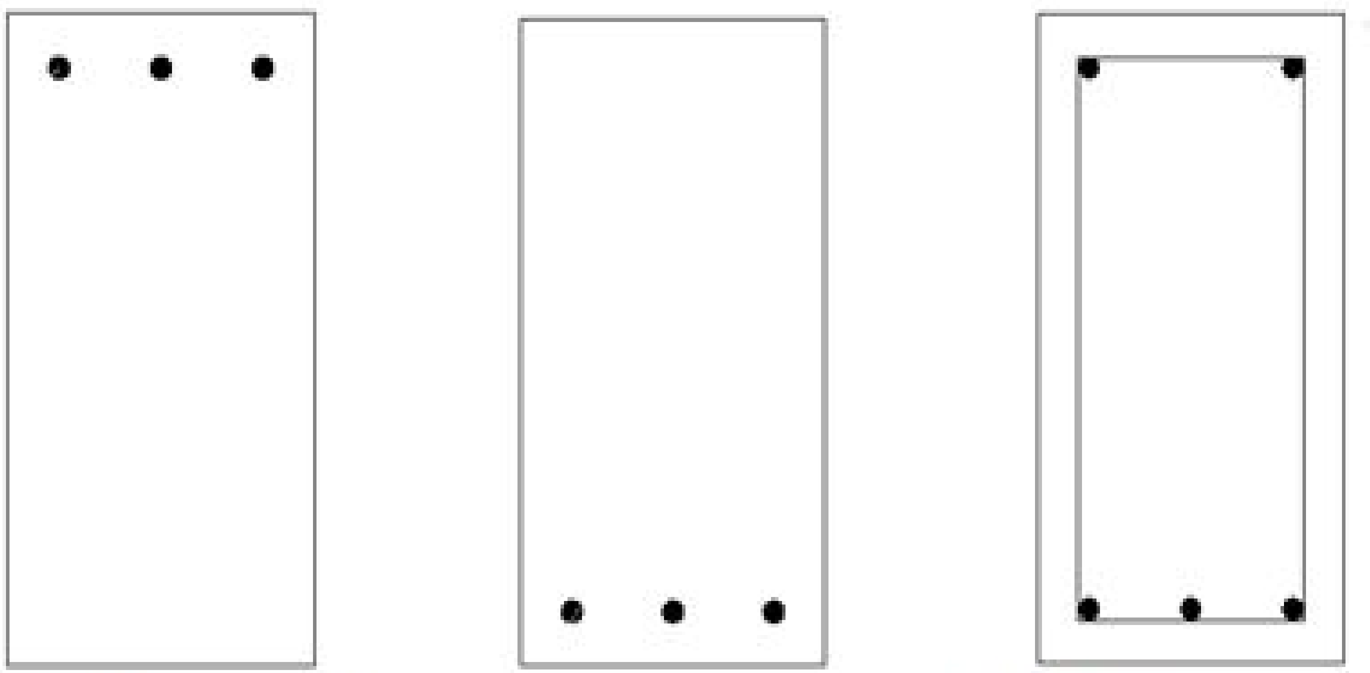
Design of rectangular beam section:

a. Singly reinforced beam (reinforcement for tension only)

b. Doubly reinforced beam (reinforcement for both tension and compression)



Singly Reinforced section M-   Singly Reinforced section M+   Doubly Reinforced section M+

Single Reinforced Section

ACI code provision:

1. $\rho < \rho_{max} = 0.85\beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004}$

   ACI code 10.3.5 for non pre stressed flexural member and with axial load $\leq 0.1Ag\,fc'$

2. $\rho > \rho_{min} = \max\left(\frac{\sqrt{fc'}}{4fy}, \frac{1.4}{fy}\right)$ fy in MPa

7.7.1 — Cast-in-place concrete (nonprestressed)

The following minimum concrete cover shall be provided for reinforcement, but shall not be less than required by 7.7.5 and 7.7.7:

	Minimum cover, mm
(a) Concrete cast against and permanently exposed to earth	75
(b) Concrete exposed to earth or weather:	
No. 19 through No. 57 bars	50
No. 16 bar, MW 200 or MD 200 wire, and smaller	40
(c) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists:	
No. 43 and No. 57 bars	40
No. 36 bar and smaller	20

Beams, columns:

Primary reinforcement, ties,
stirrups, spirals 40

Shells, folded plate members:

No. 19 bar and larger............................. 20
No. 16 bar, MW 200 or MD 200 wire,
and smaller 13

$$Sc_{min} \geq \begin{bmatrix} d_b = 25 \text{ mm} \\ 25 \text{ mm} \\ \frac{4}{3}\text{max. size of agg} \end{bmatrix} = 25mm$$

*Clear spacing between rows(layers) of reinforcement ≥ 25mm,

ACI 7.6.2

10.3.5 — For nonprestressed flexural members and nonprestressed members with factored axial compressive load less than $\mathbf{0.10}\boldsymbol{f_c'A_g}$, ε_t at nominal strength shall not be less than 0.004.

H.W: Show that

$$\rho_{max}=0.85\beta_1\frac{fc'}{fy}\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004}$$

$\varepsilon_c=\varepsilon_{cu}=0.003$ $\varepsilon_S=0.004$, $fs=fy$ $\rho=\dfrac{As}{bd}$

$$\sum Fx=0$$

$$Asfy=0.85\ fc^{'}b.a]\div bd$$

$$\frac{As_t}{bd} fy=0.85\ fc^{'} \frac{a}{d} \rightarrow \qquad \rho_{max}=0.85 \frac{fc^{'}}{fy} \frac{a}{d} \ldots\ldots.(1)$$

From strain diagram:

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_{cu} + \varepsilon_s}{d} \rightarrow c = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} * d$$

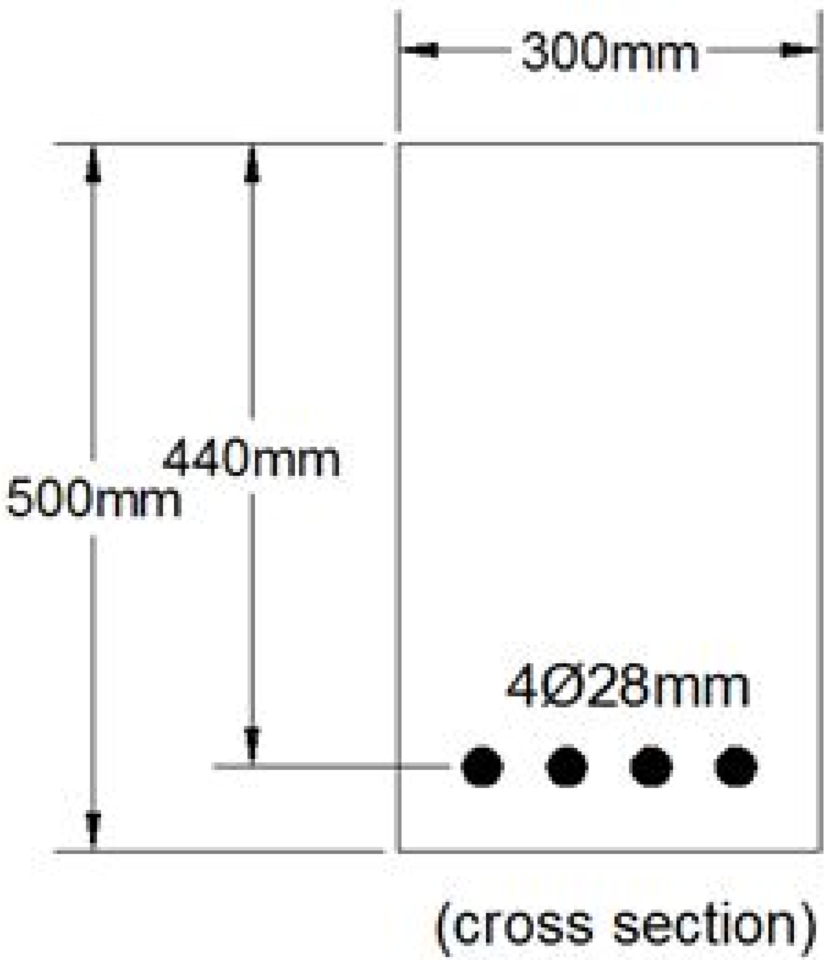
$$a=\beta_1 c=\beta_1 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} * d \ldots..(2)$$

$$sub(2) in\ (1)$$

$$\rho_{\max}=0.85\,\frac{fc^{'}}{fy}\,\frac{\beta_1\,\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004}*d}{d}$$

$$\rho_{\max}=0.85\beta_1\,\frac{fc^{'}}{fy}\,\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004}$$

Ex1: For given section calculate moment capacity(Mn) fc'=27MPa , fy=400MPa.



Solution:

$$As = 4\frac{\pi}{4}(28)^2 = 2463mm^2$$

$$\rho = \frac{As}{bd} = \frac{2463}{300 * 440} = 0.01866$$

$$\rho_b = 0.85 * \beta_1 \frac{fc'}{fy} * \frac{600}{600 + fy}$$

$\beta_1 = 0.85$ $fc' = 27MPa < 28\ MPa$

$$\rho_b = 0.85 * 0.85 * \frac{27}{400} * \frac{600}{600 + 400} = 0.0292$$

$\rho < \rho_b$ under reinforce section(tensile failure)

$$\rho_{max} = 0.85*\beta_1 \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004}$$

$$= 0.85 * 0.85 * \frac{27}{400} * \frac{0.003}{0.003 + 0.004} = 0.0209$$

$$\rho_{min} = \max\left(\frac{1.4}{fy} = \frac{1.4}{400} = 0.0035, \frac{\sqrt{fc'}}{4fy} = \frac{\sqrt{27}}{4*400} = 0.00324\right)$$

$$= 0.0035$$

$$\rho_{min} < \rho = 0.01866 < \rho_{max} \quad \therefore o.k$$

$$\rho_t = 0.85*\beta_1 \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005}$$

$$=0.85*0.85*\frac{27}{400}*\frac{0.003}{0.003+0.005}=0.0183$$

$$\rho > \rho_t \,, \qquad \varepsilon_t < 0.005 \quad \therefore \emptyset = 0.483 + 83.3 * \varepsilon_t$$

$$dt = 440mm$$

$$\varepsilon_t = \varepsilon_{cu} * \frac{dt - c}{c}$$

$$\sum Fx = 0$$

$$0.85fc^{'}b * a=As * fy$$

$$a=\frac{As * fy}{0.85fcb}=\frac{2463*10^{-6}*400}{0.85*27*0.3}=0.143m$$

$$a = \beta_1 c$$

$$c = \frac{a}{\beta_1} = \frac{0.143}{0.85} = 0.168m$$

$$\varepsilon_t = 0.003 * \frac{0.44 - 0.163}{0.163} = 0.00486 < 0.005$$

$$\emptyset = 0.483 + 83.3 * 0.00486 = 0.887$$

$$\mathrm{Mn=As\ fy}\left(\mathrm{d}-\frac{\mathrm{a}}{2}\right)=2463*10^{-6}*400*\left(0.44-\frac{0.143}{2}\right)=0.363\ \mathrm{MN.m}$$

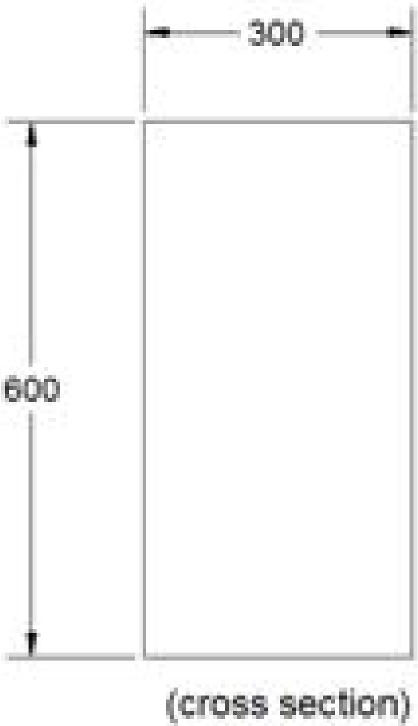
$$\mathrm{or\ Mn}=\rho \mathrm{bd}^2\mathrm{fy}\left(1-0.59\rho\frac{\mathrm{fy}}{\mathrm{fc}'}\right)$$

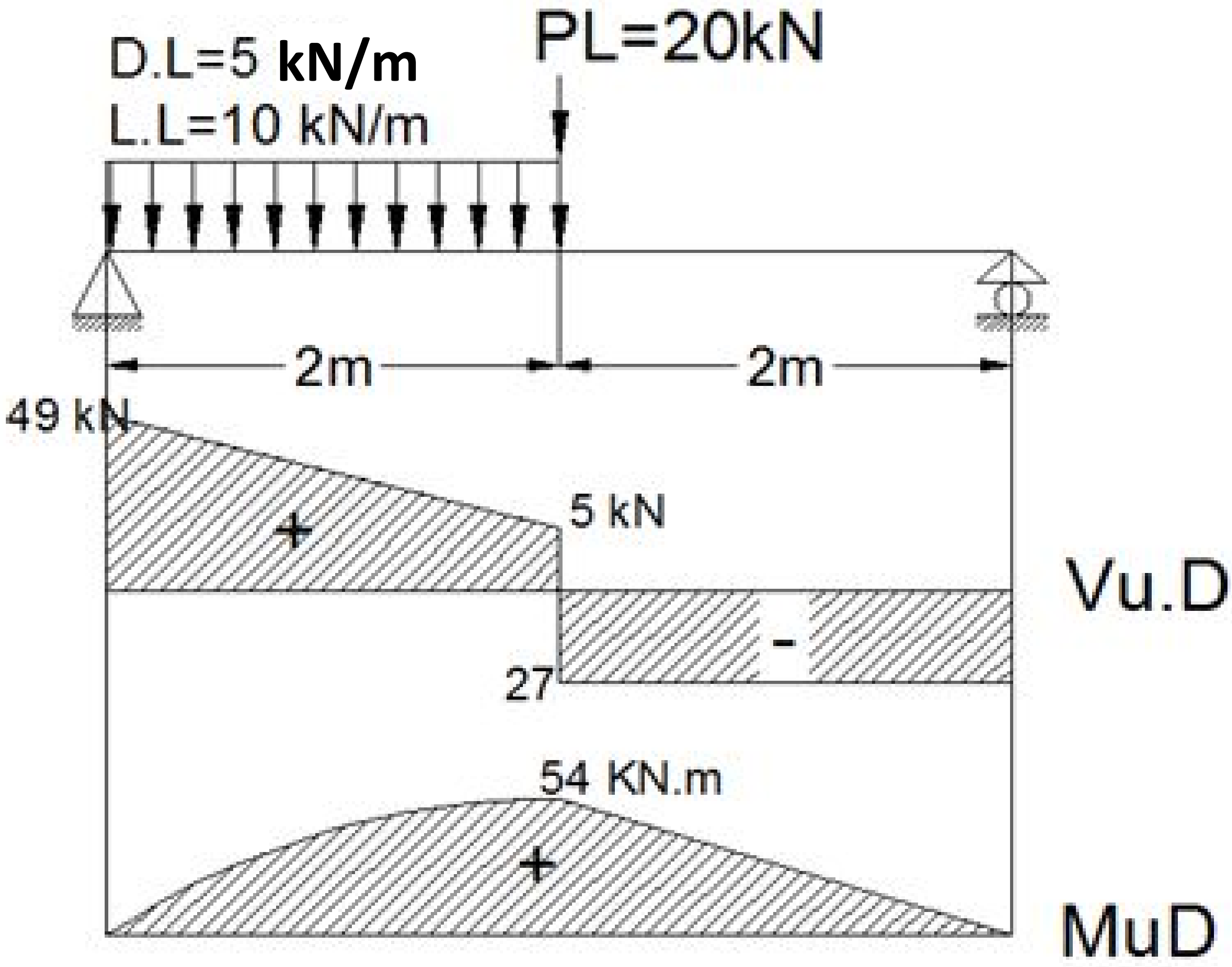
$$=0.01866*0.3*0.44^2*400*\left(1-0.59*0.01866*\frac{400}{27}\right)$$

=0.363 MN.m

$$Mu = ØMn = 0.887 * 0.363 = 0.322 MN.m = 322 \text{ kN.m}$$

Ex2: For the given applied load and section dimensions find As? $\frac{S}{C} = \frac{300}{35}$





Solution:

$$W_u = 1.2D.L + 1.6LL = 1.2 * 5 + 1.6 * 10 \quad = 22kN/m$$

$$P_{uL} = 1.6 * 20 = 32kN$$

$$Mu_{ext}. = 54\ kN.m = 0.054\ MN.m$$

$$Mu_{ext} = ØMn = Øρ\ b\ d^2 fy \left(1\text{-}0.59ρ\frac{fy}{fc'}\right)$$

$d = h - cover - d_{stirrup} - d_{bar}/2 = 600 - 40 - 10 - 25/2 = 537mm$, (db, ds are assumed)

Assume $Ø = 0.9$ to be checked later

$$0.054 = 0.9 * \rho * 0.3 * 0.537^2 * 300 * \left(1 - 0.59 * \rho * \frac{300}{35}\right)$$

$$118.12\rho^2 - 23.35\rho + 0.054 = 0$$

$$\rho = 0.00234\text{(used) and } \rho = 0.195 \; > \rho_{max}, (ignored)$$

$$\rho_{max.} = 0.85\beta_1 \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004}$$

$$fc' = 35 > 28MPa \rightarrow \beta_1 = 0.85 - \frac{fc' - 28}{7} * 0.05$$

$$= 0.80 > 0.65 \; ok$$

$$\rho_{max.} = 0.85 * 0.80 * \frac{35}{300} * \frac{0.003}{0.003 + 0.004}$$

$$\rho_{max.} = 0.0344 > \rho \quad o.k$$

$$\rho_{min.} = \max\left(\frac{\sqrt{fc'}}{4fy} = 0.00493, \frac{1.4}{fy} = 0.0047\right) = 0.00493$$

$$\rho < \rho_{min.} \quad \therefore use\ \rho = \rho_{min.}$$

$$since\ \rho = \rho_{min.} \rightarrow \emptyset = 0.9$$

$$\rho_t = 0.85\ \beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} = 0.85 * 0.80 * \frac{35}{300} * \frac{0.003}{0.003 + 0.005}$$

$$\rho_t = 0.03 > \rho, \quad \therefore \emptyset = 0.9$$

$$As = \rho\ b\ d = 0.00493 * 300 * 537$$

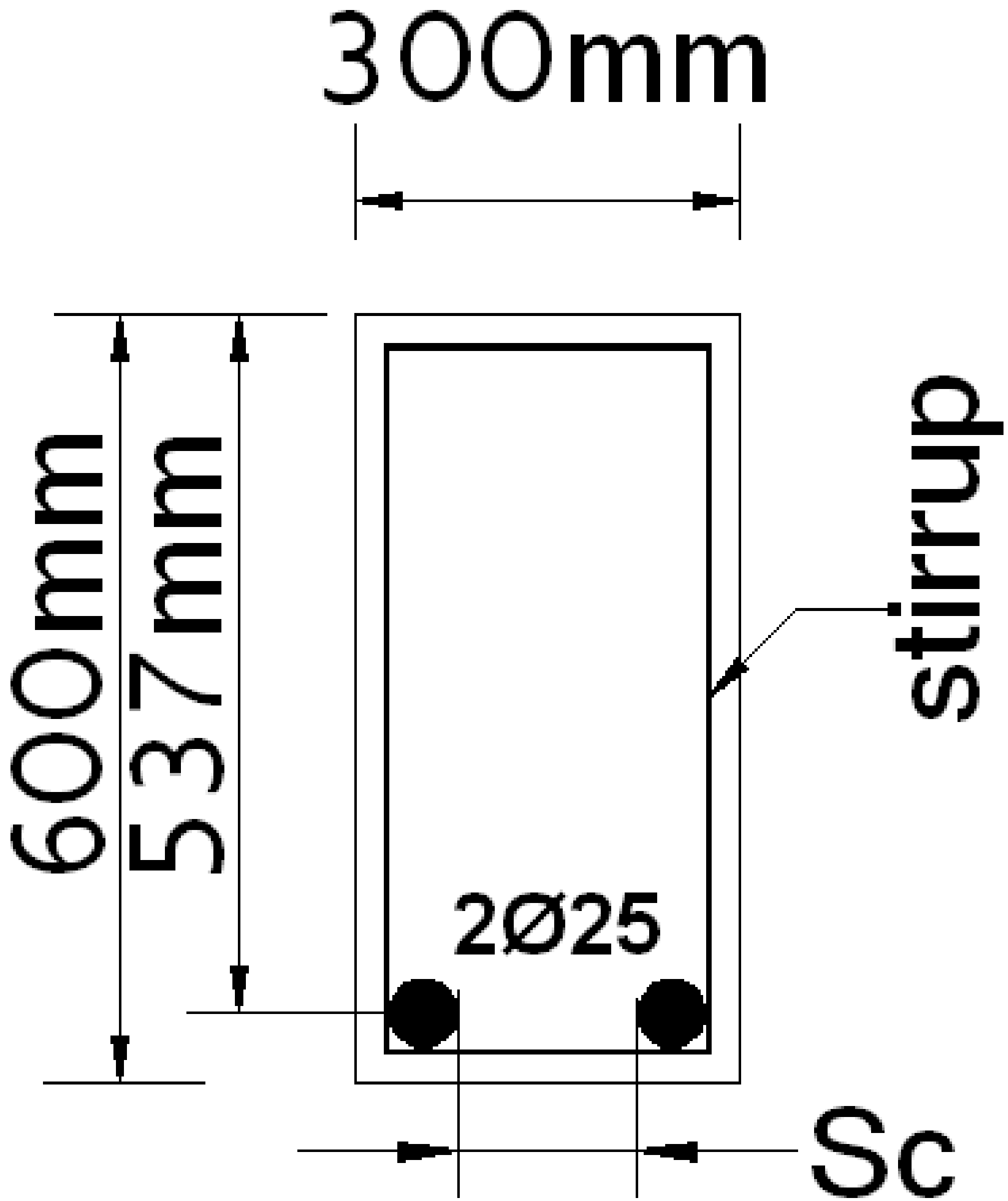
$$As = 795mm^2$$

$$n = \frac{As}{As_b} = \frac{795}{\frac{\pi}{4}25^2} = 1.6 \cong 2 \rightarrow use\ 2\emptyset25$$

$$Sc_{min} \geq \begin{bmatrix} d_b = 25\text{ mm} \\ 25\text{ mm} \\ \frac{4}{3}\text{max. size of agg} \end{bmatrix} = 25mm$$

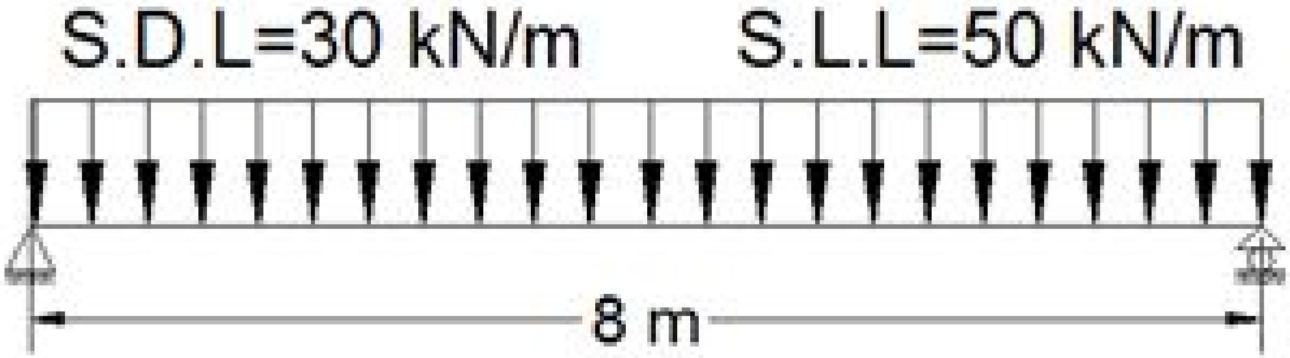
$$Sc = \frac{b - 2*cover - 2*\text{dstirrup} - \text{n}*\text{dbar}}{n-1}$$

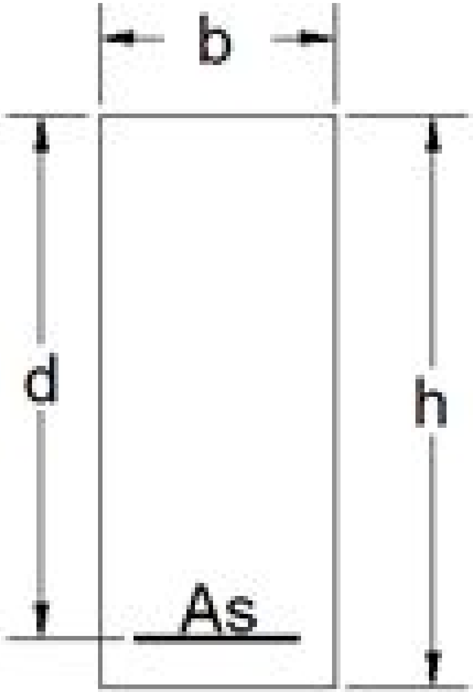
$$\text{Sc} = \frac{300-2*40-2*10-2*25}{2-1} = 150\text{mm} > Sc_{min}. = 25\text{mm}$$


300mm
600mm
537mm
stirrup
2Ø25
Sc

EX3: For the given applied load calculate (b, d, h and As).

$$\frac{S}{C} = \frac{400}{35}$$





Solution:

$$W_u = 1.2\,D.l + 1.6 * LL = 1.2 * 30 + 1.6 * 50 = 116\,\frac{kN}{m}$$

$$Mu_{ext} = \frac{W_u l^2}{8} = \frac{116 * 8^2}{8} = 928kN.m$$

$$Assume\ \rho[0.5\rho_{max}. \leftrightarrow 0.75\rho_{max}]$$

$$\rho_{max}.=0.85*\beta_1\frac{fc'}{fy}\frac{0.003}{0.003+0.004}$$

$$\text{for } fc'=35 \rightarrow \beta_1=0.85-\frac{(fc'-28)}{7}*0.05=0.80>0.65 \text{ o.k}$$

$$\rho_{max.} = 0.85 * 0.80 * \frac{35}{400} * \frac{0.003}{0.007} = 0.0255$$

$$let\ \rho = 0.6\rho_{max.}$$

$$\therefore \rho = 0.6 * 0.0255 = 0.0153$$

$$\rho_t = 0.85 * \beta_1 \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005}$$

$$\rho_t = 0.85 * 0.80 * \frac{35}{400} * \frac{0.003}{0.003 + 0.005} = 0.0223 > \rho$$

$$\rho = 0.0153 < \rho_t \quad \therefore \emptyset = 0.9$$

$$Mu=\emptyset\ \rho\ b\ d^2\ fy\left(1\text{-}0.59\rho\frac{fy}{fc'}\right)$$

0.928

$$=0.9*0.0153*(bd^2)*400*\left(1-0.59*0.0153*\frac{400}{35}\right)$$

$$\rightarrow bd^2=0.1857m^3$$

$$\text{assume}\quad d=(2\leftrightarrow 3)b\rightarrow \text{let } d=3b$$

$$b(3b)^2=0.1857\rightarrow b=0.274m=274mm$$

$$\text{use } b=300mm\rightarrow 0.3d^2=0.1857 \qquad \therefore\ d=787mm$$

$$As=\rho bd=0.0155*300*787=3660\ mm^2$$

use Ø30 mm bars

$$n = \frac{As}{A_b} = \frac{3660}{\frac{\pi}{4}30^2} = 5.18 \rightarrow \text{use } 6\text{Ø}30mm$$

Assume one layer of reinforcment

min. concrete cover= 40 mm (ACI 7.7.1)

$$Sc_{min.} = max \begin{bmatrix} db = 30mm \\ 25mm \\ \frac{4}{3} max.\, size\ of\ agg \end{bmatrix} = 30mm$$

$$Sc = \frac{300-2*40-2*10-6*30}{6-1} = 4mm < Sc_{min} \quad \text{N.G}$$

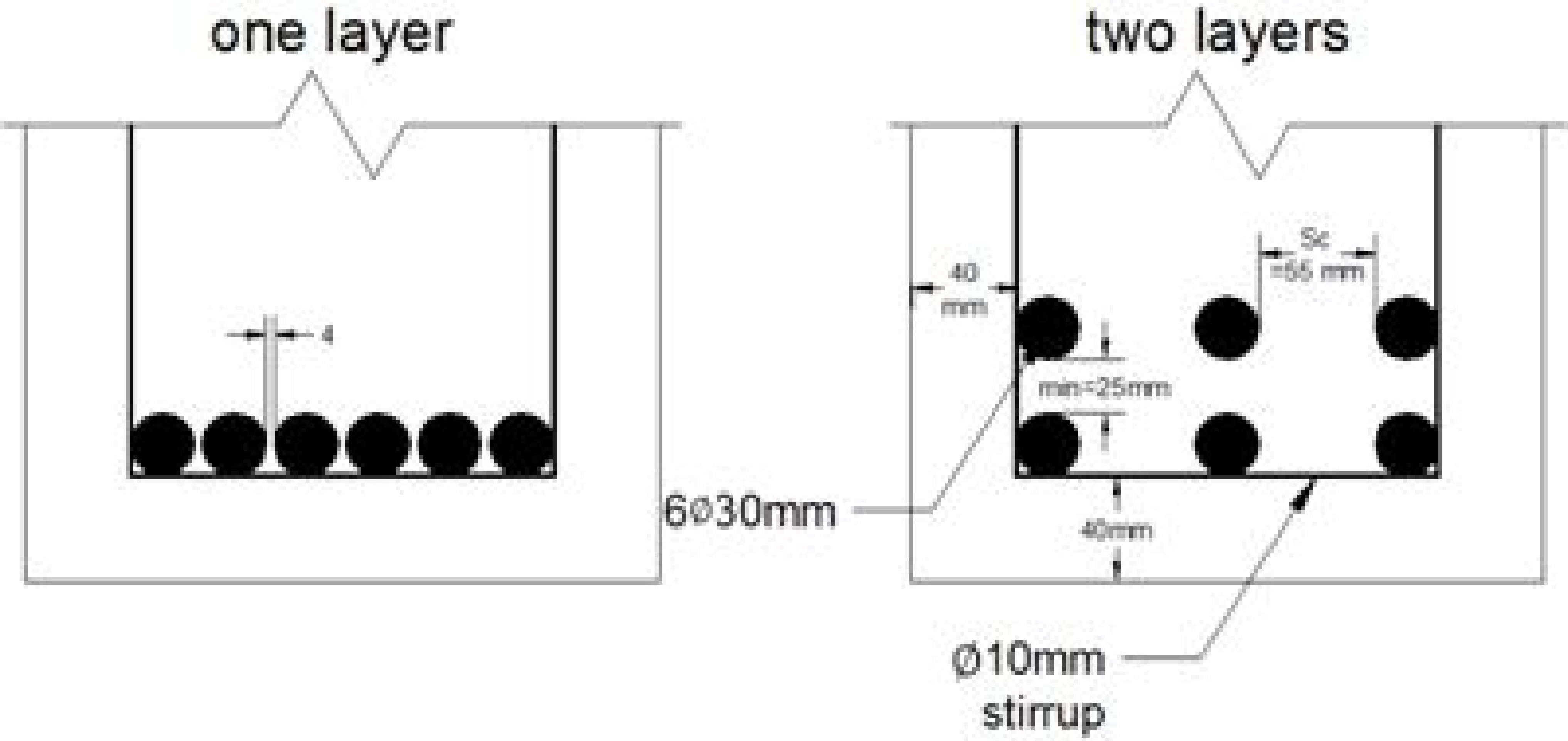
$\therefore$ use two layers of reinforcement

$$Sc = \frac{300 - 2 * 40 - 2 * 10 - 3 * 30}{(3 - 1)} = 55mm > 30mm$$

$$\rightarrow o.k$$

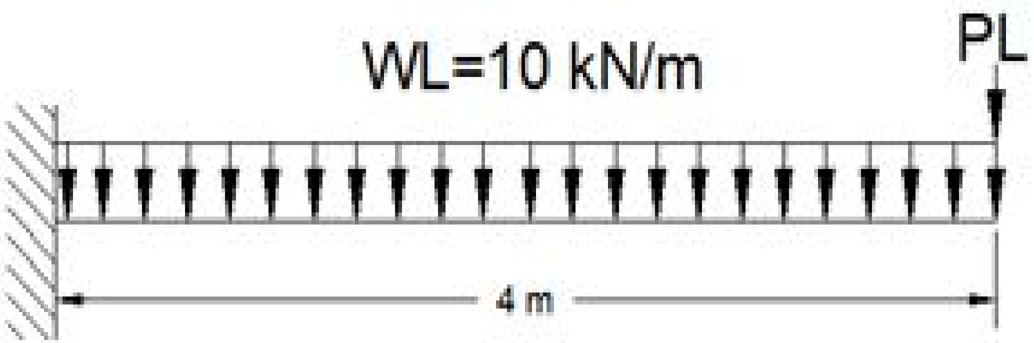
$$h = d + \frac{25}{2} + db + 10 + 40 = 879.5$$

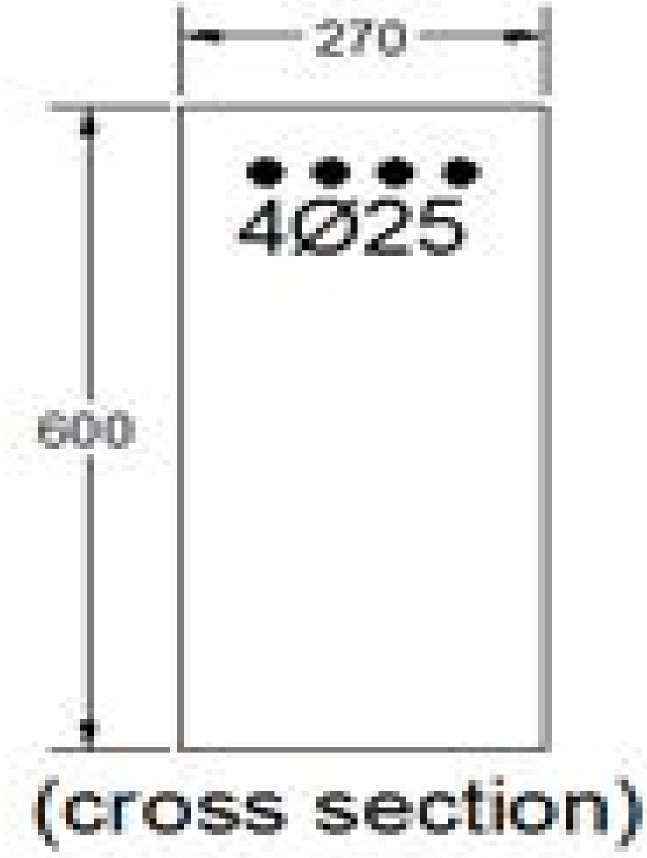
$$use\ h = 880\ mm$$


one layer
two layers
4
40
mm
Sc
=55 mm
min=25mm
6ø30mm
40mm
Ø10mm
stirrup

<u>Ex</u>4: $\frac{S}{C} = \frac{300}{20}$ MPa, D.L=self-weight of the beam.

Fined max. value of P_L?





Solution:

$$W_u = 1.2D.L + 1.6 * L.L$$

$$= 1.2 * (0.27 * 0.6 * 24) + 1.6 * 10 = 20.67 \text{ kN/m}$$

$$Mu_{ext} = \frac{W_u * l^2}{2} + PL * l = \frac{20.67 * 4^2}{2} + 1.6PL * 4$$

$$= 165.36 + 6.4PL$$

$$d = 600 - 40 - 10 - \frac{25}{2} = 537mm$$

$$\rho = \frac{4\left(\frac{\pi}{4} * 25^2\right)}{270 * 537} = 0.0135$$

$$\rho_{max}=0.85\beta_1 * \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004}$$

$$\rho_{max}=0.850*.85*\frac{20}{300}*\frac{0.003}{0.003+0.004} = 0.0206$$

$$\rho_{min} = \max\left(\frac{1.4}{fy} = 0.0047 , \frac{\sqrt{20}}{4 * 300} = 0.0037\right) = 0.0047$$

$$\rho_{min} < \rho < \rho_{max}$$

$$\rho_t=0.85\ \beta_1 * \frac{fc'}{fy} * \frac{0.003}{0.003+0.005} =0.01806>\rho \rightarrow \therefore \emptyset=0.9$$

$$Mu=\emptyset Mn=\emptyset\ \rho\ b\ d^2 fy\left(1-0.59\rho\frac{fy}{fc'}\right)$$

$$Mu = 0.9 * 0.0135 * 0.27 * 0.537^2 * 300\left(1 - 0.59 * 0.0135 * \frac{300}{20}\right) = 0.25MN.m = 250kN.m$$

$$Mu_{int} = Mu_{ext} = 250 = 165.36 + 6.4PL$$

$$PL = 13.225kN$$

Ex5: The same previous example, if PL=40kN fined As

Solution:

$$Mu_{ext} = 165.56 + 6.4 * 40 = 421kN.m$$

Let $\emptyset = 0.9$ to be checked later

$$Mu=\emptyset\ Mn=\emptyset\rho\ b\ d^2 fy\left(1\text{-}0.59*\rho\frac{fy}{fc'}\right)$$

$$0.421 = 0.9 * \rho * 0.27 * 0.537^2 * 300\left(1 - 0.59\ \rho\ \frac{300}{20}\right)$$

$$186.04\ \rho^2 - 21.02\ \rho + 0.421 = 0$$

$$\rho=\begin{bmatrix}0.026\\0.086\end{bmatrix}=0.026$$ (choose min positive value)

$$\rho_{max.} = 0.85\ \beta_1 * \frac{fc'}{fy} * \frac{0.003}{0.007} = 0.0206$$

$$\rho > \rho_{max.} \quad N.G$$

To solve the problem

1- Increase (h) if it’s allowable

2- Use Doubly reinforced section