3.7 Local Buckling

If the elements of the cross section so thin may occur local buckling. If it occurs, the cross section is no longer fully effective and the member has failed. The measure of susceptibility is the width- thickness ratio (λ) of each cross-sectional. Limiting values of width-thickness rations are given in AISC **Table B4.1** "Classification of Sections for local buckling" where cross-sectional shaped are classified as:

i. **Compact Sections.**

For a section to qualify as compact its flanges must be continuously connected to the web or webs and **the width-thickness ratios λ of its compression elements must not exceed** the limiting width-thickness ratios **λP** from **Table B4.1**.

$$\lambda \leq \lambda P$$

ii. **Noncompact Sections**

If the width - thickness ratio λ of compression elements **exceeds λp, but does not exceed λr** from Table B4.1, the section is noncompact.

$$\lambda P < \lambda \leq \lambda r$$

iii. **Slender-Element Sections**

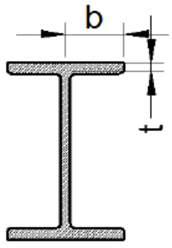
If **the width-thickness ratio of any element exceeds λr,** the section is referred to as a slender-element section.

$$\lambda > \lambda r$$

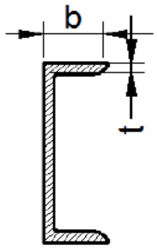
3.7.1 Type of Element

a) Unstiffened Elements

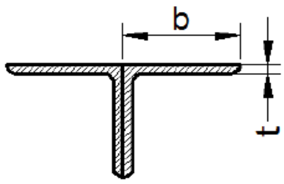
- Uniform compression in flanges of rolled I-shaped sections, outstanding legs of pairs of angles in continuous contact and flanges of channels. (Case 3)



$\lambda = \frac{b_f}{2t_f}$



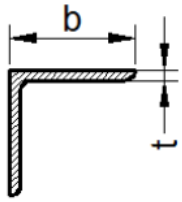
$\lambda = \frac{b_f}{t_f}$

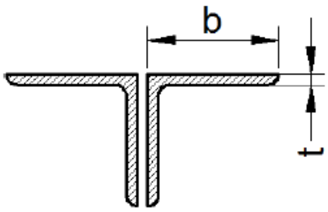


$\lambda = \frac{b}{t}$

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$

- Uniform compression in legs of single angles, legs of double angles with separators, and all other unstiffened elements (Case 5).

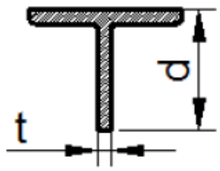




$\lambda = \frac{b}{t}$

$$\lambda_r = 0.45\sqrt{\frac{E}{F_y}}$$

- Uniform compression in stems of tees (Case 8).

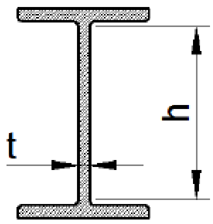


$\lambda = \frac{d}{t}$

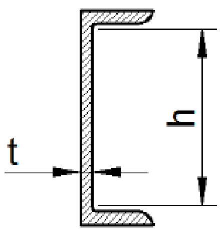
$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$

b) Stiffened Elements

- Uniform compression in webs of doubly symmetric I-shaped sections and channels. (case 10)

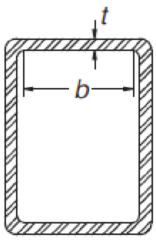


$\lambda = \frac{h}{t_w}$



$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$

❖ Uniform compression in flanges of rectangular box and hollow structural section (HSS). (case 12)



$$\lambda = \frac{b}{t}$$

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$

Example No. 1: Check the local buckling of the section $W12 \times 96$ column which has buckling length $KL = 15\ ft$ and made from **A36** steel.

Solve:

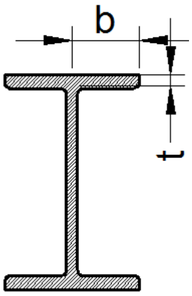
Steel and Section Properties:

$A36: Fy = 36\ ksi,\ \ Fu = 58\ ksi$

$W12 \times 96: bf = 12.2'',\ \ tf = 0.9'',\ \ d = 12.7'',\ \ tw = 0.55'',\ \ k_{des} = 1.5''$

1) Check Local Buckling of Unstiffened Parts.

$$\lambda = \frac{bf}{2\ tf} = \frac{12.2}{2 \times 0.9} = 6.78$$

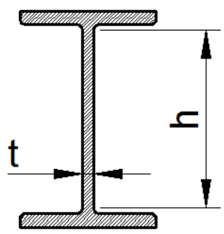


$$\lambda_r = 0.56\sqrt{\frac{E}{Fy}} = 0.56\sqrt{\frac{29000}{36}} = 15.89$$

$\lambda < \lambda_r \quad \therefore ok$

2) Check Local Buckling of Stiffened Parts.

$$\lambda = \frac{h}{tw} = \frac{d - 2\ k_{des}}{tw} = \frac{12.7 - 2 \times 1.5}{0.55} = 17.63$$



$$\lambda_r = 1.49\sqrt{\frac{E}{Fy}} = 1.49\sqrt{\frac{29000}{36}} = 42.2$$

$\lambda < \lambda_r \quad \therefore ok$

No local buckling occurs and the flexure buckling is control

H.W: Check the local buckling of the section $\boldsymbol{W14 \times 74}$ column which has length $\boldsymbol{20\ ft}$ and pinned end. Use **A992** steel.

3.7.2 Local Buckling for Members with Slender Elements

For cross sections with slender elements, local buckling occurs (when $\boldsymbol{\lambda > \lambda r}$) and the overall yield strength $\boldsymbol{Fy}$ will be reduced by the factor $\boldsymbol{Q}$, and the flexure buckling critical strength will be:

a) If $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{QFy}}$ $\quad or \;\; F_e \geq 0.44\,QFy,$ then:

$$\boldsymbol{F_{cr}} = \left[\boldsymbol{0.658^{\frac{QFy}{F_e}}}\right]\boldsymbol{QFy} \ldots\ldots\ldots\ldots \text{AISC Equation E3} - 2$$

b) If $\frac{KL}{r} > 4.71\sqrt{\frac{E}{QFy}}$ $\quad or \;\; F_e < 0.44\,QFy,$ then:

$$\boldsymbol{F_{cr}} = \boldsymbol{0.877\,F_e} \ldots\ldots\ldots\ldots \text{AISC Equation E3} - 3$$

Where:

$$\boldsymbol{F_e} = \frac{\boldsymbol{\pi^2 E}}{\left(\frac{\boldsymbol{KL}}{\boldsymbol{r}}\right)^2} \quad \ldots\ldots\ldots\ldots \text{ AISC Equation E3} - 4$$

Where:

$\boldsymbol{Q = 1.0}$: For members with compact and noncompact sections.

$\boldsymbol{Q = Qs \cdot Qa}$: For members with slender-element sections.

- For cross sections consisted of only unstiffened slender elements:

$$\boldsymbol{Q = Qs}, \qquad \boldsymbol{Qa = 1}$$

- For cross sections consisted of only stiffened slender elements:

$$\boldsymbol{Q = Qa}, \qquad \boldsymbol{Qs = 1}$$

- For cross sections consisted of both unstiffened & stiffened slender elements:

$$\boldsymbol{Q = Qa \cdot Qs}$$

3.7.3 How to Calculated of Qs and Qa: Chapter E (E7.)

Slender Unstiffened Elements, Q_s

The reduction factor Q_s for slender *unstiffened elements* is defined as follows:

(a) For flanges, angles, and plates projecting from rolled *columns* or other compression members:

(i) When $\frac{b}{t} \leq 0.56\sqrt{\frac{E}{F_y}}$

$$Q_s = 1.0 \qquad \text{(E7-4)}$$

(ii) When $0.56\sqrt{E/F_y} < b/t < 1.03\sqrt{E/F_y}$

$$Q_s = 1.415 - 0.74\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}} \qquad \text{(E7-5)}$$

(iii) When $b/t \geq 1.03\sqrt{E/F_y}$

$$Q_s = \frac{0.69E}{F_y\left(\frac{b}{t}\right)^2} \qquad \text{(E7-6)}$$

(c) For single angles

(i) When $\frac{b}{t} \leq 0.45\sqrt{\frac{E}{F_y}}$

$$Q_s = 1.0 \qquad \text{(E7-10)}$$

(ii) When $0.45\sqrt{E/F_y} < b/t \leq 0.91\sqrt{E/F_y}$

$$Q_s = 1.34 - 0.76\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}} \qquad \text{(E7-11)}$$

(iii) When $b/t > 0.91\sqrt{E/F_y}$

$$Q_s = \frac{0.53E}{F_y\left(\frac{b}{t}\right)^2} \qquad \text{(E7-12)}$$

where

b = full width of longest angle leg, in. (mm)

(d) For stems of tees

(i) When $\frac{d}{t} \leq 0.75\sqrt{\frac{E}{F_y}}$

$$Q_s = 1.0 \tag{E7-13}$$

(ii) When $0.75\sqrt{\frac{E}{F_y}} < d/t \leq 1.03\sqrt{\frac{E}{F_y}}$

$$Q_s = 1.908 - 1.22\left(\frac{d}{t}\right)\sqrt{\frac{F_y}{E}} \tag{E7-14}$$

(iii) When $d/t > 1.03\sqrt{\frac{E}{F_y}}$

$$Q_s = \frac{0.69E}{F_y\left(\frac{d}{t}\right)^2} \tag{E7-15}$$

Slender Stiffened Elements, Q_a

The reduction factor, Q_a, for slender *stiffened elements* is defined as follows:

$$Q_a = \frac{A_{eff}}{A} \tag{E7-16}$$

where

A = total cross-sectional area of member, in.^{2} (mm^2)

A_{eff} = summation of the effective areas of the cross section based on the reduced *effective width*, b_e, in.^{2} (mm^2)

(a) For uniformly compressed slender elements, with $\frac{b}{t} \geq 1.49\sqrt{\frac{E}{f}}$, except flanges of square and rectangular sections of uniform thickness:

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b \tag{E7-17}$$

where

f is taken as F_{cr} with F_{cr} calculated based on $Q = 1.0$.

(b) For flanges of square and rectangular *slender-element sections* of uniform thickness with $\frac{b}{t} \geq 1.40\sqrt{\frac{E}{f}}$:

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b \tag{E7-18}$$

f may be taken equal to Fy.

Example No. 2: A $\boldsymbol{W21\times 101}$ is used as compression member with one end fixed and the other end free. The length is $\boldsymbol{10\ ft}$. what is the nominal compressive strength if $\boldsymbol{Fy=\ 50\ ksi}$. Note that this is a slender-element compression member, and the equations of AISC Section E7 must be used.

Solve:

Steel and Section Properties:

$\boldsymbol{W21\times 101}$: $A_g = 29.8\ in^2,\ \ b_f = 12.3'',\ \ t_f = 0.8'',\ \ d = 21.4'',\ \ t_w = 0.5'',$

$r_x = 9.02'', r_y = 2.89'',\ \ k_{des} = 1.3''$

1) Calculate $\boldsymbol{Fcr}$ use $\boldsymbol{Q = 1.00}$

$$\frac{KL}{r} = \frac{2.1\times 10\times 12}{2.89} = 87.2 < 4.71\sqrt{\frac{E}{Fy}} = 4.71\times\sqrt{\frac{29000}{50}} = 113.43$$

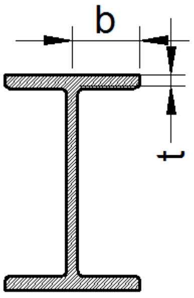
$$F_{cr} = \left[0.658^{\frac{Fy}{F_e}}\right]Fy$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2\times 29000}{(87.2)^2} = 37.64\ ksi$$

$$F_{cr} = \left[0.658^{\frac{50}{37.64}}\right]\times 50 = 28.68\ ksi$$

2) Check Local Buckling of Unstiffened Parts.

$$\lambda = \frac{bf}{2\ tf} = \frac{12.3}{2\times 0.8} = 7.69$$

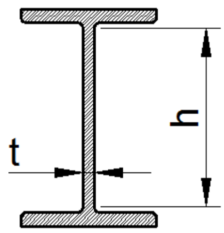


$$\lambda_r = 0.56\sqrt{\frac{E}{Fy}} = 0.56\sqrt{\frac{29000}{50}} = 13.487$$

$\lambda < \lambda_r \quad \therefore ok$

3) Check Local Buckling of Stiffened Parts.

$$\lambda = \frac{h}{t_w} = \frac{d - 2\,k_{des}}{t_w} = \frac{21.4 - 2 \times 1.3}{0.5} = 37.6$$



$$\lambda_r = 1.49\sqrt{\frac{E}{Fy}} = 1.49\sqrt{\frac{29000}{50}} = 35.88$$

$\lambda > \lambda_r$ $\quad\therefore$ local buckling occurs in web

Because this cross sectional element is a stiffened element $\therefore$ $\boldsymbol{Q = Qa, \quad Qs = 1}$

4) Calculate local buckling reduction factors $\boldsymbol{Qa}$

$$Qa = \frac{A_{eff}}{A_g}$$

$$h_e = 1.92\,t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{\lambda}\sqrt{\frac{E}{f}}\right] \le h, \qquad f = F_{cr}$$

$$h = 21.4 - 2 \times 1.3 = 18.8''$$

$$h_e = 1.92 \times 0.5\sqrt{\frac{29000}{28.68}}\left[1 - \frac{0.34}{37.5}\sqrt{\frac{29000}{28.68}}\right] = 21.72'' > h \qquad \therefore h_e = h$$

$$A_{eff} = A_g - (h - h_e)t_w = A_g$$

then:

$$Qa = 1.0 \;\; and \; F_{cr} = 28.68\; ksi$$

$$P_n = F_{cr}\,A_g = 28.68 \times 29.8 = 854.664\; kips$$

H.W: Repeat Ex.2 use $\boldsymbol{W36 \times 231}$

Example No. 3: An $\boldsymbol{HSS\ 10 \times 8 \times \frac{3}{16}}$ is used as compression member. If the maximum slenderness ratio is $\mathbf{35.8}$, what is the nominal compressive strength if the section made from $\boldsymbol{A500}$, $\boldsymbol{Fy = 46\ ksi}$. Note that this is a slender-element compression member, and the equations of AISC Section E7 must be used.

Solve:

Steel and Section Properties:

$\boldsymbol{HSS\ 10 \times 8 \times \frac{3}{16}}$: $A_g = 6.06\ in^2,\ \ t = 0.174'',\ \ \frac{b}{t} = 43.0'',\ \ \frac{h}{t} = 54.5'',$

$h = 9.48'', \quad b = 7.48''$

1) Calculate $\boldsymbol{Fcr}$ use $\boldsymbol{Q = 1.00}$

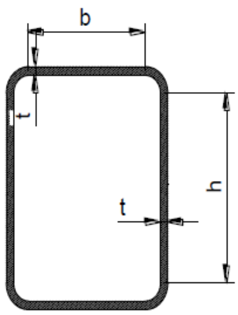
$$\frac{KL}{r} = 35.8 < 4.71\sqrt{\frac{E}{Fy}} = 4.71 \times \sqrt{\frac{29000}{46}} = 118.26$$

$$F_{cr} = \left[0.658^{\frac{Fy}{F_e}}\right] Fy$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000}{(35.8)^2} = 223.32\ ksi$$

$$F_{cr} = \left[0.658^{\frac{46}{223.322}}\right] \times 46 = 42.2\ ksi$$

2) Check Local Buckling



$$\lambda = \frac{b}{t} = 43'', \quad and \quad \lambda = \frac{h}{t} = 54.5''$$

$$\lambda_r = 1.40\sqrt{\frac{E}{Fy}} = 1.40\sqrt{\frac{29000}{46}} = 35.15$$

$\lambda > \lambda_r \quad \therefore$ local buckling occur in both sides

Because this cross sectional element is a stiffened element $\therefore \boldsymbol{Q = Qa,\quad Qs = 1}$

3) Calculate local buckling reduction factors $\boldsymbol{Qa}$

$Qa = A_{eff}/A_g$

$$b_e = 1.92\,t\sqrt{\frac{E}{f}}\left[1-\frac{0.38}{\lambda}\sqrt{\frac{E}{f}}\right] \le b, \qquad f = Fy$$

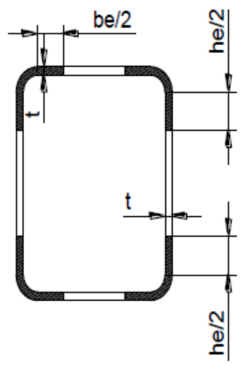
$$b_e = 1.92 \times 0.174\sqrt{\frac{29000}{46}}\left[1-\frac{0.38}{43}\sqrt{\frac{29000}{46}}\right] = 6.52'' < b = 7.48''$$

$$h_e = 1.92 \times 0.174\sqrt{\frac{29000}{46}}\left[1-\frac{0.38}{54.5}\sqrt{\frac{29000}{46}}\right] = 6.92'' < h = 9.48''$$

$$A_{eff} = A_g - 2[(b-b_e)+(h-h_e)]t$$

$$A_{eff} = 6.06 - 2[(7.48-6.52)+(9.48-6.92)] \times 0.174 = 4.84\ in^2$$

$$\therefore Qa = \frac{A_{eff}}{A_g} = \frac{4.84}{6.06} = 0.799 = Q$$



4) Calculate Fcr use $Q = 0.799$

$$\frac{KL}{r} = 35.8 < 4.71\sqrt{\frac{E}{QFy}} = 4.71 \times \sqrt{\frac{29000}{0.799 \times 46}} = 133$$

$$F_{cr} = \left[0.658^{\frac{QFy}{F_e}}\right]QFy, \quad F_e = 223.32\ ksi$$

$$F_{cr} = \left[0.658^{\frac{0.799\times46}{223.32}}\right] \times 0.799 \times 46 = 34.3\ ksi$$

then:

$$P_n = F_{cr}\,A_g = 34.3 \times 6.06 = 207.8\ kips$$

H.W: Determine the axial compressive strength of an $\boldsymbol{HSS\ 8 \times 4 \times \frac{1}{8}}$ with an effective length $\boldsymbol{15\ ft}$. Use $\boldsymbol{Fy = 46\ ksi}$.

Ans: $P_n = 53.35\ kips,\ \ (P_u = \emptyset P_n = 48\ kips),\ \ (P_a = P_n/\Omega = 32\ kips)$