Design of concrete structure

Syllabus :

1. Concrete and reinforcing steel-Introduction
2. Mechanical properties and behavior of reinforced concrete beams
3. Beams
   a. Design for flexural (ultimate and working stress method)
   b. Design for shear and diagonal tension

c. Bond, Anchorage, development length, bar cut off and bent point and bar splice in flexure members.

d. Control of cracking and deflection at service loads (Serviceability requirement).

e. Torsion and torsion plus shear.

f. ACI code moment and shear coefficients method for continues beam.

4. Slabs

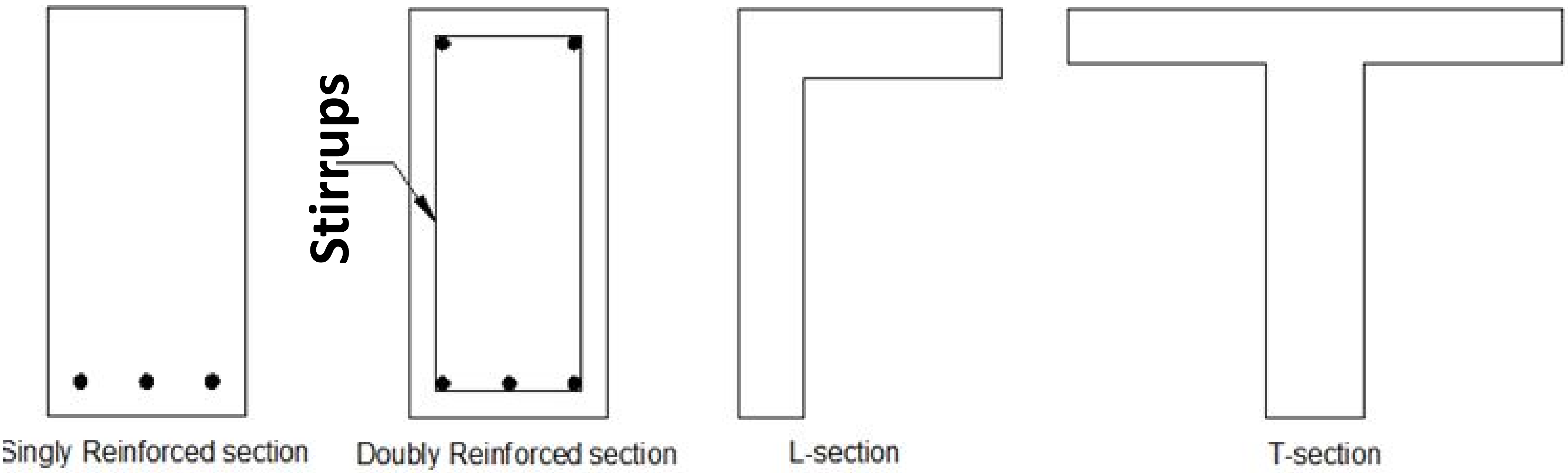
a. One way solid slab

b. One way ribbed slab

c. Two way solid slab

d. Two way ribbed slab

5. Compression member and columns
   a. Compression plus bending.
   b. Rectangular columns, circular columns(with spiral reinforcement)
   c. Safety provisions
   d. Rectangular columns in biaxial bending.
6. Stairways.
   a. Type of concrete stair ways
   b. Building code requirements.


Stirrups
Singly Reinforced section
Doubly Reinforced section
L-section
T-section

References

1. Design of concrete structure by **(Winter and Nilson)**
2. Reinforced concrete fundamentals by **Ferguson**.
3. Design of concrete structure by **Wang and Solomon**.
4. Reinforced concrete Structure by **Park and Pouly**.
5. Building code requirement for structural concrete **(ACI 318 M-02)**

SI unit System International

Length: meter (m)

Area: meter square (m^2)

Force: Newton N

Stress, Pressure and Modulus of Elasticity: Pascal $Pa = 1\,\frac{N}{m^2}$

Moment: N.m

$m: milli\ 10^{-3}$

$\mu: micro\ 10^{-6}$

$$k: kilo\ 10^3$$

$$M: mega\ 10^6$$

$$Mpa = 10^6 Pa = 10^6 \frac{N}{m^2} = M \frac{N}{m^2}$$

$$kPa = 10^3 Pa = k \frac{N}{m^2}$$

Introduction

Concrete: mixture of Portland cement or any other hydraulic cement, fine aggregate (sand), coarse aggregate and water with or without admixture.

Concrete as a building material:

1. High compressive strength
2. High degree of formability
3. Availability of indigenous materials
4. Fire resistance weather endurance
5. Low tensile strength

Constituent material of concrete

1-Cement	2-Aggregate	3-Water
a. Portland cement type I b. Portland cement type III c. Portland cement type V	a. Fine aggregate (sand) b. Coarse aggregate (gravel)	

Max size of gravel in reinforced concrete:-

$$\leq \frac{1}{5}\ narrowest\ dimension\ of\ the\ frames$$

$$\leq \frac{1}{3}\ depth\ of\ slab$$

$$\leq \frac{3}{4}\ min.\ distance\ between\ reinforcement.$$

Density of concrete:-

a. Natural concrete

$(Wc)\gamma_{concrete}(22-24)\frac{kN}{m^3} \rightarrow fc' \geq 17MPa$ ACI 1.1.1

& 5.1.1

b. Light weight concrete.

1. $\gamma_c \leq 8\frac{kN}{m^3}$ used for insulation.

2. $\gamma_c = 9.5 - 13.5\frac{kN}{m^3}\ modreate\ strength$

(fc'=7-17 MPa)

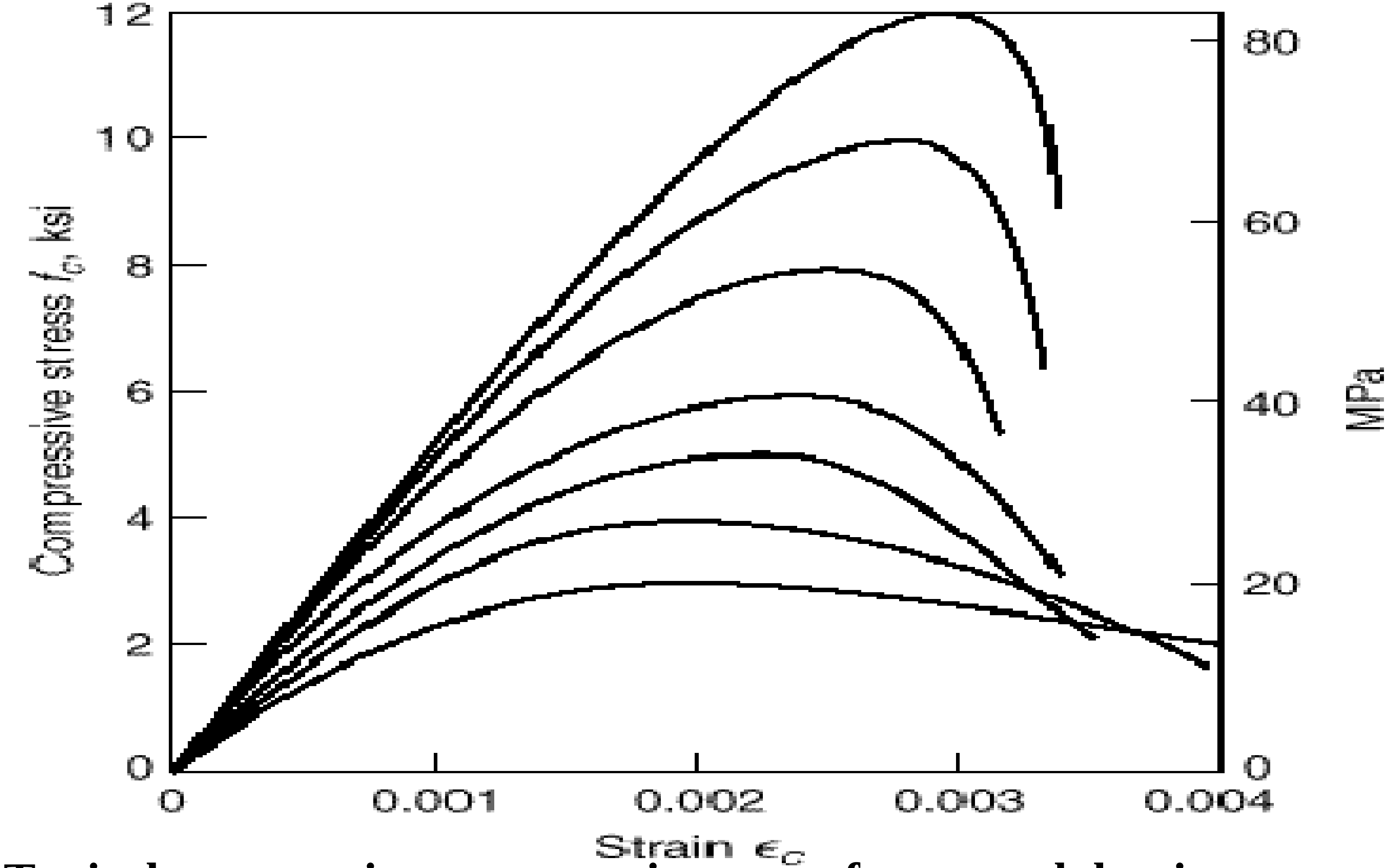
3. $\gamma_c = 14 - 19\,\frac{kN}{m^3}$

c. Heavy weight concrete.

$\gamma_c = 13.5 - 36.5\,\frac{kN}{m^3}$ Used for shielding against gamma and x-radiation in nuclear reactors

Mechanical properties of concrete:

1. Specified compressive strength of concrete fc':



Typical compressive stress-strain curves for normal density concrete with γc=23kN/m^3

fc'=20 MPa for normal density cast in place concrete

fc'=55 MPa for precast pre-stressed concrete

} Normal concrete

For light weight concrete strength are somewhat below these values generally.

fc'>103MPa for high strength concrete used with increasing frequency Particularly for heavily loaded columns in high rise buildings and for long span bridges (mostly pre-stressed) where a significant reduction in dead load may be realized by minimizing member cross section dimensions.

2. Modulus of elasticity of concrete E_c:

Ratio of normal stress to corresponding strain for tensile or compressive stress below proportianal limit of material.

Ec=Slop of fc-$\epsilon_c$diagram

Ec Constance up to (fc=0.5fc')

$$Ec = \tan(\alpha) = \frac{\Delta f_c}{\Delta \epsilon_c}$$

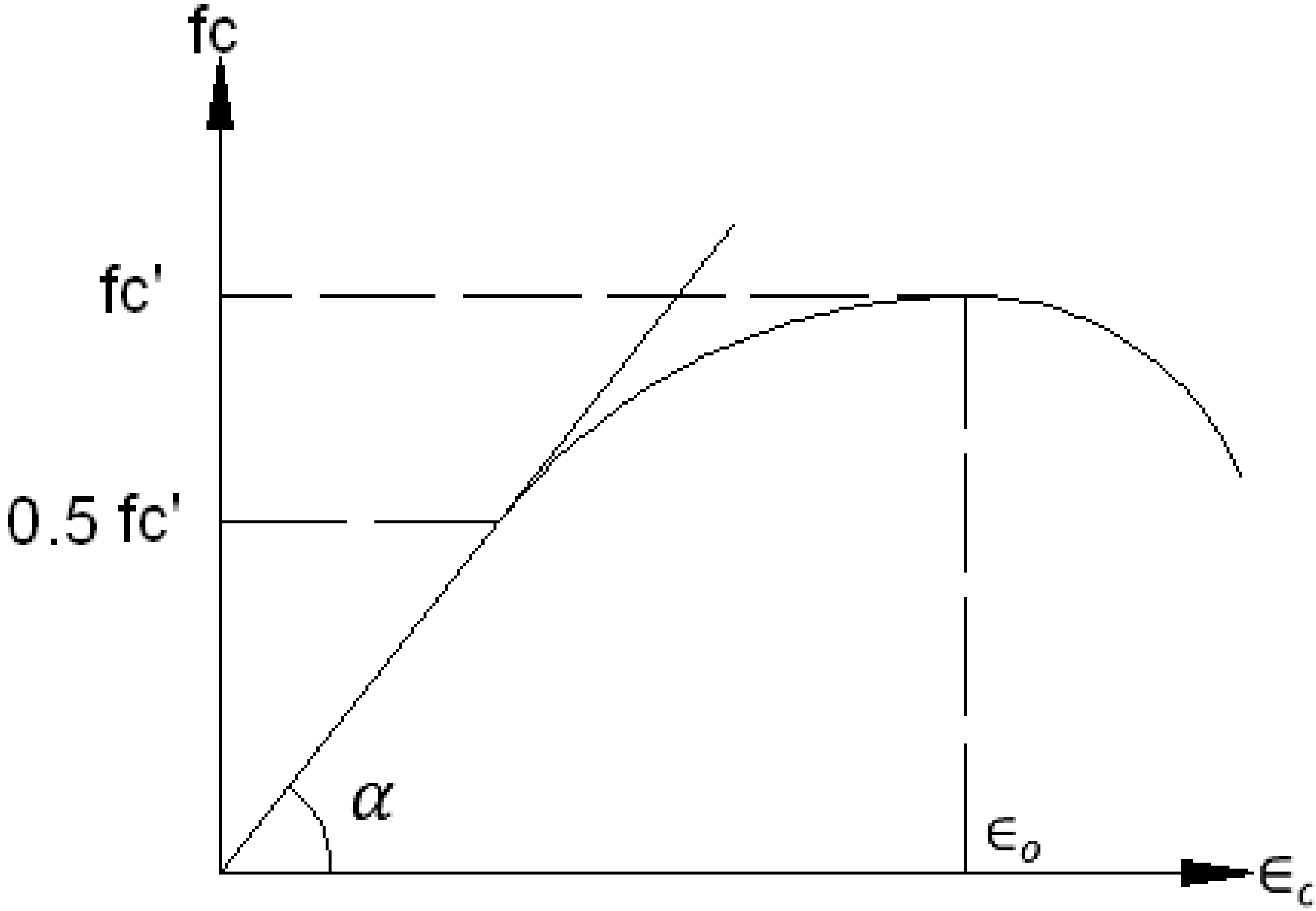
$=0.043\, w_c^{1.5}\sqrt{fc'}$ initial modulus of elasticity (empirical eq.)

For $W_c = 1500 - 2500\,\frac{kg}{m^3}$

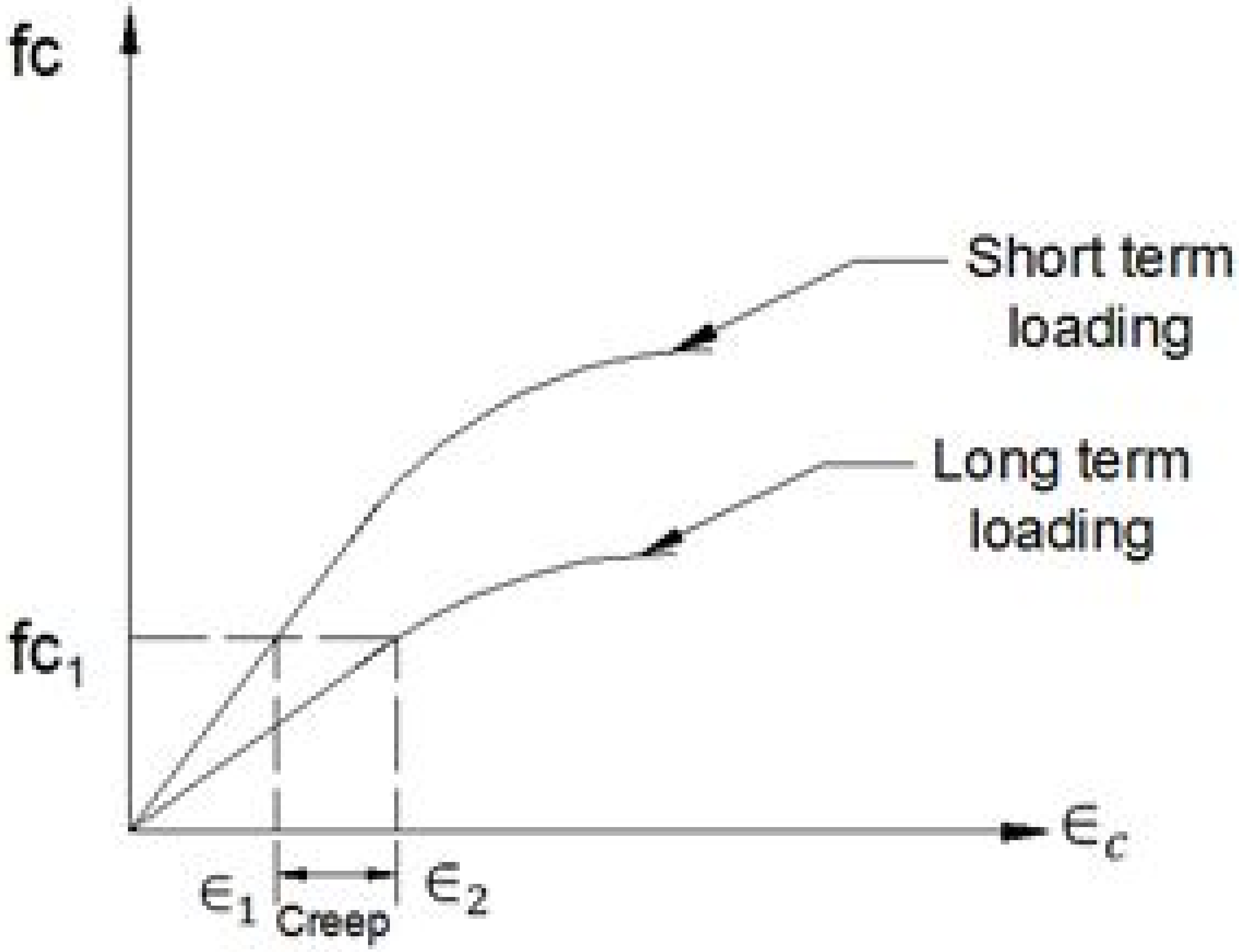
If $\gamma_c = 2400\,\frac{kg}{m^3}$ $(24\,\frac{kN}{m^3})$ and fc' in MPa

$E_c=0.043(2400)^{1.5}\sqrt{fc'}$

$E_c=4700\sqrt{fc'}$ ACI code 8.5.1


fc
fc'
0.5 fc'
α
∈o
∈c

Creep: is the slow deformation of a material of considerable length of time at a constant stress or load



H.W

$$fc = fc' \left[\frac{2\varepsilon_c}{\varepsilon_o} - \left(\frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right]$$ stress –strain equation for normal concrete

$$\varepsilon_o = \frac{2fc'}{E_c}$$

$$E = \frac{dfc}{d\varepsilon_c}$$

let $fc' = 20$ Mpa, $E_c = 4700\sqrt{fc'}$

Required

a. Plot fc vs ε_c diagram

b. Plot $\frac{E}{E_c}$ vs ε_c diagram

c. Discuss these diagram

$$E_c = 4700\sqrt{20} = 21019\ MPa$$

$$\varepsilon_o = \frac{2 * 20}{21019} = 0.0019$$

$$fc = 20\left[\frac{2\varepsilon_c}{0.0019} - (\frac{\varepsilon_c}{0.0019})^2\right]$$

$$\varepsilon_{c1} = 0.0005$$

$$fc1 = 20\left[\frac{2 * 0.0005}{0.0019} - \left(\frac{0.0005}{0.0019}\right)^2\right] = 9MPa$$

ε_c	fc	$E = \frac{dFc}{d\varepsilon_c}$	$\frac{E}{Ec}$
0.0005	9.12		1
-			-
-			-
-			-
0.0019	fc'	0	0
-			-
-			-
-			-
0.004			-

$$E = \frac{\Delta fc}{\Delta \varepsilon_c}$$

3. Poissonons ratio μ

Ratio of transverse to longitudinal strain

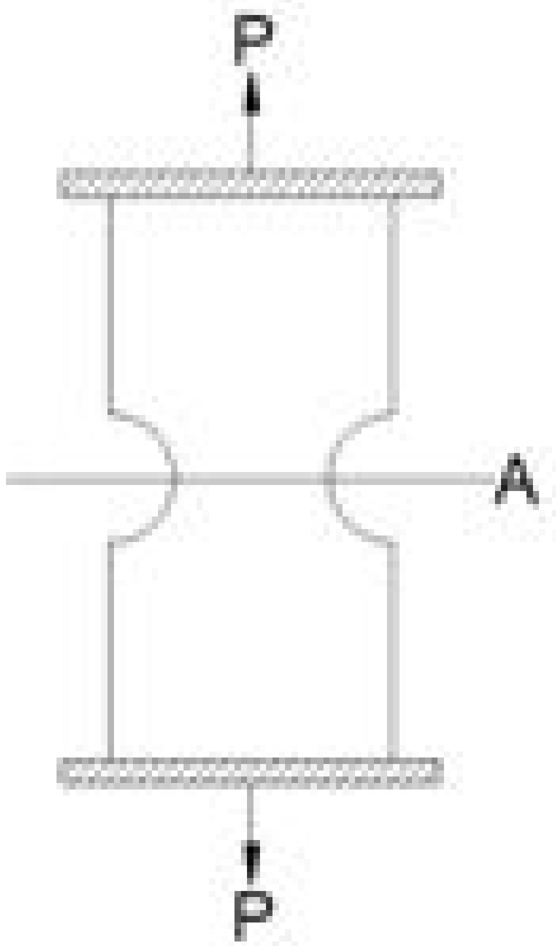
$$\mu = \frac{lateral\ strain}{long.strain}$$

$$\mu \rightarrow 0.15(tension) - 0.20(compression), \qquad \mu_{av} = 0.18$$

4. Tensile strength

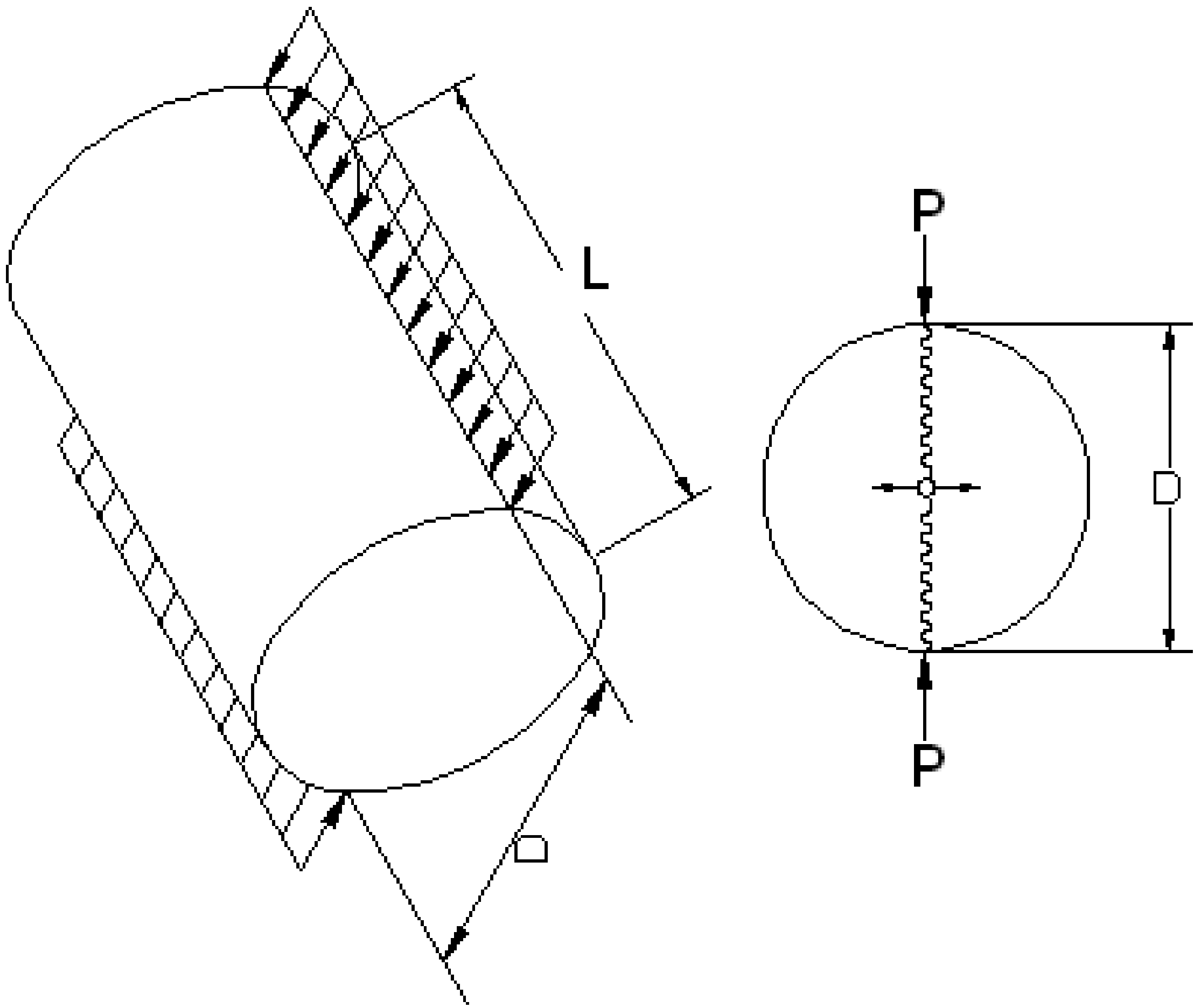
a. Direct tensile strength (ft')

$$ft' = \frac{P}{A}$$



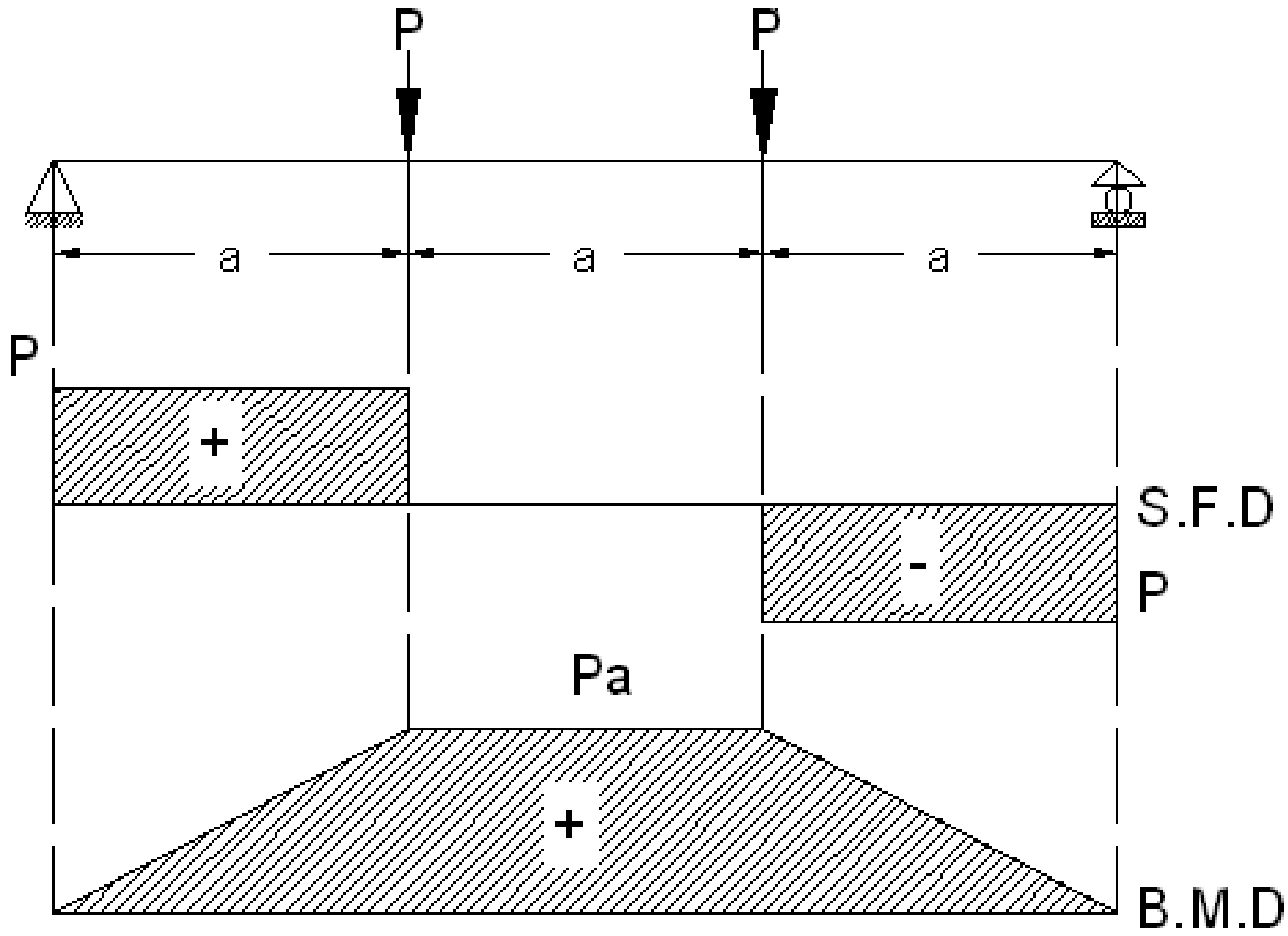
b. Split Cylinder test f_{sp}'

$$f_{sp}' = \frac{2P}{\pi DL}$$



c. Modulus of rupture test (fr)

$$fr = \frac{Mc}{I} = \frac{pa.\frac{h}{2}}{\frac{bh^3}{12}}, \quad (take\ b = h) = \frac{6pa}{h^3}$$



$$fr > f'_{sp} > Ft'$$

$$ft' = (0.5\text{-}0.625) f'_{sp}$$

$$fr = (1.33\text{-}1.5) f'_{sp}$$

$fr = 0.62\sqrt{fc'}$ (fc' in MPa) for normal concrete ACI 9.5.2.3

$fr = 0.62\sqrt{fc'} * 0.85 = 0.53\sqrt{fc'}$ for sand light weight

$fr = 0.62\sqrt{fc'} * 0.7 = 0.46\sqrt{fc'}$ For all light weight concrete

ACI 1.1.1 , 5.1.1 and Table 4.2.2

$f_c' \geq 17$ MPa for structural concrete

$f_c' \geq 28$ MPa concrete intended to have low permeability when exposed to water

$f_c' \geq 31$ MPa concrete exposed to freezing or in a moist condition or to deicing chemicals

$f_c' \geq 35$ MPa for corrosion protection of reinforcement in concrete exposed to chlorides salt water sea water

No max limit for fc'

Mechanical properties of reinforcement bars:

fy: yield strength of steel Grade (300,400)MPa have high ductility.

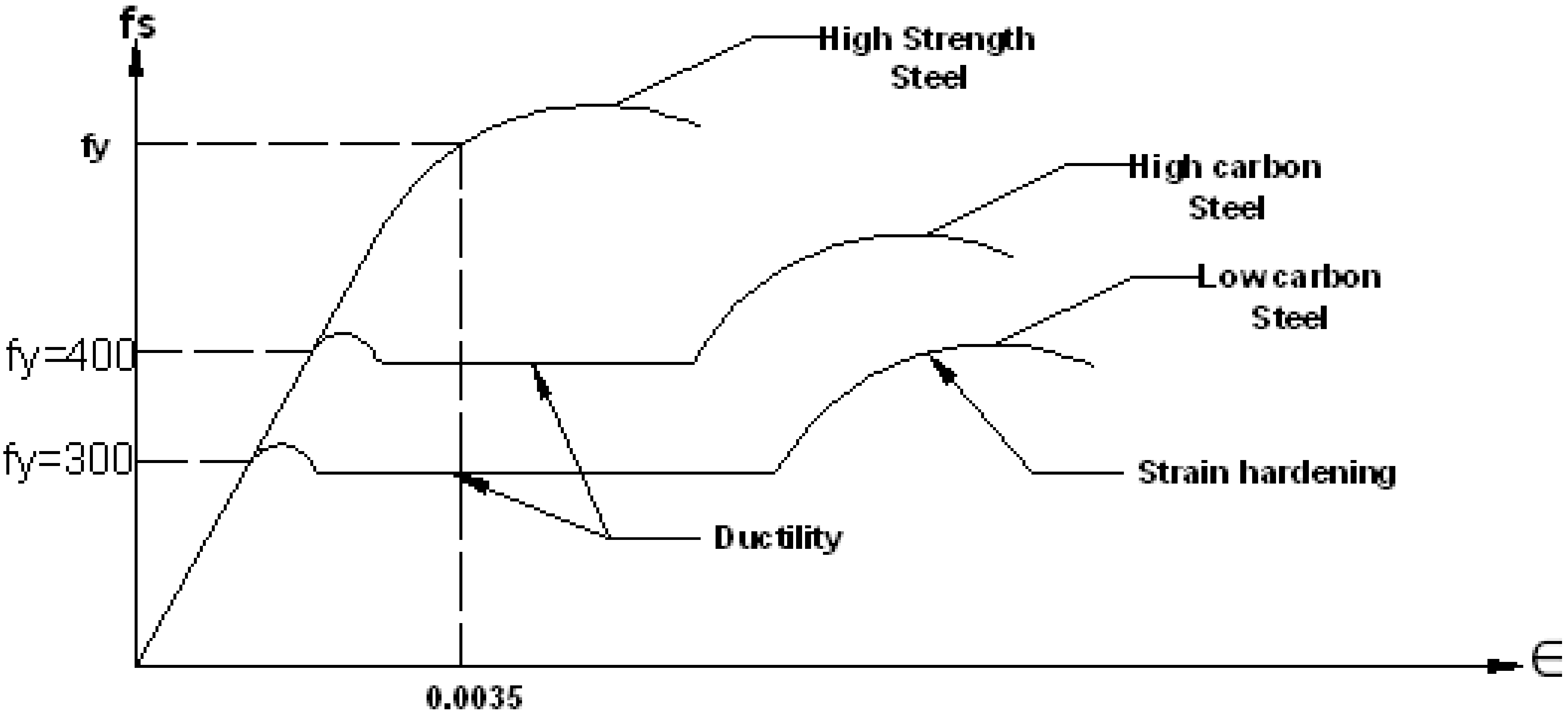
a. According to fy

fy=300 MPa low carbon steel

fy=400 MPa high carbon steel

*ACI 9.4 fy≤550 MPa ,except for prestressing steel and for spiral transverse reinforcement

*Higher-strength carbon steels e.g., those with 400 MPa yield stress or higher, either have a yield plateau or much shorter length or enter strain-hardening immediately without any continued yielding at constant stress. In the latter case, the ACI code(3.5.3.2) specifies that yield stress fy be the stress corresponding to a strain of 0.0035



Stress-Strain relation ship for steel reinforcement

b. According to shape

- Plain bars
- Deformed bars

ACI 3.5.1: Reinforcement shall be deformed reinforcement except that plain reinforcement shall be permitted for spirals or pre-stressing steel.

c. According to diameter $\emptyset, dia,$ # (6-55)mm

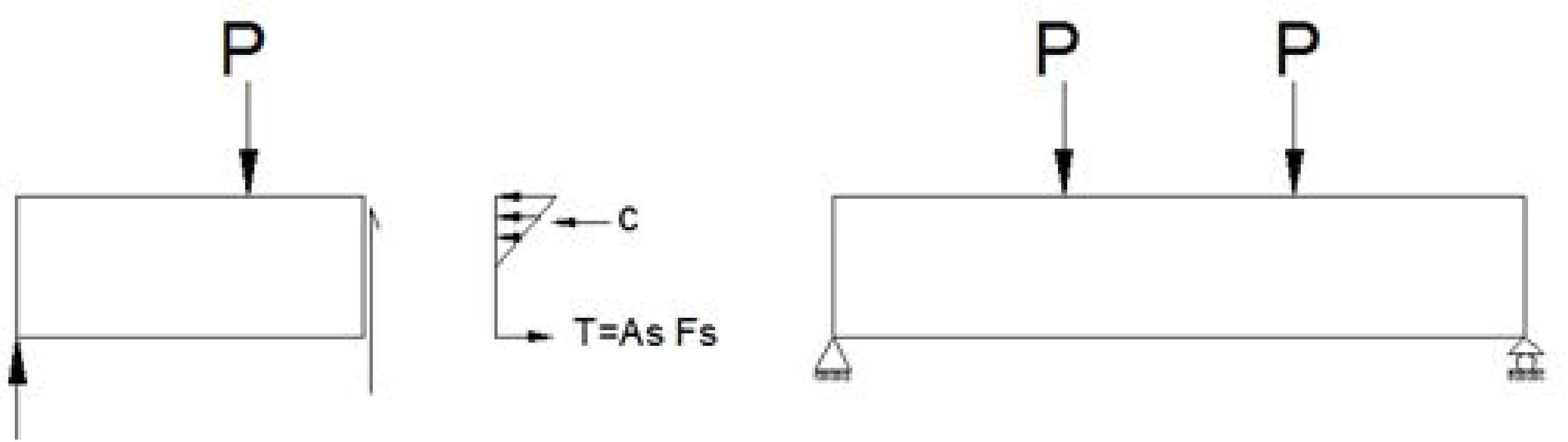
$$\text{Design of Concrete Structure}\begin{cases}\text{strength requirements: flexture, shear, torsion,}\\ \text{axial force effect. We find best dimensions and}\\ \text{reinforcement}\\ \\ \text{Servicibility requirements: control of deflection}\\ \text{and crack width. Some time we change strength}\\ \text{requirement to satisfy the servicibility conditions}\end{cases}$$

Basic assumptions of Design for Strength Method:-

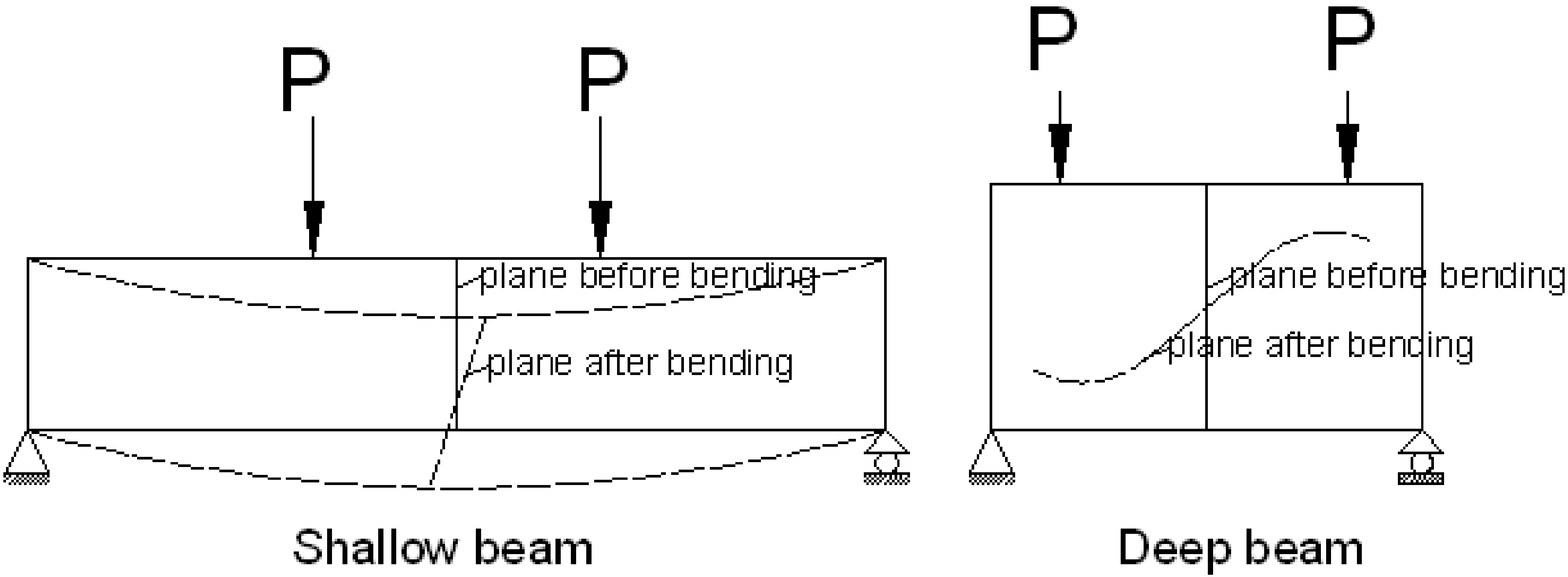
1. External forces must be in a state of equilibrium with internal stresses.

$\sum Fx = 0$ axial forces, $\sum Fy = 0$ Shear Forces

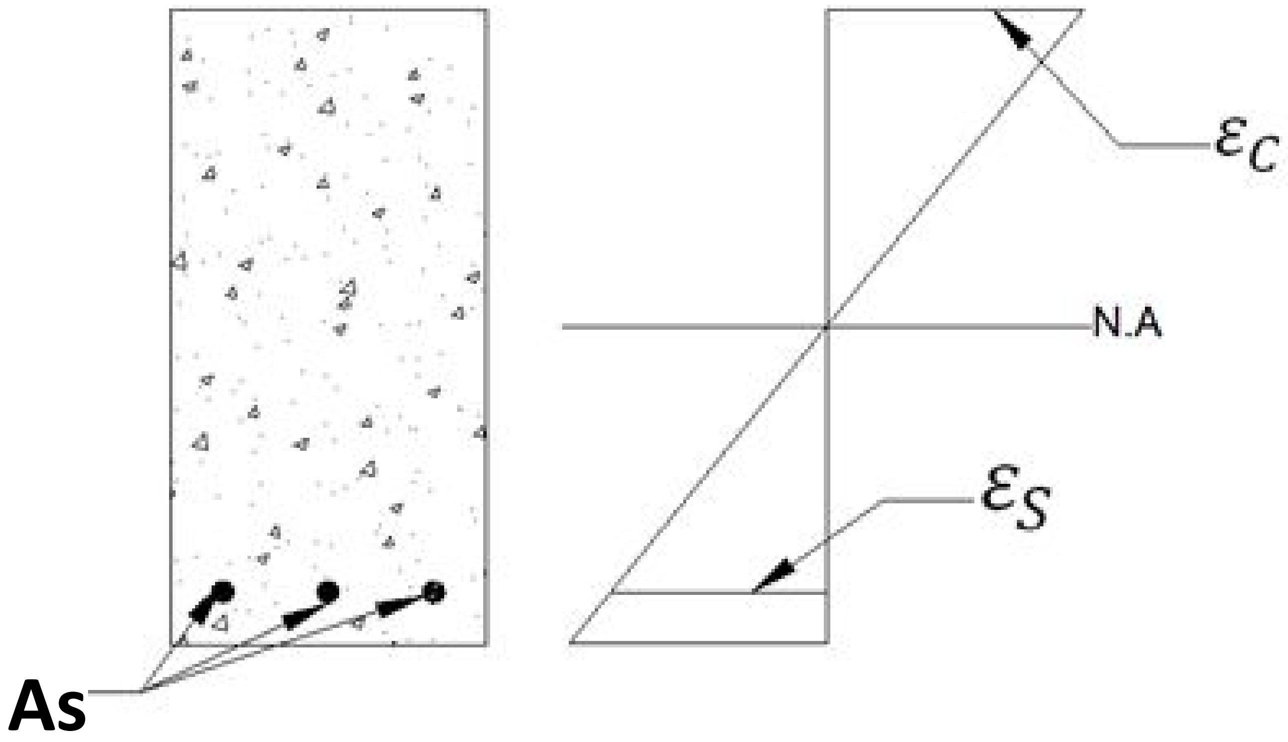
$\sum M = 0$ bending moment(Flexure)



2. Plane section before loading remain plane under loading [strain in beam above and below the N.A are proportional to distance from that axis (N.A)].

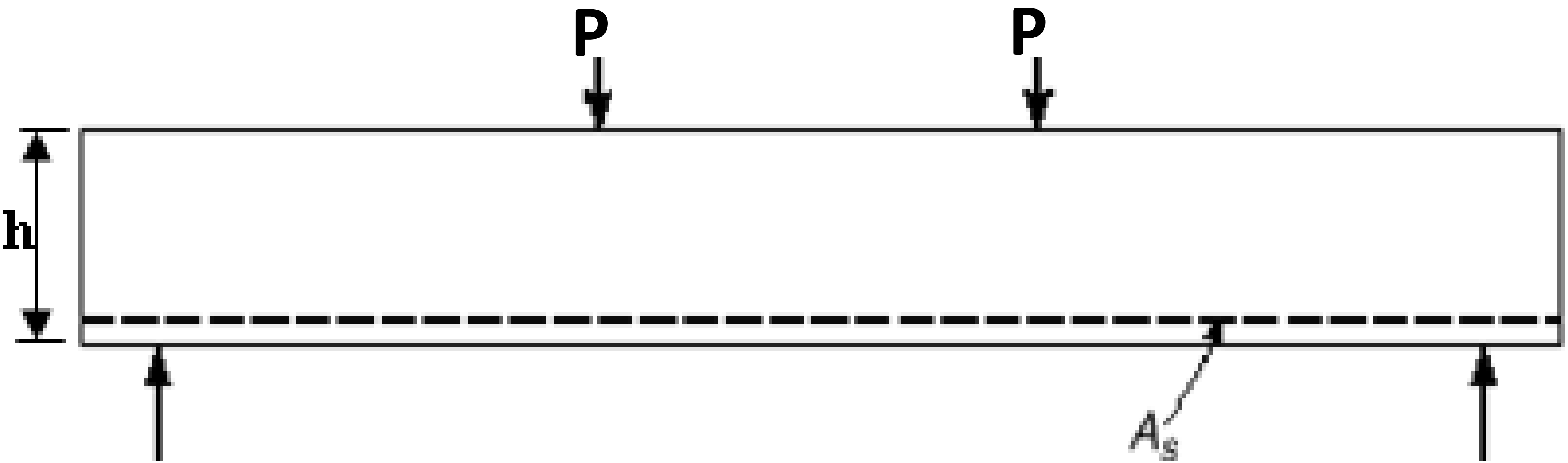


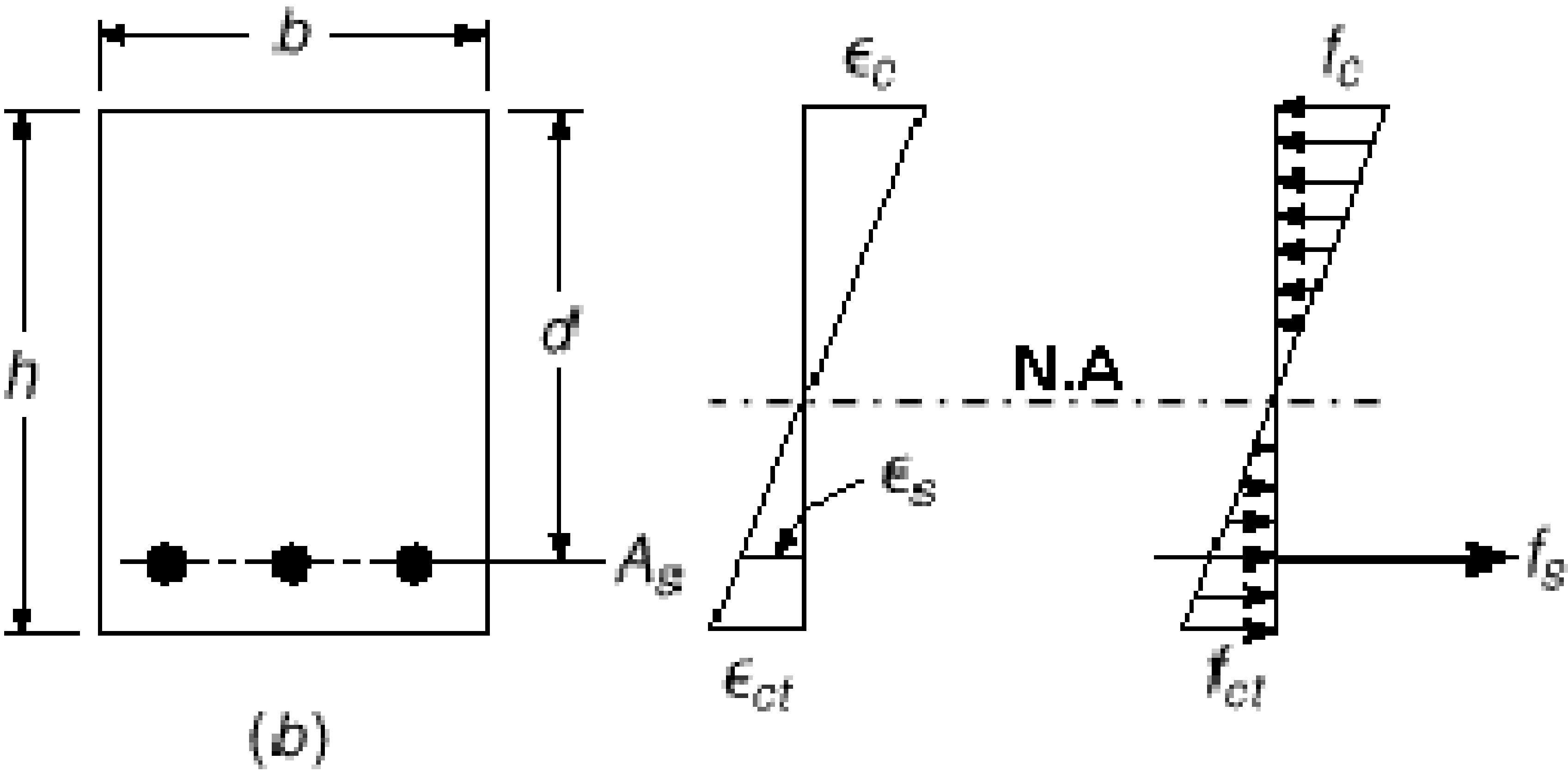
3. Strain in steel is the same as that in concrete at the level of steel (strong bond between steel and concrete)



4. Concrete in tension is assumed in capable of carrying that tension (Ignore tension stresses carried by concrete).
5. Allowable stress in steel $\leq fy$
6. Allowable compression in concrete $\leq fc'$

Behavior of Reinforced concrete Beam under Bending stresses:-

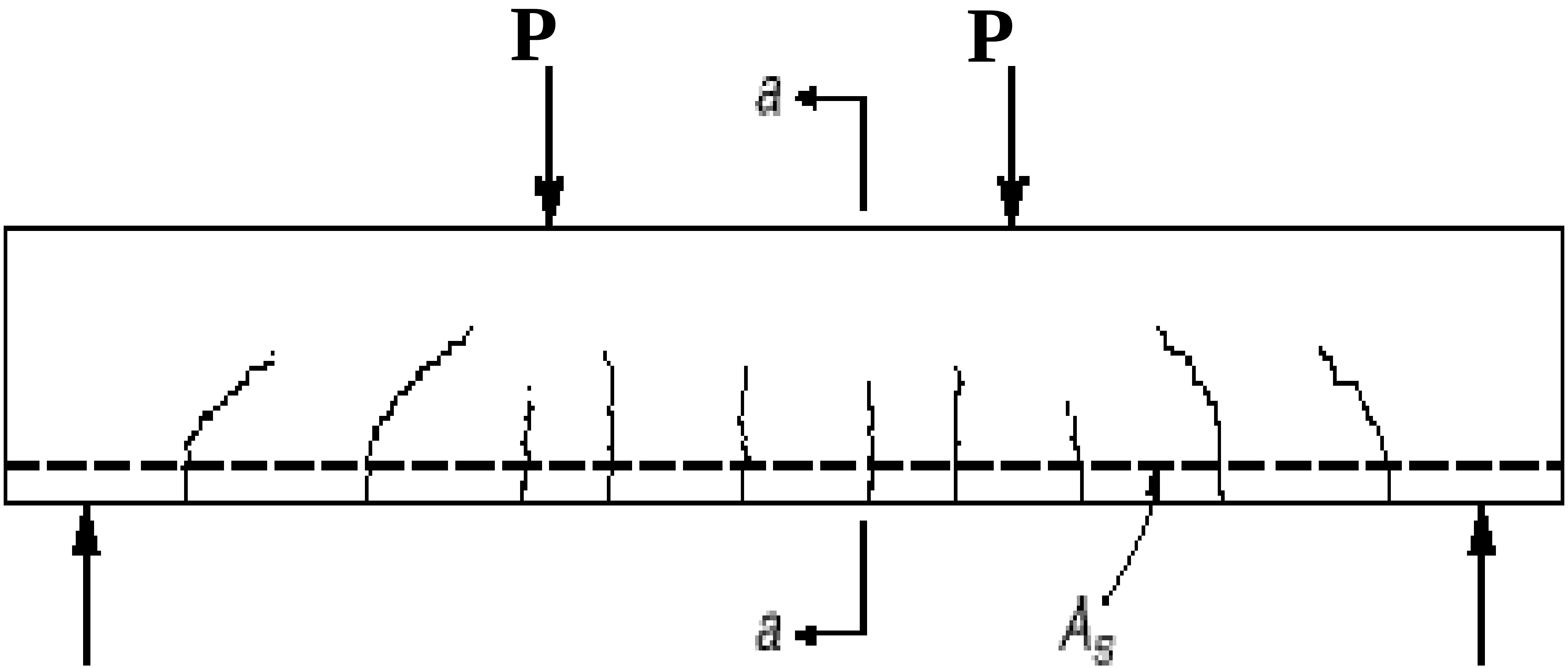




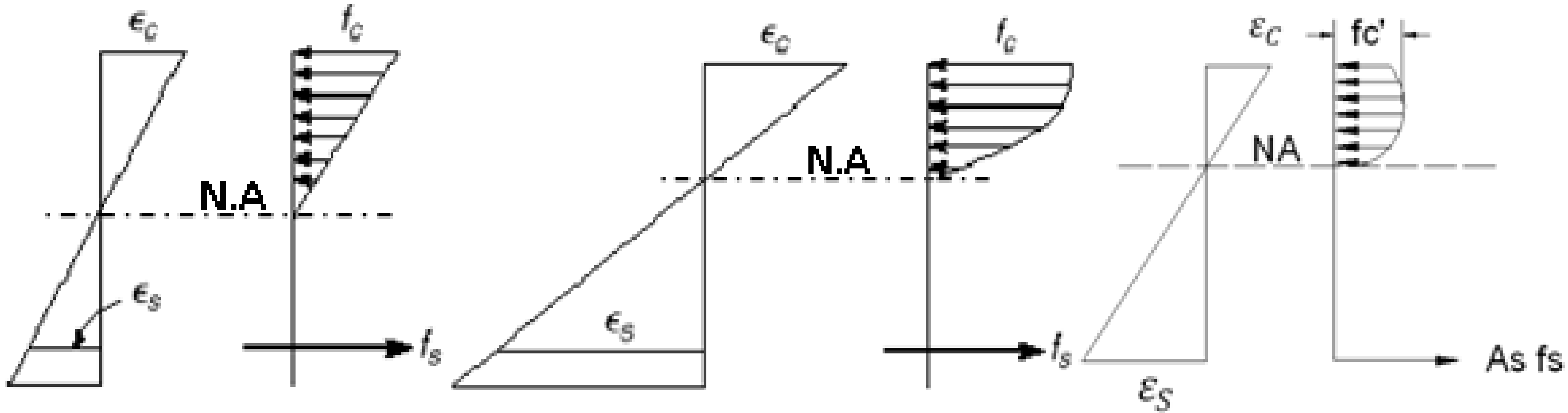
$f_{ct} < f_r$, $f_c \leq f_c'/2$

uncracked elastic

Increasing load P:



(d)



fct ≥f r , fc≤fc'/2
cracked elastic

fct ≥ fr , fc > fc'/2
cracked unelastic

fc > fc'
failure stage

1. Un cracked elastic section $\left[fc_{t} < fr \;\&\; fc \le \frac{fc'}{2}\right]$
2. Cracked elastic section $\left[fc_{t} \ge fr \;\&\; fc \le \frac{fc'}{2}\right]$
3. Cracked unelastic section $\left[fc_{t} \ge fr \;\&\; fc > \frac{fc'}{2}\right]$
4. Failure stage

Types of Failure

1. Balanced failure condition

$\varepsilon_c = \varepsilon_{cu}$, $\varepsilon_S = \varepsilon_y$, As_b:Balance steel area → sudden failure

2. Primary compression failure

$\varepsilon_c=\varepsilon_{cu}$, $\varepsilon_S<\varepsilon_y \rightarrow f_s<f_y$, $As>As_b$ Over reinforced section→ (sudden failure)

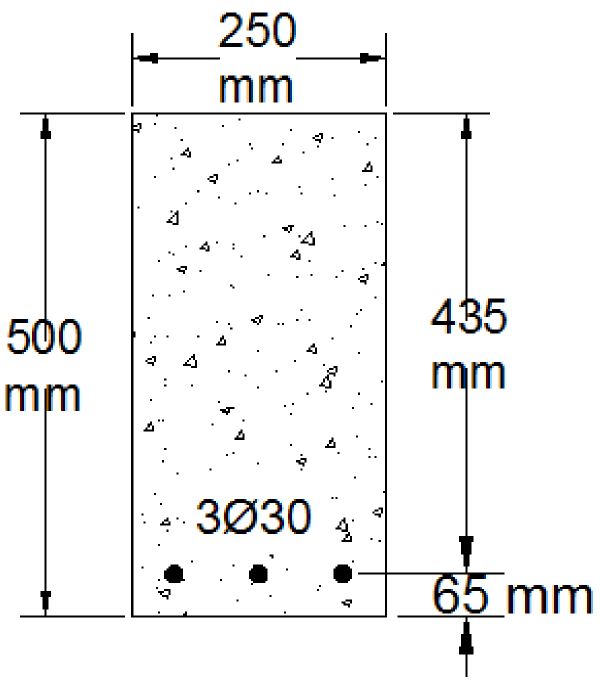
3. Tensile failure followed by secondary compression failure

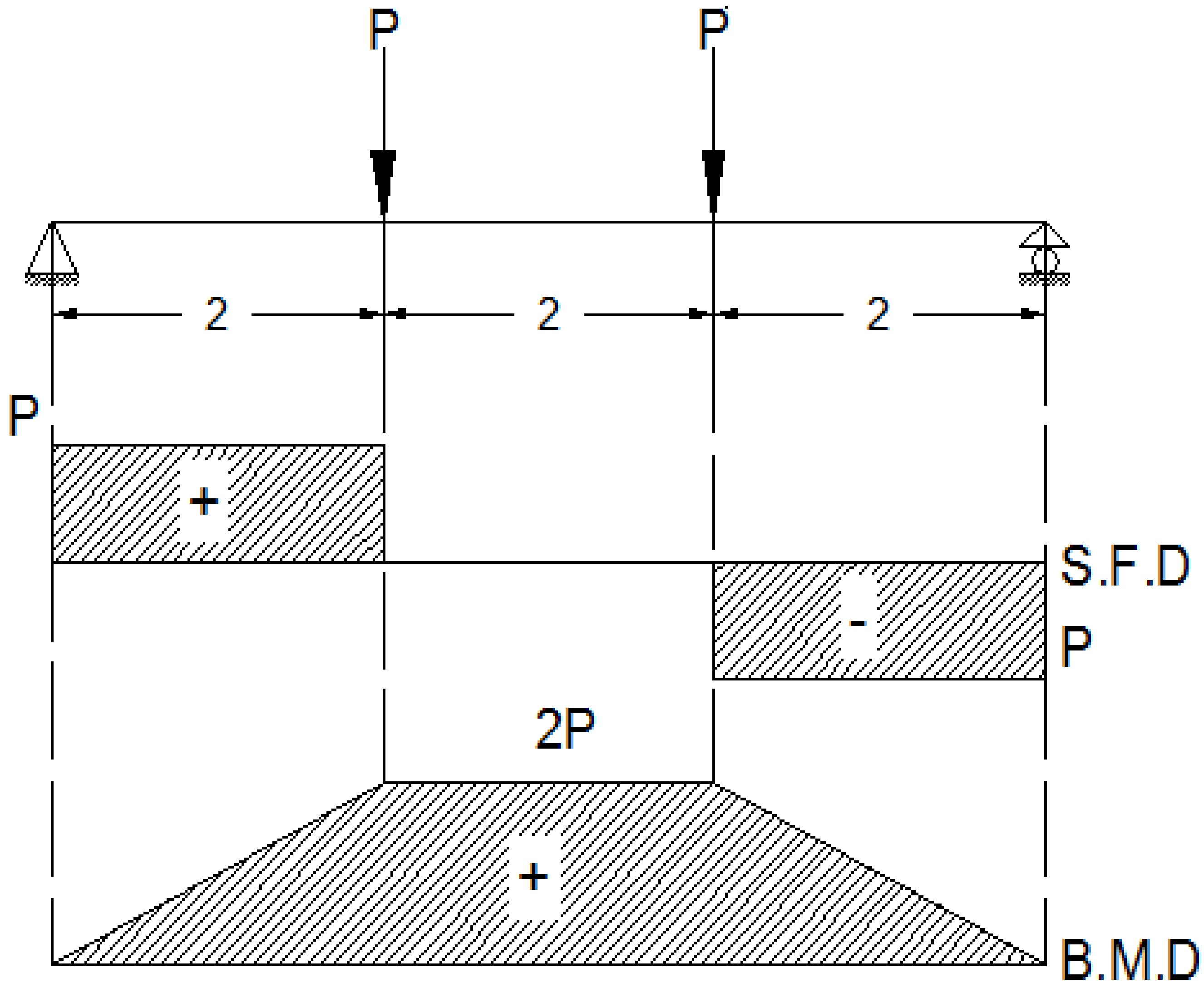
$$\begin{matrix} \varepsilon_S=\varepsilon_y,\ f_s=f_y \\ \varepsilon_c<\varepsilon_{cu} \end{matrix} \xrightarrow{\text{increasing P}} \begin{matrix} \varepsilon_S>\varepsilon_y,\ f_s=f_y \\ \varepsilon_c=\varepsilon_{cu} \end{matrix}$$

$As < As_b$Under reinforced section →(gradual failure a desirable type).

Ex: Concrete grade fc'=30 MPa ,steel grade fy=400 MPa, check stress if:

1. P=17 kN
2. P=34kN
3. P=90kN





$(1) at\ P = 17\ kN$

$M = 2P = 34\ kN.m$

$E_c = 4700\sqrt{fc'} = 25743$ MPa$E_S = 200000$MPa

$$n = \frac{E_S}{E_c} = \frac{200000}{25743} = 7.8 \cong 8$$

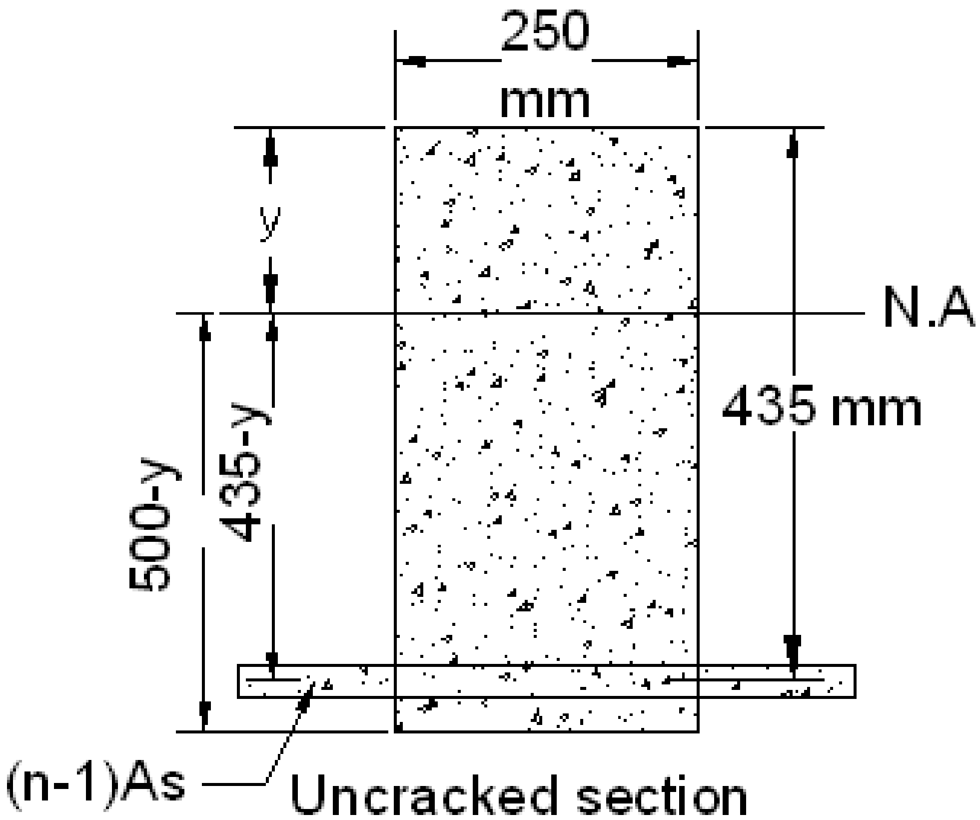
$$Assume\ uncracked\ elastic\ section \rightarrow \delta = \frac{Mc}{I}$$

$Find\ N.A\ location$

$As = 3\emptyset 30 = 2120\ mm^2, (n-1)As = 14840\ mm^2$

$$\sum M_{N.A} = 0 \rightarrow 250(y)\left(\frac{y}{2}\right) = 250 * (500 - y) * \frac{(500-y)}{2} + (8-1)(2120)(435-y) \rightarrow$$

$$y = 270\ mm$$



I_{un}

$$= \frac{0.25 * 0.27^3}{3} + \frac{0.25 * (0.5 - 0.27)^3}{3} + (8-1)(2120) * 10^{-6} * (0.435 - 0.27)^2$$

$$I_{un} = 3.058 * 10^{-3} m^4$$

$$f_{ct} = \frac{Mc}{I} = \frac{34*(0.5-0.27)}{3.058*10^{-3}} *10^{-3}$$

$$=2.56 \text{ MPa} < fr = 0.62\sqrt{fc'} = 3.83 \text{ MPa} \rightarrow$$

$$\therefore \textit{uncracked section}$$

$$fc_{\text{comp.}} = \left[\frac{34*0.27}{3.058*10^{-3}}\right]*10^{-3} = 3 \text{ MPa} < \frac{fc'}{2} = 15 \text{ MPa} \rightarrow$$

$$\therefore \textit{elastic section}$$

$$fs = \left[\frac{34*(0.435-0.27)}{3.058*10^{-3}}\right]*10^{-3}*8 = 14.67 \text{ MPa}$$

$\underline{(2) at\ P = 34\ kN\ , M = 2P = 68\ kN.m}$

$Assume\ elastic\ uncracked\ section$

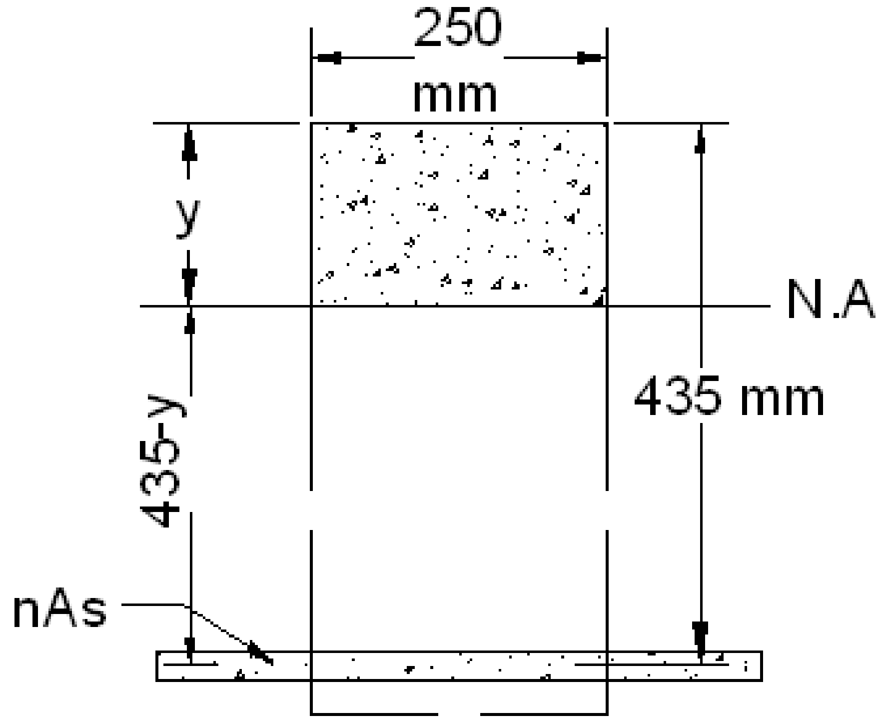
$$fct=\left[\frac{68*(0.5-0.27)}{3.058*10^{-3}}\right]*10^{-3}=5.1\ MPa>fr=3.83\ MPa$$

$\therefore\ cracked\ section$

$Find\ N.A\ location\ (cracked\ section)$

$$\sum M_{N.A} = 0$$

$$250\left(\frac{y^2}{2}\right) = 8(2120)(435 - y)$$



Cracked section

$$Solve\ for\ y \rightarrow y = 184\ mm$$

$$I_{cr} = \frac{0.25 * 0.184^3}{3} + 8 * 2120 * 10^{-6}(0.435 - 0.184)^2$$

$$I_{cr} = 1.587 * 10^{-3} m^4$$

$$fct=\left[\frac{68*(0.5\text{-}0.184)}{1.587*10^{-3}}\right]*10^{-3}=13.5\ \text{MPa} > fr=3.83\ MPa$$

$$\rightarrow\ cracked\ section$$

$$fc_{comp}=\left[\frac{68*(0.184)}{1.587*10^{-3}}\right]*10^{-3}=7.88\ \text{MPa}<\frac{fc^{'}}{2}=15\ \ \text{MPa}$$

$$\therefore\ elastic\ section$$

$$fs=\left[\frac{68*(0.435\text{-}0.184)}{1.587*10^{-3}}\right]*10^{-3}*8=86\ \text{MPa}$$

$(3) P = 90\ kN$

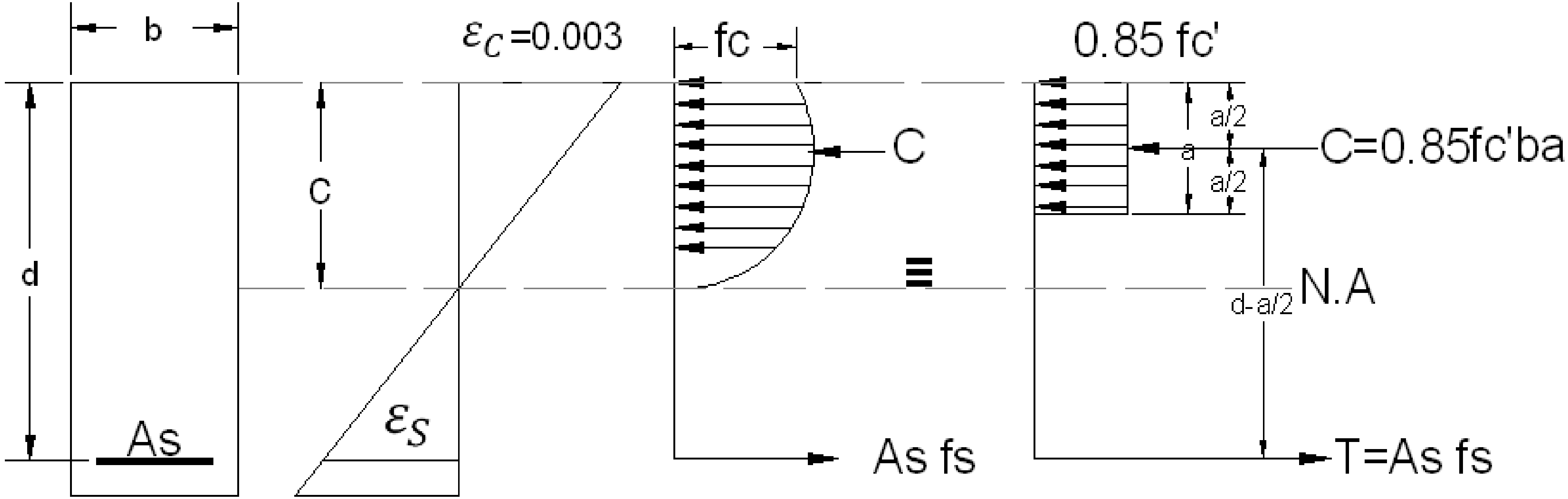
$M = 180\ kN.m$

$P = 90kN > 34kN \therefore cracked\ section$

$$fc_{comp} = \left[\frac{180*0.184}{1.587*10^{-3}}\right]*10^{-3} = 21\ \text{MPa} > \frac{fc'}{2}$$

$$\therefore In\ elastic\ section, \delta \neq \frac{M.c}{I}$$

General Analysis of ultimate strength



Equivalent rectangular stress block or (Whitney stress block)

10.2.7.1 — Concrete stress of $0.85 f_c'$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fiber of maximum compressive strain.

10.2.7.2 — Distance from the fiber of maximum strain to the neutral axis, c, shall be measured in a direction perpendicular to the neutral axis.

$$a=\beta_1 c$$

$$\beta_1=0.85 \text{ for } fc' \leq 28 \text{ MPa} \quad \text{ACI } 10.2.7.3$$

$$\beta_1=0.85-\frac{fc'-28}{7}*0.05 \geq 0.65 \qquad \text{for } fc'>28 \text{ MPa}$$

$$Mn = Asfs\left(d-\frac{a}{2}\right)$$

Or $\quad Mn = 0.85fc'ba\left(d-\frac{a}{2}\right)$

1. Balance failure

10.3.2 — Balanced strain conditions exist at a cross section when tension reinforcement reaches the strain corresponding to f_y just as concrete in compression reaches its assumed ultimate strain of 0.003.

$$\varepsilon_c=\varepsilon_{cu}=0.003, \quad \varepsilon_S=\varepsilon_y \rightarrow fs=fy \qquad \rho=\frac{As}{bd}$$

$$\sum Fx = 0$$

$$As\ fy=0.85\ fc'b.a \qquad]\div bd$$

$$\frac{As_b}{bd} fy=0.85fc' \frac{a}{d} \qquad \rightarrow a=\rho_b \frac{fy}{0.85fc'} d \ldots\ldots\ldots\ldots\ldots(1)$$

ρ_b: blanced steel ratio= $\frac{As_b}{bd}$

From strain diagram:

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_{cu} + \varepsilon_S}{d} \rightarrow c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} * d = \frac{0.003}{0.003 + \frac{f_y}{Es}} * d$$

$$E_s = 200000\ MPa$$

$$c = \frac{600}{600+fy} * d$$

$$\therefore a = \beta_1 c = \frac{600\beta_1}{600+fy} d \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$eq(2) \equiv eq(1)$

$$\rho_b = 0.85\, \beta_1 * \frac{fc'}{fy} * \frac{600}{600+fy}$$ for rectangular section,

dimensions not appeared at the equation

2. $If\ \rho > \rho_b$ over reinforced section ,→ compression failure

$\varepsilon_c = \varepsilon_{cu} = 0.003, \qquad \varepsilon_S < \varepsilon_y \rightarrow f_s < f_y$

$$\sum Fx = 0$$

$$Asfs = 0.85\, fc' b.a \qquad]\div bd$$

$$\therefore \rho fs = 0.85\, fc' * \frac{a}{d} \rightarrow a = \frac{\rho * fs * d}{0.85 * fc'} \ldots\ldots\ldots(1)$$

From strain diagram

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_{cu} + \varepsilon_s}{d} \rightarrow c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_s} * d = \frac{0.003}{0.003 + \frac{f_s}{Es}} * d$$

$$c=\frac{600}{600+fs}*d \quad , \quad a=\beta_1\frac{600}{600+fs}*d\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$eq(1)\equiv eq(2)$$

$$\frac{\rho.fs.d}{0.85fc'}=\beta_1\frac{600}{600+fs}*d$$

2^{nd} order equation Solve for

fs$\rightarrow$ sub. fs in eq(1 or 2)to calculate (a)

$$\rightarrow Mn=As\ fs\left(d-\frac{a}{2}\right)$$

3. If $\rho < \rho_b$ (under reinforced section , tensile failure followed by compression failure)

$$\begin{matrix} \varepsilon_S = \varepsilon_y \\ f_s = f_y \\ \varepsilon_c < \varepsilon_{cu} \end{matrix} \quad \xrightarrow{increasing\ P} \quad \begin{matrix} \varepsilon_S > \varepsilon_y \\ f_s = f_y \\ \varepsilon_c = \varepsilon_{cu} \end{matrix}$$

$$\sum Fx = 0$$

$$Asfy = 0.85\ fc' b.a \qquad]\div bd$$

$$\rho f y = 0.85\ f c' * \frac{a}{d}$$

$$a = \frac{\rho.fy.d}{0.85\ fc'}$$

$$Mn=Asfy\left(d-\frac{a}{2}\right)$$

$$Mn=\rho\ .b\ .d\ fy\left(d-\rho\frac{fyd}{1.7\ fc'}\right)=\rho\ b\ d^2fy\left(1\text{-}0.59\rho\frac{fy}{fc'}\right)$$

The most important equation in analysis and design according to ACI code strength method

Ex: The same previous example calculate

1. Max moment capacity of the beam Mn
2. The corresponding Pn

1. Estimate type of failure

$$\rho_b = 0.85\beta_1 \frac{fc'}{fy} \cdot \frac{600}{600+fy}$$

$$fc' = 30\ MPa > 28MPa \rightarrow \beta_1 = 0.85 - \frac{30-28}{7} * 0.05 = 0.83 \geq 0.65$$

$$\rho_b = 0.85 * 0.83 * \frac{30}{400} * \frac{600}{600+400} = 0.0317$$

$\rho = \frac{2120}{250*435} = 0.0195 < \rho_b$ (Under reinforced section tensile failure followed by compression failure)

$$\sum Fx = 0$$

Asfy=0.85 fc'b.a→a= $\frac{2120*10^{-6}*400}{0.85*30*0.25}$ =0.133 m

Mn=As fy $\left(d-\frac{a}{2}\right)$ =$2120*10^{-6}$ $*400\left(0.435-\frac{0.133}{2}\right)$ =0.312 MN.m

Or

$$Mn=\rho.bd^2fy\left(1\text{-}0.59\rho\frac{fy}{fc'}\right)=0.0195*0.25*0.435^2*400*$$

$$\left(1\text{-}0.59*0.0195*\frac{400}{30}\right)=0.312MN.m=312kN.m$$

2. External moment=Internal moment

$$2pn = Mn \rightarrow 2pn = 312 \rightarrow pn = 156kN$$

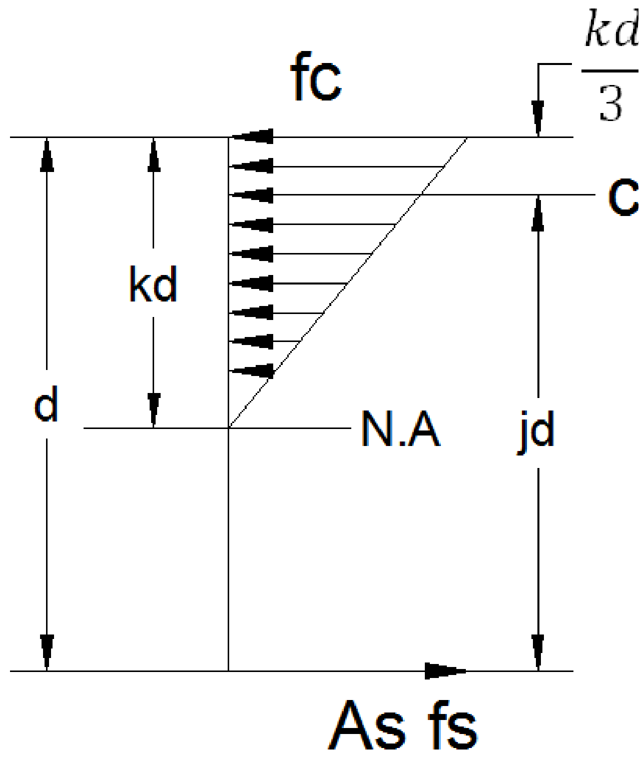
$$c = \frac{a}{\beta_1} = \frac{0.133}{0.83} = 0.16m$$

Stage of loading	P (kN)	M (kN.m)	N.A location (mm)	I (m^4)	fc (MPa)	fs (MPa)
Un cracked elastic section	17	34	270	$3.058 * 10^{-3}$	3	14.67
Cracked elastic section	34	68	184	$1.58 * 10^{-3}$	7.88	86
Ultimate (failure)stage cracked unelastic	156	312	160	-	30	400

Methods of Design

1. Service Load Design method (SLD) (working stress method)

ACI code 1955 , 1963. and British Standard (B.S) CP114 Based on cracked elastic section.



$W = Wd + Wl$

$Wd: dead\ load(service\ load)$

$Wl: live\ load\ (service\ load)$

$fc \leq \alpha_1 fc'$, $\alpha_1 = 0.45$

$fs \le \alpha_2 fy$, $\alpha_2 = 0.5$ $fs \le 170$ Mpa for fy=400 MPa

$fs \le 140$ Mpa for fy=300 , 350 MPa

2. Ultimate Strength Method (USD) or strength method

ACI code 1971, British code (B.S) CP110 Based on failure stage.(fs=fy, fc=fc')

Factor of safety

a. Load factors (ACI 9.2)

$$W_u = C_1 * W_d + C_2 W_l$$

$$C_1, C_2 > 1.0$$

$C_1 = 1.2$ For dead load

$C_2 = 1.6$ For live load

b. Capacity reduction factor $\emptyset$ (ACI 9.3)

$$Mn = \rho b d^2 Fy\left(1 - 0.59\rho\frac{fy}{fc'}\right)$$

$$Mu = \emptyset Mn \qquad \emptyset < 1.0$$

$\emptyset$=0.9 for $\varepsilon_t \geq 0.005$ ($\rho \leq \rho_t$) tension control

$\emptyset$=$0.483 + 83.3\varepsilon_t$ for $0.002 \leq \varepsilon_t \leq 0.005$ ($\rho > \rho_t$) transition

$\emptyset$=0.65 for $\varepsilon_t \leq \varepsilon_y$=0.002 compression control

$$\rho_t = 0.85\,\beta_1\frac{fc'}{fy}\,\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005}$$

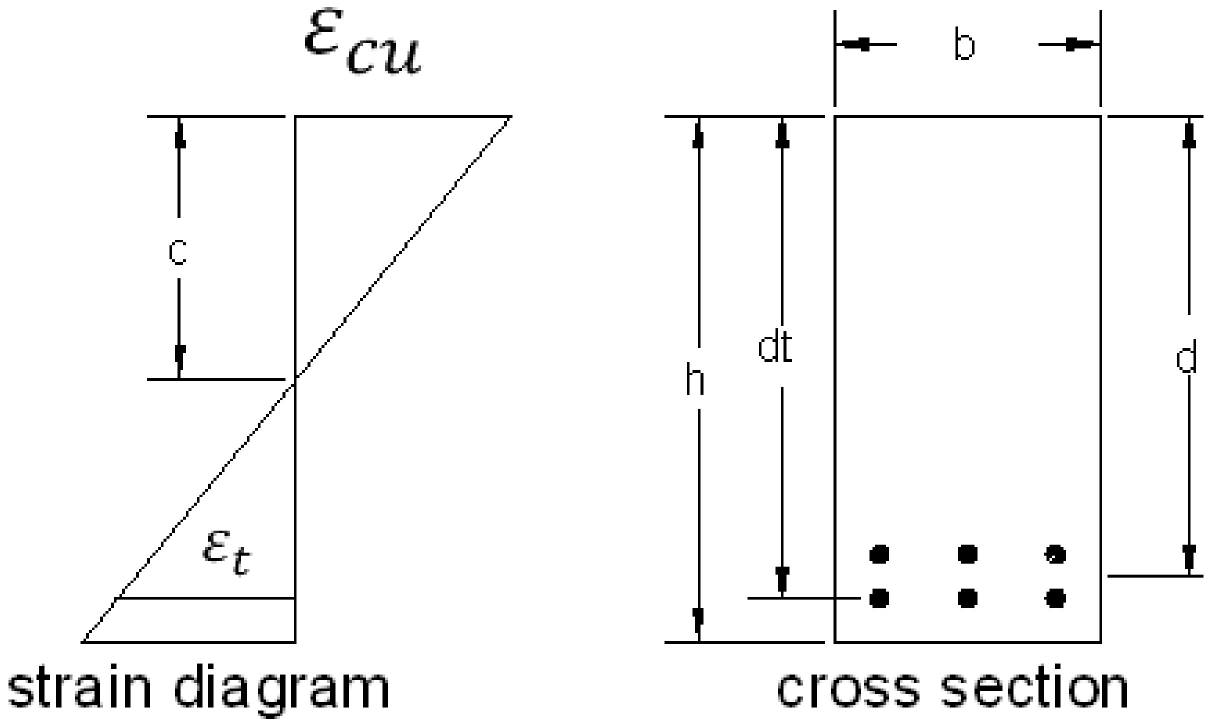
$$\varepsilon_{cu} = 0.003$$

ε_t: net tensile strain in the extreme tension steel

$$\varepsilon_t = \varepsilon_{cu}\frac{dt-c}{c}$$

$dt = d$ for one layer of reinforcement

$dt \cong d$ for two layers of reinforcement and more.



9.3.2.1 — Tension-controlled sections
as defined in 10.3.4 .... 0.90
(See also 9.3.2.7)

9.3.2.2 — Compression-controlled sections, as defined in 10.3.3:

(a) Members with spiral reinforcement
conforming to 10.9.3 .... 0.70

(b) Other reinforced members 0.65

For sections in which the net tensile strain in the extreme tension steel at nominal strength, ε_t, is between the limits for compression-controlled and tension-controlled sections, ϕ shall be permitted to be linearly increased from that for compression-controlled sections to 0.90 as ε_t increases from the compression-controlled strain limit to 0.005.

10.3.3 — Sections are compression-controlled if the net tensile strain in the extreme tension steel, ε_t, is equal to or less than the compression-controlled strain limit when the concrete in compression reaches its assumed strain limit of 0.003. The compression-controlled strain limit is the net tensile strain in the reinforcement at balanced strain conditions. For Grade 420 reinforcement, and for all prestressed reinforcement, it shall be permitted to set the compression-controlled strain limit equal to 0.002.

10.3.4 — Sections are tension-controlled if the net tensile strain in the extreme tension steel, ε_t, is equal to or greater than 0.005 when the concrete in compression reaches its assumed strain limit of 0.003. Sections with ε_t between the compression-controlled strain limit and 0.005 constitute a transition region between compression-controlled and tension-controlled sections.

H.W Show that $\rho_t = 0.85\beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005}$

$$\sum Fx = 0$$

$As\ fy = 0.85\ fc'\, b.a$ $\quad\quad\quad]\div bd$

$\frac{As_t}{bd} fy = 0.85 fc' \frac{a}{d}$ $\quad\quad \rightarrow a = \rho_t \frac{fy}{0.85fc'} d$……………………(1)

ρ_t: max. steel ratio at which net steel tensile strain

exceed $0.005 = \frac{As_t}{bd}$

From strain diagram:

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_{cu} + \varepsilon_S}{d} \rightarrow c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_t} * d \; = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} * d$$

$$a = \beta_1 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} * d \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$(2) \equiv (1)$$

$$\rho_t \frac{\text{fy}}{0.85\text{fc'}} \text{d} = \beta_1 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} * d$$

$$\rho_t = 0.85\beta_1 \frac{\text{fc'}}{\text{fy}} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005}$$