

Ex; for the cylindrical tank shown, determine the tangential and longitudinal stress in the wall, the thickness of the wall is 1 mm and the internal pressure is 2 MPa







Sol:-

1) Tangential stress

$$\sigma_H = \frac{P_o * \text{مساحة الفراغ}}{t * \text{محيط القص}}$$

$$\text{مساحة الفراغ} = 1000 * 2000 = 2*10^6\ mm^2$$

$$\text{المحيط} = (2*1000)+(2*2000) = 6*10^3\ mm$$

$$\therefore \sigma_H = \frac{2*2*10^6}{1*6*10^3} = \boxed{666.6\ MPa}$$

2) Longtudinal stress

$$\sigma_L = \frac{P_o * \text{مساحة الفراغ}}{t * \text{محيط القص}}$$

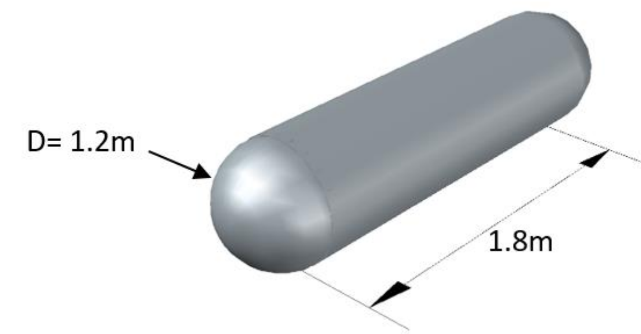
$$\text{مساحة الفراغ} = \frac{\pi}{4} D^2$$

$$= \frac{\pi}{4} *(1000)^2 = 785*10^3\ mm^2$$

$$\text{المحيط} = \pi * D = \pi * 1000 = 3140\ mm$$

$$\therefore \sigma_L = \frac{2*785*10^3}{1*3140} = \boxed{500\ MPa}$$

Ex; for the cylindrical tank shown, determine the tangential and longitudinal stress in the wall, the thickness of the wall is 6.4 mm and the internal pressure is 0.9 MPa

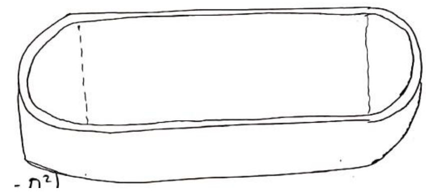


Sol:-

① tangential section:-

as F.B.D

$$\sigma_H = \frac{P_o * \text{مساحة الفراغ}}{t * \text{المحيط}}$$



1- مساحة الفراغ = مساحة مستطيل + مساحة نصفي دائرة

$$\text{مساحة الفراغ} = D*L + \frac{\pi}{4}D^2$$

2- المحيط = محيط نصفي دائرة + ضلعين (L)

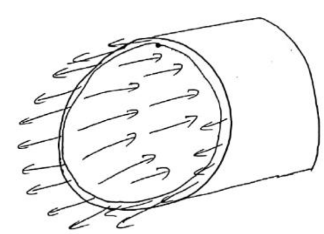
$$\text{المحيط} = \pi D + 2L$$

$$\therefore \sigma_H = \frac{P_o * (D*L + \frac{\pi}{4}D^2)}{t * (\pi D + 2L)}$$

$$= \frac{0,9 * (1200 * 1800 + \frac{\pi}{4} * 1200^2)}{6,4 * (1200 * \pi + 2 * 1800)} = 62,8 \text{ MPa}$$

② Longitudinal stress:

as F.B.D



$$\sigma_L = \frac{P_o * \text{مساحة الفراغ}}{t * \text{المحيط}}$$

1- مساحة الفراغ = مساحة دائرة

$$\therefore \text{مساحة الفراغ} = \frac{\pi}{4}D^2$$

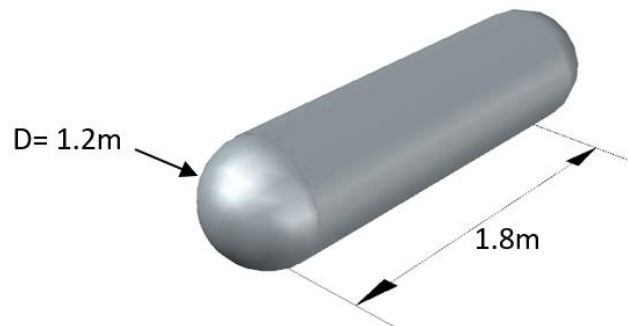
2- المحيط = محيط الدائرة

$$\therefore \text{المحيط} = \pi D$$

$$\therefore \sigma_L = \frac{P_o * \frac{\pi}{4} D^2}{t * \pi D}$$

$$= \frac{0{,}9 * \frac{\pi}{4} * 1200^2}{6{,}4 * \pi * 1200} = 42{,}187 \text{ MPa}$$

Ex; for the cylindrical tank shown, determine the minimum thickness of the wall so that the stress in the tangential and longitudinal will not exceed 42 MPa, the internal pressure is 0.9 MPa



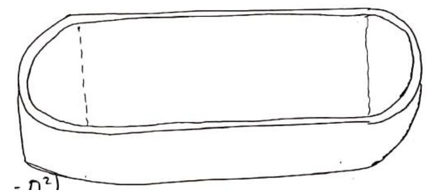
Sol;-

① Tangential section;-

as F.B.D

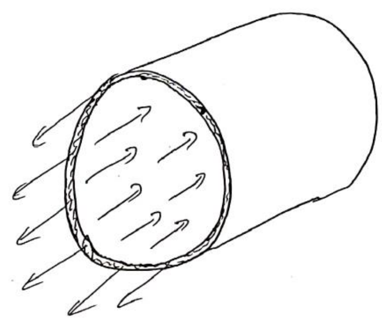
$$\sigma_H = \frac{P_o * \text{مساحة الفراغ}}{t * \text{المحيط}}$$

مساحة الفراغ = مساحة مستطيل + مساحة نصفي دائرة



$$\therefore \text{مساحة الفراغ} = D*L + \frac{\pi}{4} D^2$$

$$\text{المحيط} = \pi D + 2L$$



$$\therefore \sigma_H = 42 = \frac{0{,}9 * \left(D*L + \frac{\pi}{4} D^2\right)}{t * (\pi D + 2L)}$$

$$\therefore 42 = \frac{0{,}9 * \left(1200 * 1800 + \frac{\pi}{4} 1200^2\right)}{t * (2*1800 + 3{,}14 * 1200)}$$

$$\therefore t_1 = 9{,}56 \text{ mm}$$

② Longitudinal section:

$$\sigma_L = \frac{P_o * \text{مساحة الفراغ}}{t * \text{المحيط}}$$

$\therefore$ مساحة الفراغ $= \frac{\pi}{4} D^2$

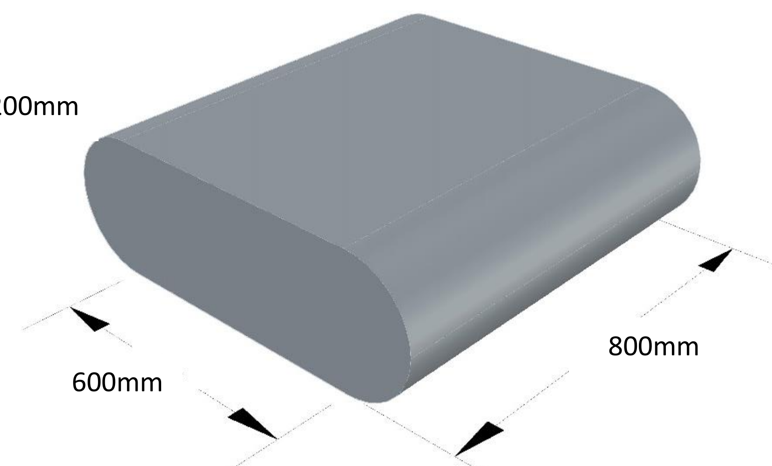
المحيط $= \pi D$

$$\therefore \sigma_L = 42 = \frac{0,9 * \frac{\pi}{4} * 1200^2}{t * \pi * 1200}$$

$$\therefore t_2 = 6,42 \text{ mm}$$

$$\therefore \text{use } t = 9,56 \text{ mm}$$

Ex; for the cylindrical tank shown, determine the minimum thickness of the wall so that the stress in the tangential and longitudinal will not exceed 40 MPa, the internal pressure is 1.5 MPa



① Tangential section

$$\sigma_H = \frac{P_o * \text{مساحة الفراغ}}{t * \text{المحيط}}$$

مساحة الفراغ = مساحة A + مساحة B + مساحة C

$\therefore$ مساحة الفراغ $= 800*600 + (2*200*800)$

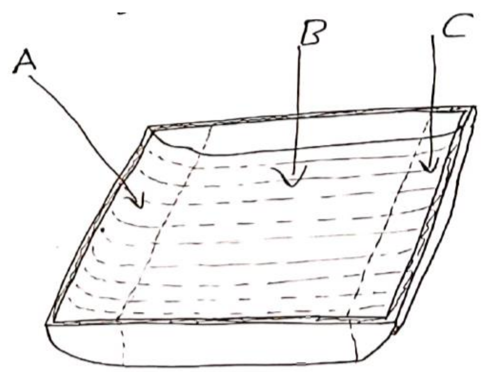
$= 800\,000 \text{ mm}^2$

المحيط $= 2*800 + 2*600 + 4*200$

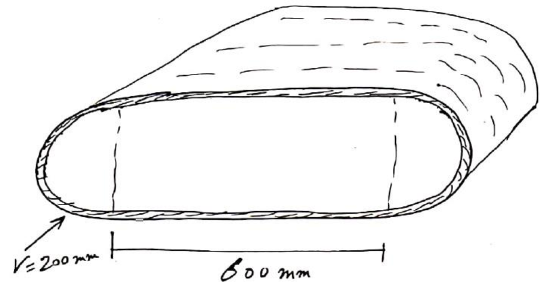
$= 3600 \text{ mm}$

$$\therefore \sigma H = \frac{1,5 * 800000}{t * 3600} = 40 \text{ MPa}$$

$$\therefore t_1 = 8,33 \text{ mm}$$



② Longtudinal section :-



$$\sigma_L = \frac{P_o * \text{مساحة الفراغ}}{t * \text{المحيط}}$$

$$\text{مساحة الفراغ} = \frac{\pi}{4} D^2 + DL$$

$$= \frac{\pi}{4} * 400^2 + 400 * 600$$

$$= 365600 \ mm^2$$

$$\text{المحيط} = \pi D + 2L$$

$$= \pi * 400 + 2 * 600$$

$$= 2456 \ mm$$

$$\therefore \sigma_L = \frac{1,5 * 365600}{t * 2456} = 40$$

$$\therefore t_2 = 5,582 \ mm$$

$$\boxed{\therefore t = 8,33 \ mm}$$