Serviceability Requirements:

$$Design \rightarrow \begin{cases} \rightarrow strength\ requirements \\ \rightarrow servceability\ requirements \end{cases}$$

servceability requirements

1. Control of cracking
2. Control of deflection

Control of deflection:

Deflections are to be calculated for service load.

Use of limiting Span/Depth Ratio:

Table 9.5-a, use for one way slabs or non-prestressed beams.

9.5.2.1 — Minimum thickness stipulated in Table 9.5(a) shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections, unless computation of deflection indicates a lesser thickness can be used without adverse effects.

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:

Values given shall be used directly for members with normalweight concrete (density w_c = 2320 kg/m^3) and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:

a) For structural lightweight concrete having unit density, w_c, in the range 1440-1920 kg/m^3, the values shall be multiplied by $(1.65 - 0.003w_c)$ but not less than 1.09.

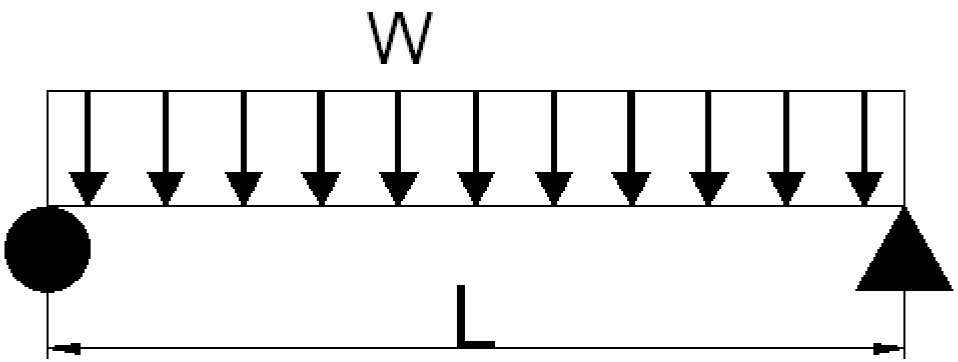
b) For f_y other than 420 MPa, the values shall be multiplied by $(0.4 + f_y/700)$.

Permissible Deflections:

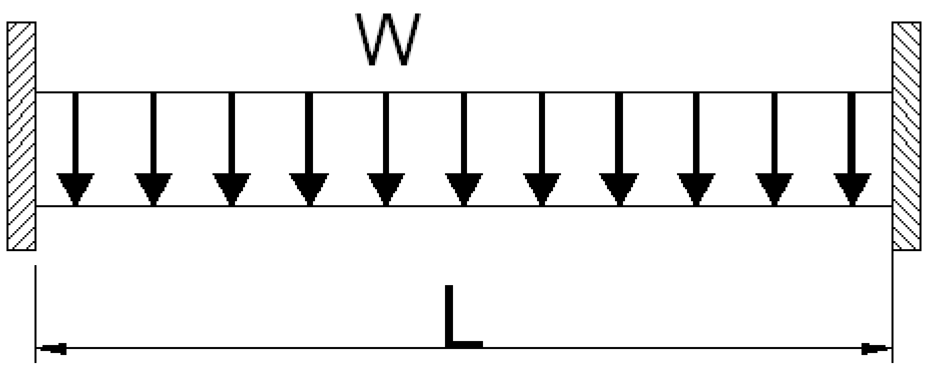
<table>
<tr><td colspan="3">Table 9.5-b, maximum permissible computed deflections.</td></tr>
<tr><th>Type of member</th><th>Deflection to be considered</th><th>Deflection limitation</th></tr>
<tr><td>Flat roofs not supporting or attached to non-structural elements likely to be damaged by large deflections</td><td>Immediate deflection due to live load</td><td>L/180</td></tr>
<tr><td>Floors not supporting or attached to non-structural elements likely to be damaged by large deflections</td><td>Immediate deflection due to live load</td><td>L/360</td></tr>
<tr><td>Roof or floors construction supporting or attached to non-structural elements likely to be damaged by large deflections</td><td rowspan="2">That part of the total deflection occurring after attachment of nonstructural elements(sum of the long term deflection due to all sustained loads and the immediate deflection due to any additional live load)</td><td>L/480</td></tr>
<tr><td>Roof or floors construction supporting or attached to non-structural elements not likely to be damaged by large deflections</td><td>L/240</td></tr>
</table>

Deflection Computations:

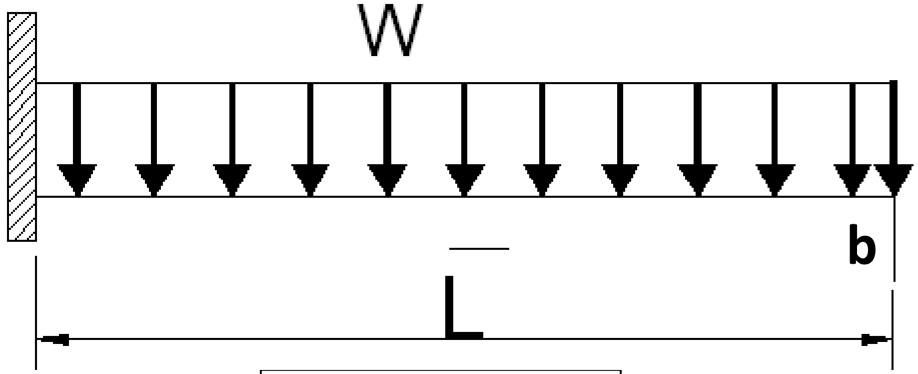
- **Immediate deflection:** deflections that occur at once upon application of load.
- **Long time deflection**: deflections that occur gradually over an extended period of time. These time dependent deflections are chiefly due to concrete creep and shrinkage, this continue over a period of several years.



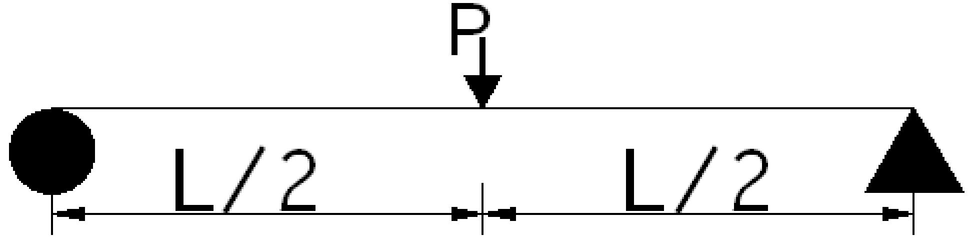
$$\delta_{CL} = \frac{5Wl^4}{384EI}$$



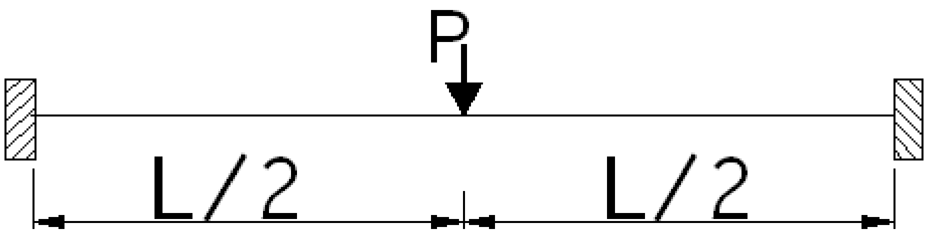
$$\delta_{CL} = \frac{Wl^4}{384EI}$$



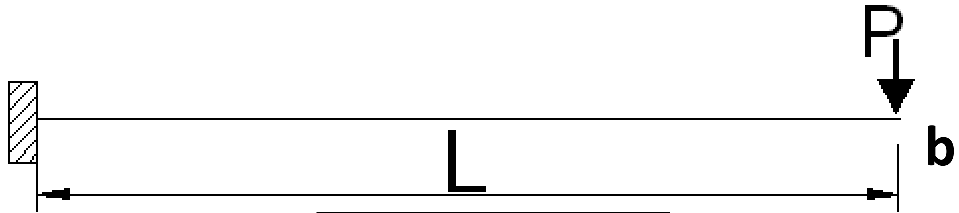
$$\delta = \frac{Wl^4}{8EI}$$



$$\delta_{CL} = \frac{Pl}{48EI}$$



$$\delta_{CL} = \frac{Pl}{192EI}$$



$$\delta = \frac{Pl}{3EI}$$

Maximum deflection equations

Immediate deflection(δi)

Effective moment of inertia for computation of immediate deflection (Ie)

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \ldots ACI\ 9-8$$

$$M_{cr} = \frac{f_r I_g}{y_t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ACI\ 9-9$$

$$f_r = 0.62\sqrt{fc'} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots ACI9-10$$

M_{cr}:cracking moment

f_r:modulus of rupture

y_t:distance from centeriodal axis to extreme fiber in tension(gross section)

I_g:moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement.

I_{cr}: moment of inertia of cracked transformed section.

M_a:maximum moment in member at stage deflection is computed.

Long time deflection(δ_{LT})

$\boldsymbol{\delta_{LT}} = \lambda\ \boldsymbol{\delta i}_{sust}$

λ:multiplier for additional long-time deflection.

$$\lambda = \frac{\zeta}{1 + 50\rho'}$$

ρ':reinforcement ratio for non-prestressed compression reinforcement

$\rho' = As'/(b\ d)$(at midspan for simple and continuous spans, or at support for cantilevers)

ζ = time dependent factor sustained load.

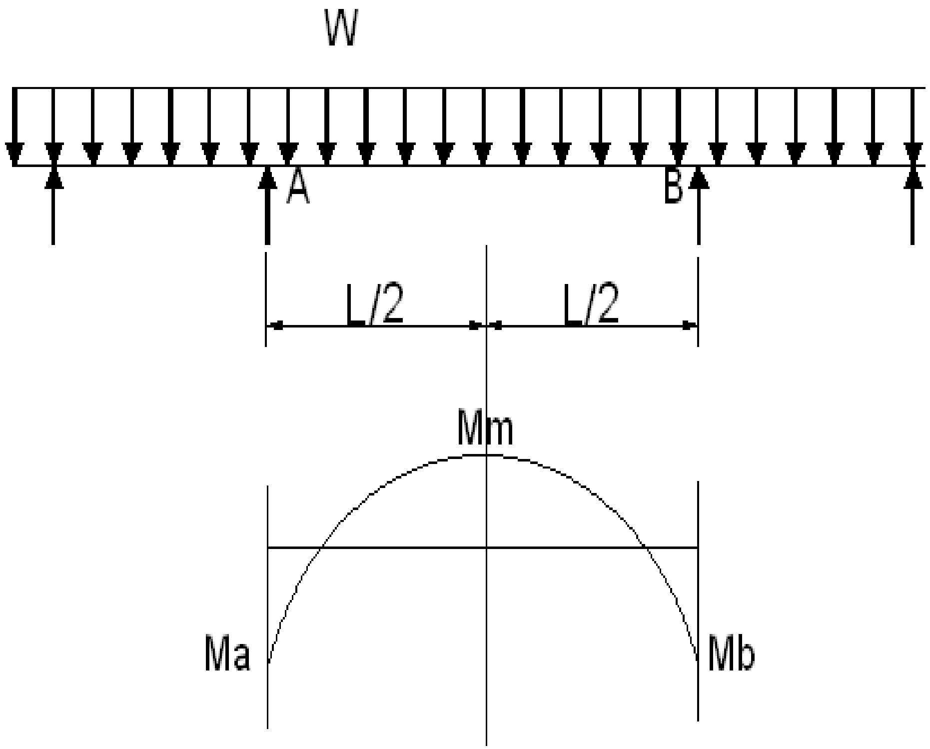
5 years or more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

Sustained load: Permanent load such: dead load, and percent of live load according to building uses (20-25% for office , 70-80% for ware house)

For continuous spans:

Deflection at midspan for one or two ends continuous under uniformly distributed load:

$$\delta_m = \frac{5l^2}{48EI_e}\left[M_m + \frac{1}{10}(M_a + M_b)\right]$$

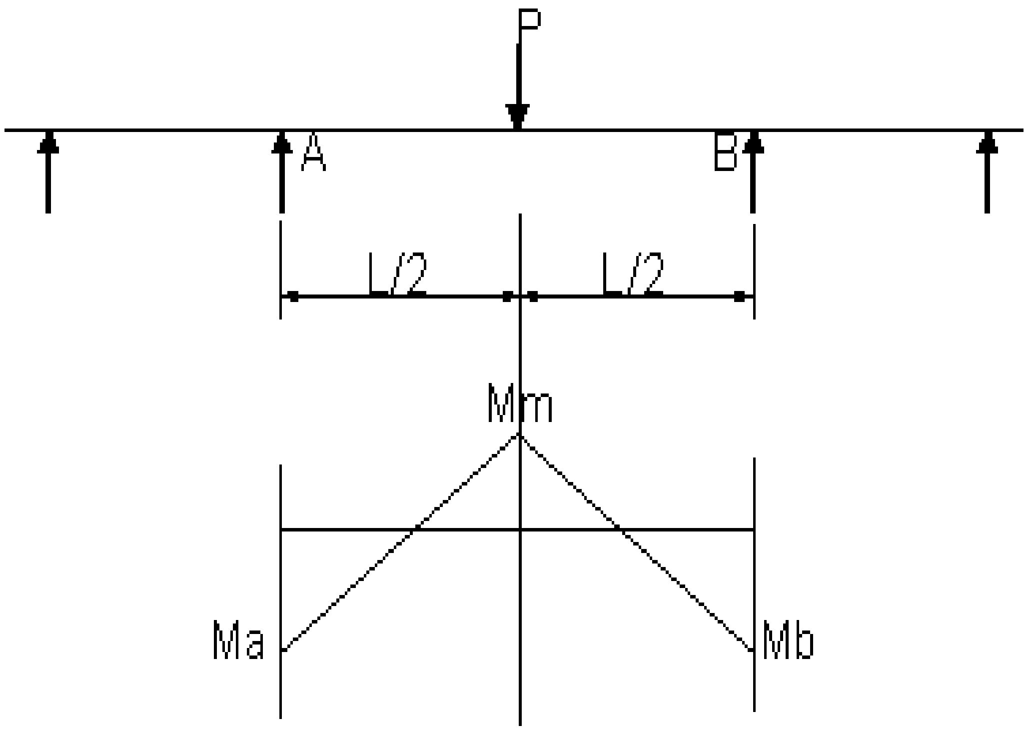


M_m:positive moment at midspan

M_a, M_b:negative moments at supports.

Deflection at midspan for one or two ends continuous under concentrated load:

$$\delta_m = \frac{l^2}{48EI_e}[4M_m + M_a + M_b]$$



M_m:positive moment at midspan

M_a, M_b:negative moments at supports.

$$I_e = 0.5I_{em} + 0.25(I_{e1} + I_{e2})$$

I_{em}:effective moment of inertia for midspan section

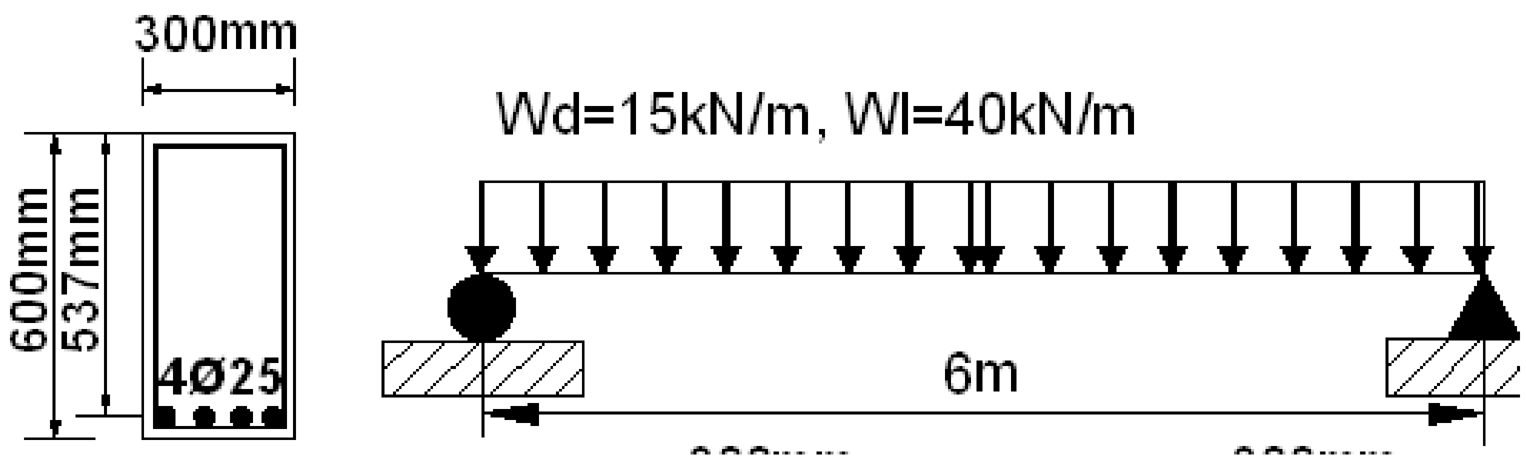
I_{e1}, I_{e2}:effective moment of inertia at negative moment sections at the beam ends.

Ex1:

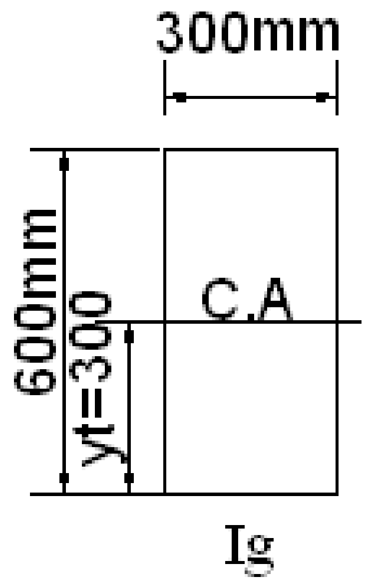
fy=400MPa, fc'=30MPa,ζ=2.0, assume 25%of L.L as sustained.

Find:

1. Immediate deflection due to (L.L+DL).
2. Maximum deflection.
3. Check deflection limit, if the member is construction supporting nonstructural elements likely to be damaged by large deflection.



Solution:



$$Ec = 4700\sqrt{30} = 25743MPa$$

$$n = \frac{Es}{Ec} = \frac{200000}{25743} = 8$$

$$Ig = \frac{bh^3}{12} = \frac{0.3 * 0.6^3}{12} = 5.4 * 10^{-3}m^4$$

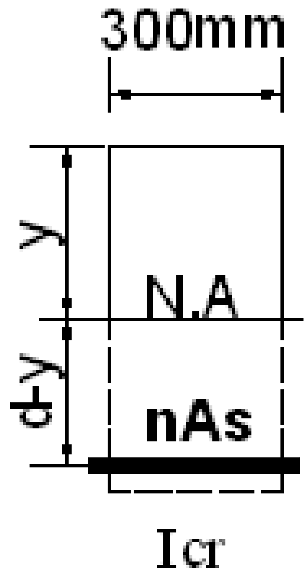
Calculation of *Icr*

Location of NA

As=4Ø25=1963mm^2

$$\sum M_{NA} = 0$$

$$b * y * \frac{y}{2} = nAs(d - y)$$



$$300 * y * \frac{y}{2} = 8 * 1963(537 - y)$$

$$150y^2 + 15704y - 8433048 = 0 \rightarrow y = 190mm$$

OR $\rho = \frac{1963}{300*537} = 0.0122$

$$k = \sqrt{(2n\rho) + (n\rho)^2} - n\rho$$

$$= \sqrt{(2 * 8 * 0.0122) + (8 * 0.0122)^2} - 8 * 0.0122$$

$$= 0.355$$

y=kd=0.355*537=190mm

$$Icr = \frac{0.3 * 0.19^3}{3} + 8 * 1963 * 10^{-6}(0.537 - 0.19)^2$$

$$= 2.577 * 10^{-3} m^4$$

$$Ma = \frac{Wl^2}{8} = \frac{(15 + 40) * 6^2}{8} = 247.5 kN.m$$

$$f_r = 0.62\sqrt{fc'} = 0.62\sqrt{30} = 3.396 MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.396 * 5.4 * 10^{-3}}{0.3} = 69 kN.m$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

$$I_e = \left(\frac{69}{247.5}\right)^3 * 5.4 * 10^{-3} + \left[1 - \left(\frac{69}{247.5}\right)^3\right] * 2.577 * 10^{-3}$$

$$= 2.638 * 10^{-3} m^4 \le I_g = 5.4 * 10^{-3} m^4$$

1. $\delta i_{(D+L)} = \frac{5Wl^4}{384EI} = \frac{5(15+45)*6^4}{384*25743*10^3*2.638*10^{-3}} * 10^3 =$

$13.66mm$

2. $\delta i_{(D+0.25L)} = 13.66 * \frac{(15+0.25*40)}{(15+40)} =$

$6.21mm \;\; due\ to\ sustaind\ load$

$$\lambda = \frac{\zeta}{1 + 50\rho'} = \frac{2.0}{1 + 50 * 0} = 2.0$$

$\delta_{LT} = \lambda\, \delta i_{sust} = 2 * 6.21 = 12.42mm$

$$\max defl. = \delta i_{(D+L)} + \delta LT = 13.66 + 12.42 = 26.08mm$$

3. $\delta i_{(0.75L)} = 13.66 * \frac{(0.75*40)}{(15+40)} = 7.45mm$

$$\delta LT_{(all\ sustaind\ load)} + \delta i_{(any\ additional\ LL)} = 12.42 + 7.45$$
$$= 19.87mm > defl.limit = \frac{l}{480} = \frac{6000}{480}$$
$$= 12.5mm\ \ N.G$$

Solution:

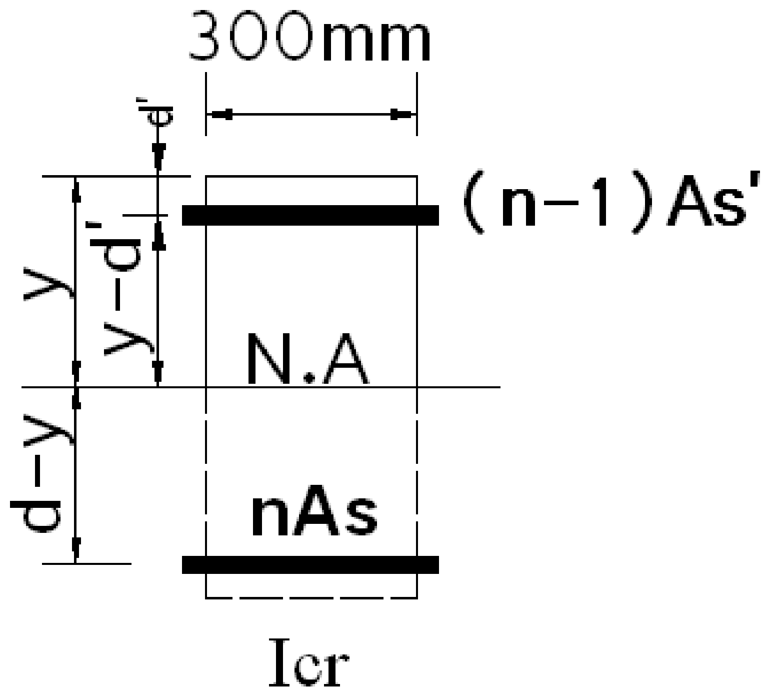
1. Use compression reinforcement if the difference is small to decrease long time deflection also it effects on Icr
2. Increase (h) if the difference between the actual and allowable deflection is large.

Ex2: The same information of example1, add $As'=2Ø20=628mm^2$, $d'=60mm$.

Solution:

Location of NA

$As=4Ø25=1963mm^2$



$$\sum M_{NA} = 0$$

$$b * y * \frac{y}{2} + (n-1)As' * (y - d') = nAs(d - y)$$

$$300 * y * \frac{y}{2} + (8-1) * 628 * (y - 60) = 8 * 1963(537 - y)$$

$$150y^2 + 20100y - 8696808 = 0 \rightarrow y = 183mm$$

$$Icr$$

$$= \frac{0.3 * 0.183^3}{3} + (8-1) * 628 * 10^{-6}(0.183 - 0.06)^2 + 8$$

$$* 1963 * 10^{-6}(0.537 - 0.183)^2 = 2.647 * 10^{-3}m^4$$

$$I_e = \left(\frac{69}{247.5}\right)^3 * 5.4 * 10^{-3} + \left[1 - \left(\frac{69}{247.5}\right)^3\right] * 2.647 * 10^{-3}$$

$$= 2.707 * 10^{-3}m^4 \leq I_g = 5.4 * 10^{-3}m^4$$

1. $\delta i_{(D+L)} = \frac{5Wl^4}{384EI} = \frac{5(15+45)*6^4}{384*25743*10^3*2.707*10^{-3}} * 10^3 =$

$13.32mm$

$$2.\ \delta i_{(D+0.25L)} = 13.32 * \frac{(15+0.25*40)}{(15+40)} =$$

$$6.05mm \ \ due\ to\ sustaind\ load$$

$$\rho' = \frac{628}{300 * 537} = 0.0039$$

$$\lambda = \frac{\zeta}{1 + 50\rho'} = \frac{2.0}{1 + 50 * 0.0039} = 1.67$$

$$\delta_{LT} = \lambda\ \delta i_{sust} = 1.67 * 6.05 = 10.12mm$$

$$\max defl. = \delta i_{(D+L)} + \delta LT = 13.32 + 10.12 = 23.44mm$$

3. $\delta i_{(0.75L)} = 13.55 * \frac{(0.75*40)}{(15+40)} = 7.26mm$

$$\delta LT_{(all\ sustaind\ load)} + \delta i_{(any\ additional\ LL)} = 10.12 + 7.26$$

$$= 17.38mm > defl.limit = \frac{l}{480} = \frac{6000}{480}$$

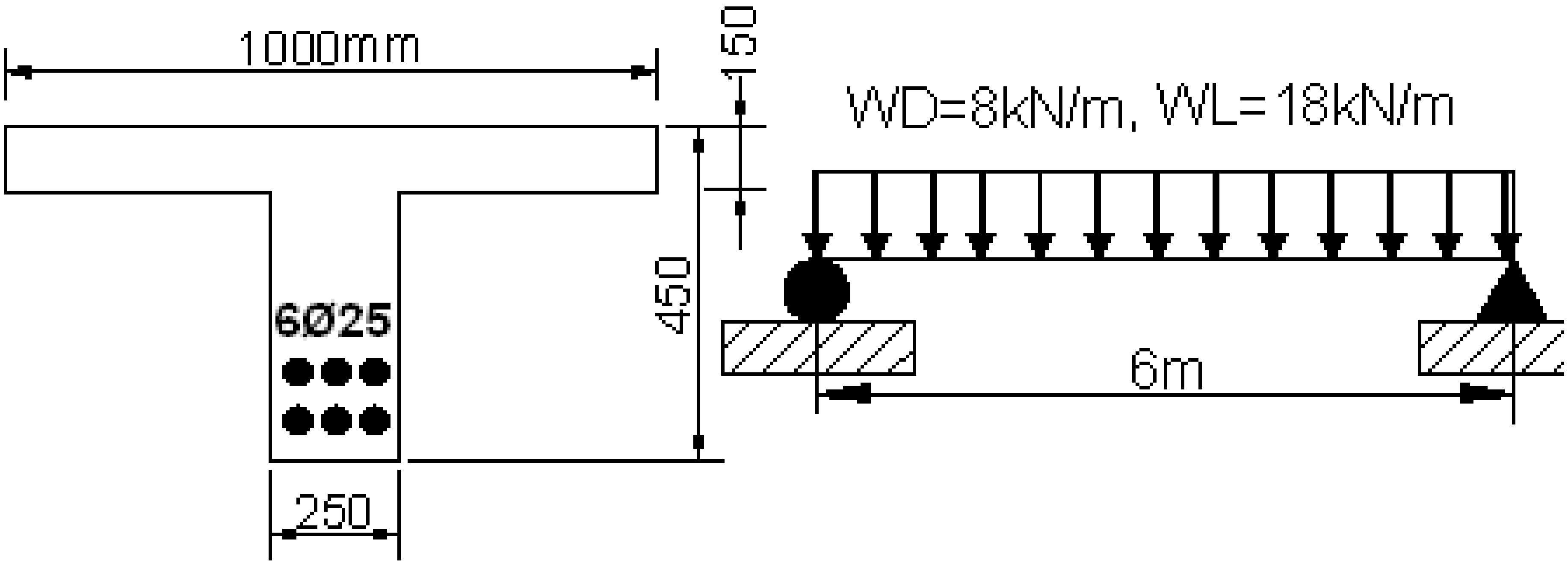
$$= 12.5mm\ \ N.G \rightarrow$$

$$\therefore The\ \ stiffness\ of\ the\ member\ is\ not\ sufficient$$

Ex3:

fy=300MPa, fc'=20MPa,ζ=2.0, assume 30%of L.L as sustained.
Find:

1. Immediate deflection and long time deflection
2. Check deflection limit, if the member is floor not supporting partitions likely to be damaged by large



deflection.

Solution:

$$Ec = 4700\sqrt{20} = 21019MPa$$

$$n = \frac{Es}{Ec} = \frac{200000}{21019} = 10$$

As(6Ø25)=2945mm^2

d=450-40-10-25-25/2=363mm

Ig, calculation

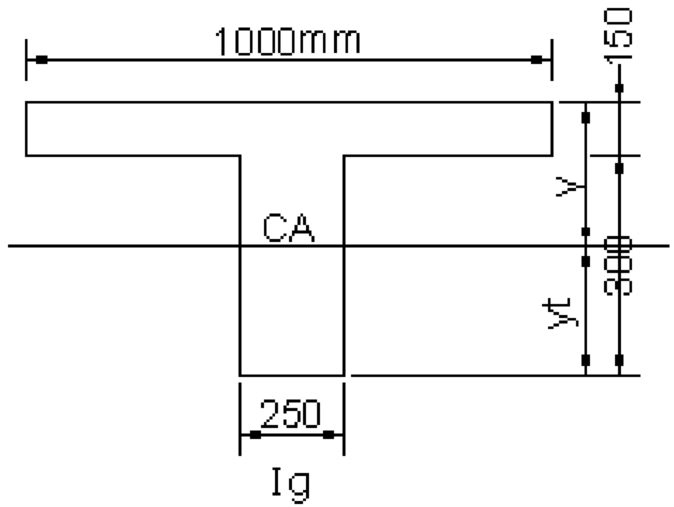
Find location of C.A

$$\sum M_{top\ fiber}$$

$$1000 * 150 * \frac{150}{2} + 250 * 300 *$$

$$\left(150 + \frac{300}{2}\right) = (1000 * 150 + 250 *$$

$$300)y \rightarrow y = 150mm$$



$$Ig = \frac{1 * 0.15^3}{3} + \frac{0.25 * 0.3^3}{3}$$

$$= 3.375 * 10^{-3} m^4$$

$$f_r = 0.62\sqrt{fc'} = 0.62\sqrt{20} = 2.773MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{2.773 * 3.375 * 10^{-3}}{(0.45 - 0.15)} = 35kN.m$$

Icr, calculation

$$M1 = mom.of\ comp.area = 1000 * 150 * \frac{150}{2}$$

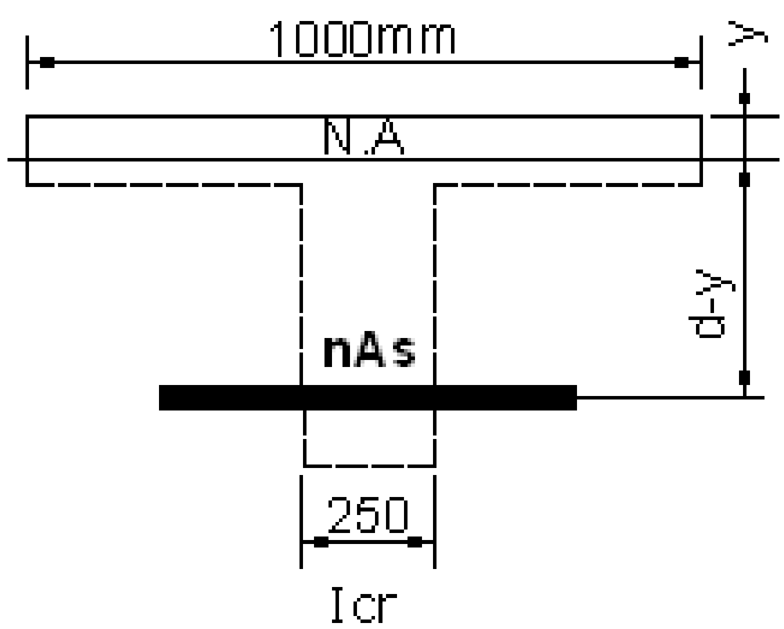
$$= 11250000mm^3$$

$$M2 = mom.of\ ten.area = 10 * 2945 * (363 - 150) =$$

$$6250000mm^3$$

Since M1>M2→N.A, within flange depth(i.e, y=kd<hf)

$$\sum M_{NA} = 0$$



$$b * y * \frac{y}{2} = nAs(d - y)$$

$$1000 * y * \frac{y}{2} = 8 * 2945(363 - y)$$

$$500y^2 + 2945y - 10690350 = 0 \rightarrow y = 120mm$$

OR $\rho = \frac{2945}{1000*363} = 0.0081$

$$k = \sqrt{(2n\rho) + (n\rho)^2} - n\rho$$

$$= \sqrt{(2 * 10 * 0.0081) + (10 * 0.0081)^2} - 10 * 0.0081$$

$$= 0.33$$

y=kd=0.33*363=120mm

$$Icr = \frac{1 * 0.12^3}{3} + 10 * 2945 * 10^{-6}(0.363 - 0.12)^2$$

$$= 2.331 * 10^{-3}m^4$$

$$Ma = \frac{Wl^2}{8} = \frac{(8 + 18) * 6^2}{8} = 117kN.m$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 \ I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

$$I_e = \left(\frac{35}{117}\right)^3 * 3.375 * 10^{-3} + \left[1 - \left(\frac{35}{117}\right)^3\right] * 2.331 * 10^{-3}$$

$$= 2.36 * 10^{-3}m^4 \leq I_g = 3.375 * 10^{-3}m^4$$

1. $\delta i_{(D+L)} = \frac{5Wl^4}{384EI} = \frac{5(18+8)*6^4}{384*21019*10^3*2.36*10^{-3}} * 10^3 =$

$8.84mm$

$$\delta i_{(D+0.3L)} = 8.84 * \frac{(8 + 0.3 * 18)}{(8 + 18)}$$

$$= 4.56mm \;\; due\ to\ sustaind\ load$$

$$\lambda = \frac{\zeta}{1 + 50\rho'} = \frac{2.0}{1 + 50 * 0} = 2.0$$

$\delta_{LT} = \lambda\ \delta i_{sust} = 2 * 4.56 = 9.116mm$

$$\delta total = \delta i_{(D+L)} + \delta LT = 8.84 + 9.116 = 17.95\ mm$$

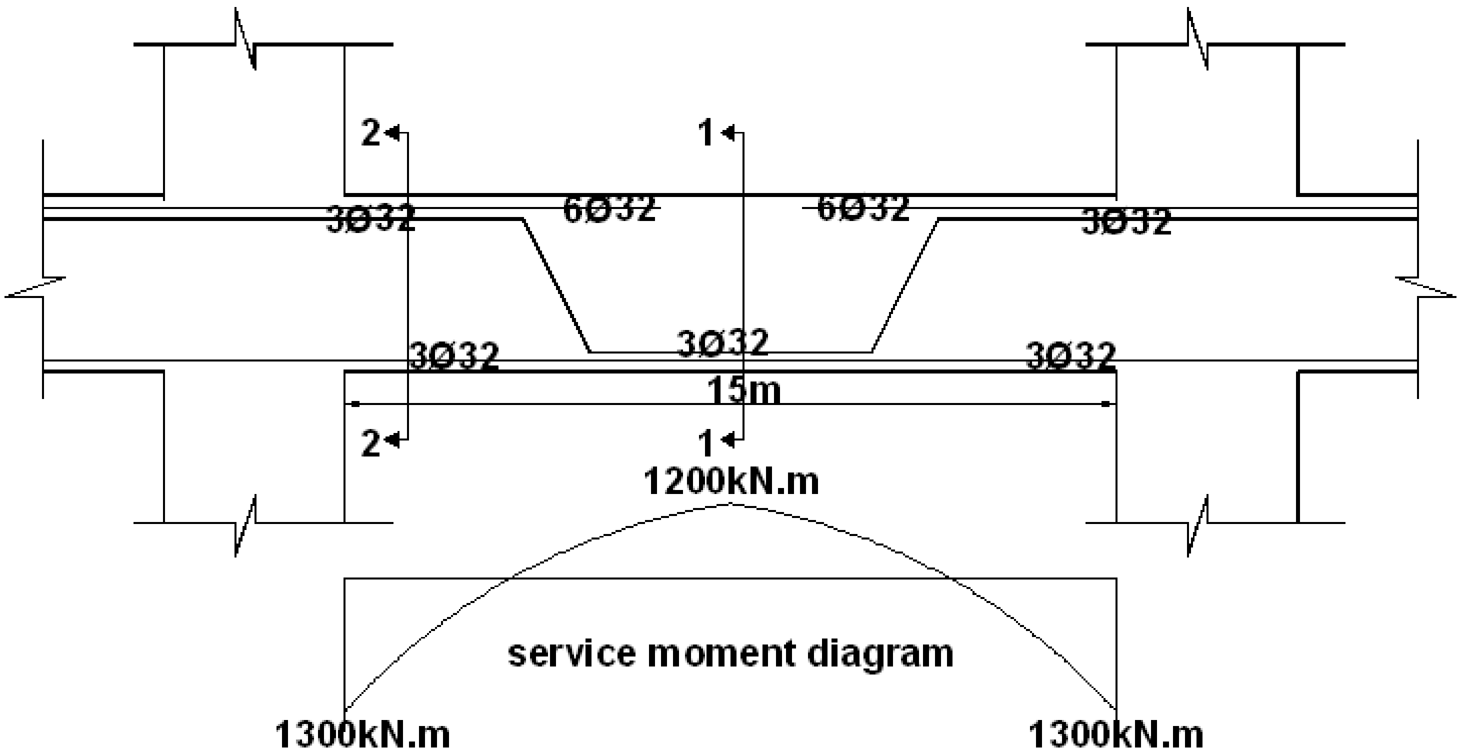
2. Check defl. Limit

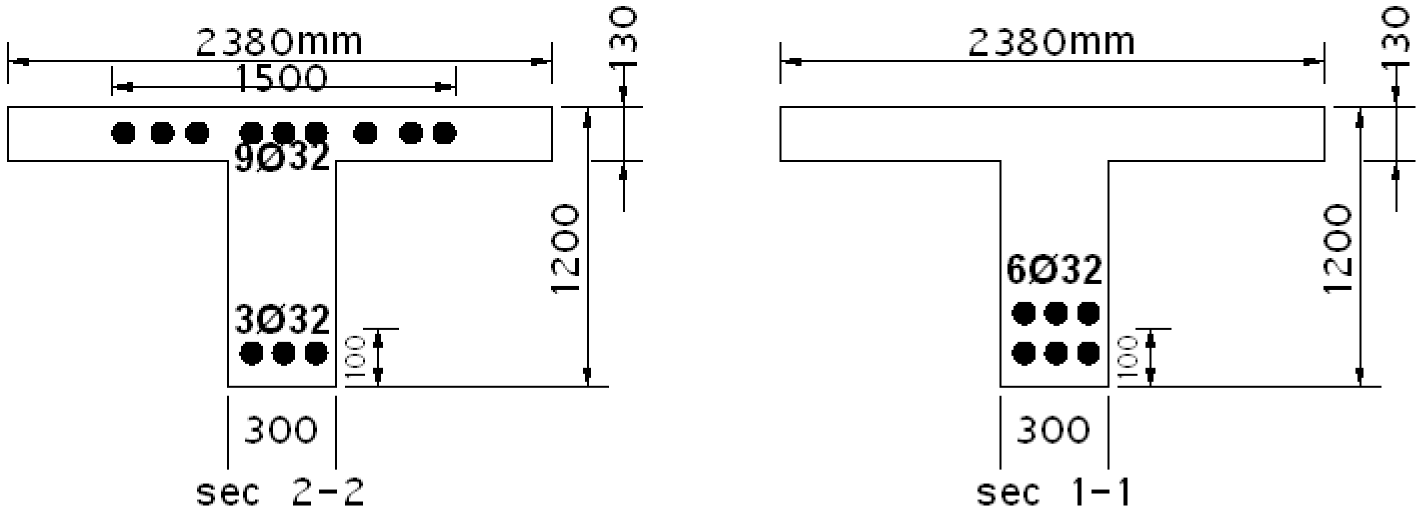
$$\delta i_{(LL)} = 8.84 * \frac{(18)}{(18+8)} = 6.12mm < defl.limit = \frac{l}{360}$$

$$= \frac{6000}{360} = 16.67mm \;\; O.K$$

Ex3: fy=400MPa, fc'=28MPa, ζ=2.0, assume 25%of L.L as sustained. Find:

1. $\delta i_{m(D+L)}$, δLT
2. Check deflection limit, if the beam will supporting non structural partitions that would be damaged if large deflections were to occur.





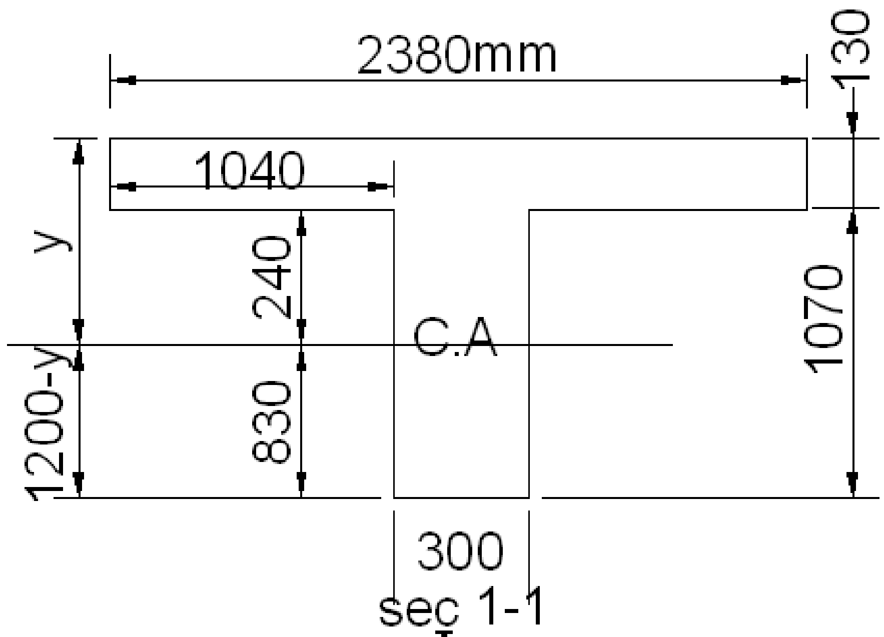
Solution:

$$Ec = 4700\sqrt{28} = 27870MPa$$

$$n = \frac{Es}{Ec} = \frac{200000}{27870} = 8$$

Positive moment sec 1-1

Ig calculation:



Assume centroid of T-section at distance (y) from top fiber

$$\sum M_{top\ fiber} = 0$$

$$2380 * 130 * \frac{130}{2} + 1070 * 300 * \left(130 + \frac{1070}{2}\right)$$

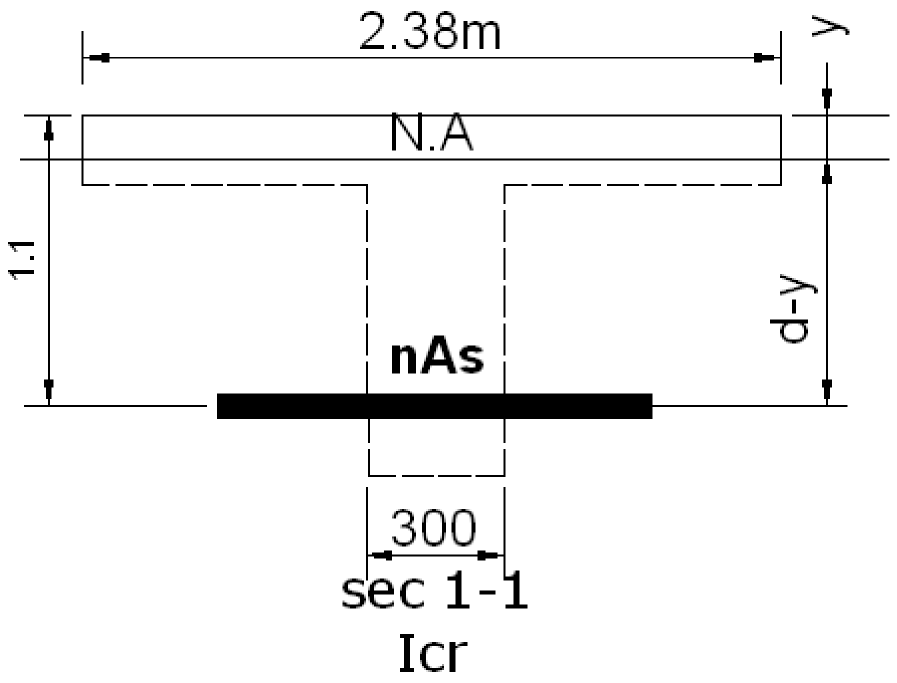
$$= (2380 * 130 + 1070 * 300)y \rightarrow y = 370mm$$

$$Ig = \frac{2.38*0.37^3}{3} - 2\left(\frac{1.04*0.24^3}{3}\right) + \frac{0.3*0.83^3}{3} = 87.778*10^{-3}m^4$$

Icr calculation:

$6Ø32=4825mm^2$

$$\sum N.A = 0$$



$$2.38 * y * \frac{y}{2}$$

$$= 8 * 4825 * 10^{-6}(1.1 - y) \rightarrow y$$

$$= 0.173m > hf \rightarrow$$

$$\therefore N.A\ within\ web$$

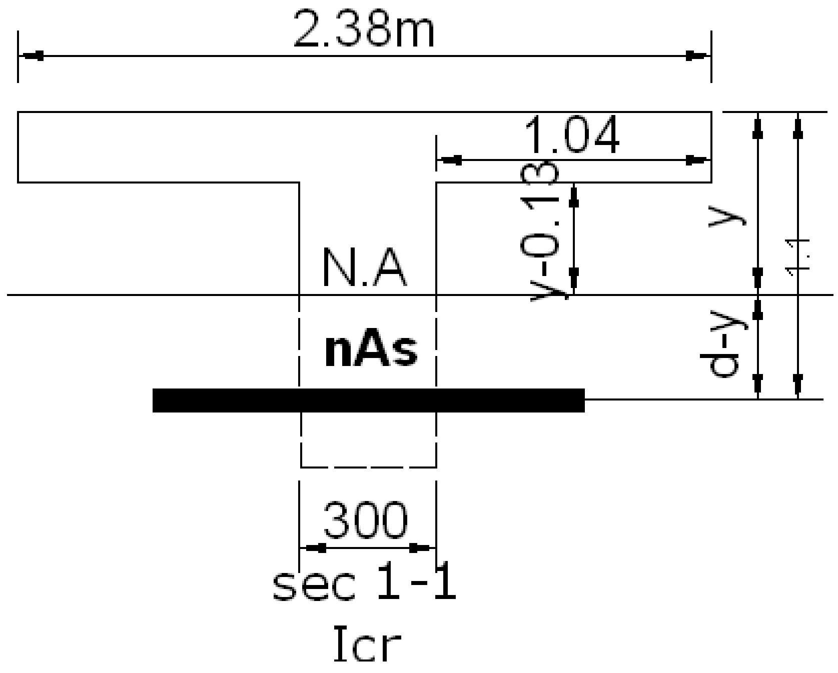
$$\sum N.A = 0$$

$$2.38 * y * \frac{y}{2}$$

$$- 2[1.04(y - 0.13)\frac{(y - 0.13)}{2}]$$

$$= 8 * 4825 * 10^{-6}(1.1 - y) \rightarrow y$$

$$= 0.179m > hf$$



Icr

$$= \frac{2.38 * 0.179^3}{3} - 2\left(\frac{1.04 * 0.049^3}{3}\right) + 8 * 4825$$

$$* 10^{-6}(1.1 - 0.179)^2 = 37.21 * 10^{-3}m^4$$

$$f_r = 0.62\sqrt{fc'} = 0.62\sqrt{28} = 3.28MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.28 * 87.778 * 10^{-3}}{(1.2 - 0.37)} * 10^3 = 392kN.m$$

Ma=1200kN.m

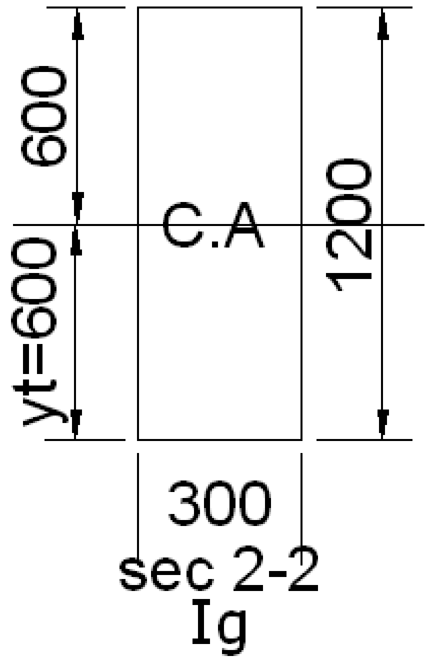
$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

$$I_{e(D+L)}$$

$$= \left(\frac{392}{1200}\right)^3 * 87.778 * 10^{-3} + \left[1 - \left(\frac{392}{1200}\right)^3\right] * 37.21$$

$$* 10^{-3} = 38.97 * 10^{-3} m^4 \leq I_g = 87.778 * 10^{-3} m^4$$

Negative moment, sec 2-2:



$$Ig = \frac{0.3 * 1.2^3}{12} = 43.2 * 10^{-3} m^4$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.28 * 43.2 * 10^{-3}}{0.6} * 10^3$$

$$= 267 kN.m$$

Icr calculation:

9Ø32=7238mm^2

3Ø32=2413mm^2

$$\sum N.A = 0$$

$$300 * y * \frac{y}{2} + (8-1) * 2413(y - 100) =$$

$$8 * 7238(1130 - y) \rightarrow y = 464\text{mm}$$

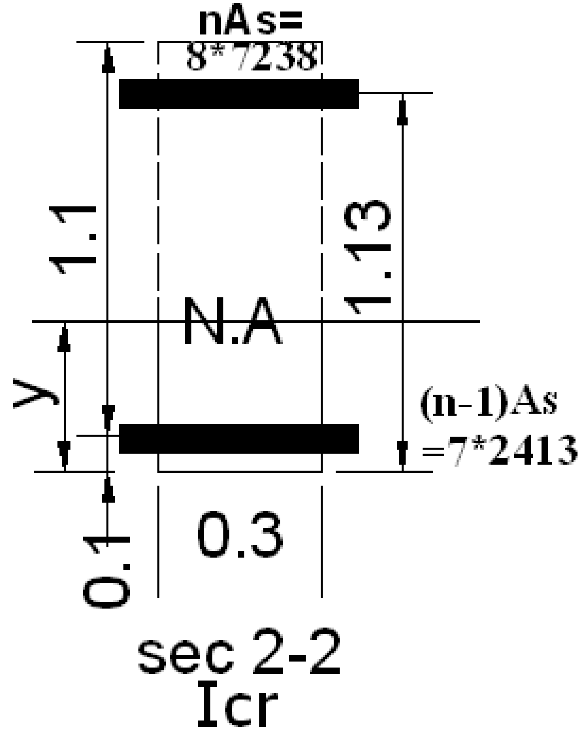
$$Icr$$

$$= \frac{0.3 * 0.464^3}{3} + (8-1) * 2413$$

$$* 10^{-6}(0.464 - 0.1)^2 + 8 * 7238$$

$$* 10^{-6}(1.13 - 0.464)^2$$

$$= 37.911 * 10^{-3} m^4$$



Ma=1300kN.m

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g$$

$$I_{e(D+L)}$$

$$= \left(\frac{267}{1300}\right)^3 * 43.2 * 10^{-3} + \left[1 - \left(\frac{267}{1300}\right)^3\right] * 37.911$$

$$* 10^{-3} = 37.957 * 10^{-3} m^4 \leq I_g = 43.2 * 10^{-3} m^4$$

$$I_{e(a)} = I_{e(b)} = 37.957 * 10^{-3} m^4$$

$$I_{e_{midspan}} = 0.5I_{em} + 0.25(I_{ea} + I_{eb})$$

$$= [0.5 * 38.97 + 0.25(37.957 + 37.957)] * 10^{-3}$$

$$= 38.46 * 10^{-3} m^4$$

1. $$\delta_m = \frac{5l^2}{48EI_e}\left[M_m + \frac{1}{10}(M_a + M_b)\right]$$

$$\delta i_{m(D+L)}$$

$$= \frac{5 * 15^2}{48 * 24870 * 10^3 * 38.46 * 10^{-3}}\left[1200 + \frac{1}{10}(-1300 - 1300)\right] 10^3 = 23.03mm$$

$$\delta i_{m(D+0.25L)} = 23.03 * \frac{(0.25 * 65 + 23)}{(65 + 23)} = 10.27mm$$

$$\lambda = \frac{\zeta}{1 + 50\rho'} = \frac{2.0}{1 + 50 * 0} = 2.0 \qquad , \rho': at\ mid\ span$$

$\delta_{LT} = \lambda\ \delta i_{sust} = 2 * 10.27 = 20.54$ mm

$$\delta total = \delta i_{(D+L)} + \delta LT = 23.03 + 20.54 = 43.57 \text{ mm}$$

2. Check defl. Limit

$$\delta i_{(0.75L)} = 23.03 * \frac{(0.75 * 65)}{(65 + 23)} = 12.75mm$$

$$\delta LT_{(all\ sustaind\ load)} + \delta i_{(any\ additional\ LL)}$$

$$= 20.54 + 12.75 = 33.29mm > defl.limit = \frac{l}{480}$$

$$= \frac{15000}{480} = 31.25mm\ \ N.G \rightarrow$$

$$\therefore The\ \ stiffness\ of\ the\ member\ not\ sufficient$$