

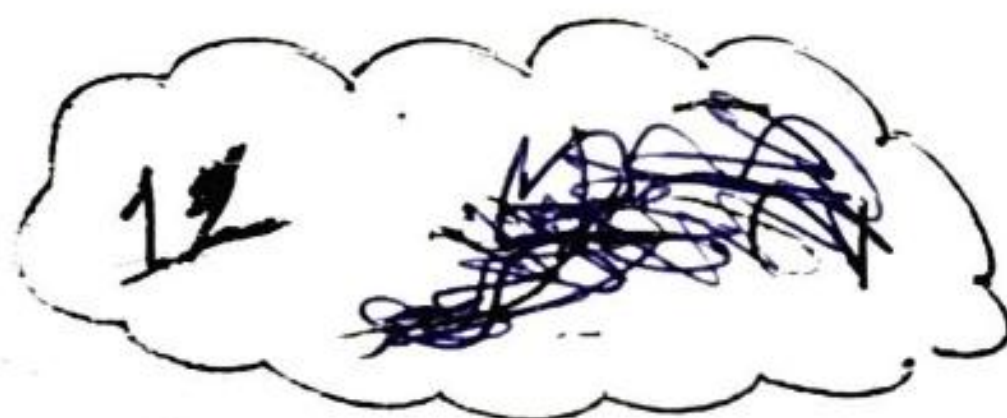
Analyze Structures by using

Moment distribution Method

تستخدم هذه الطريقة لتحليل المنشآت غير المحددة أستانيكياً من عتبات وهياكل فقط .

يكون أساليب الحل بهذه الطريقة على هيئة جدول بحيث نهاية هذا الجدول هي نهاية الحل . أي أن قيم العزوم تكون معلومة .

مكتب القيمة
طباعة عامة - استنساخ
باب المعظم معاور كلية الهندسة



~~Structure~~ ~~في حاده~~



البيد عن الدوران

4/5/2010

$$M_{AB} = \frac{-2EI\theta}{l}$$

$$M_{BA} = \frac{-4EI\theta}{l}$$



هذا الدوران عكس اتجاه الساعة لذلك العزم

سالب

$$M_{AB} = \frac{2EI\theta}{l}$$

$$M_{BA} = \frac{4EI\theta}{l}$$



هذا الدوران مطابق اتجاه الساعة لذلك العزم موجب

(MF)

④ - أمثلة (Fixed-end-Moment) وذلك حسب الحالات التالية:

$$M_{AB}^F = - \frac{wl^2}{12}$$

$$M_{BA}^F = + \frac{wl^2}{12}$$

$$M_{AB}^F = - \frac{wl^2}{30}$$

$$M_{BA}^F = + \frac{wl^2}{20}$$

$$M_{AB}^F = - \frac{Pl}{8}$$

$$M_{BA}^F = + \frac{Pl}{8}$$

$$M_{AB}^F = - \frac{P \cdot a \cdot b^2}{l^2}$$

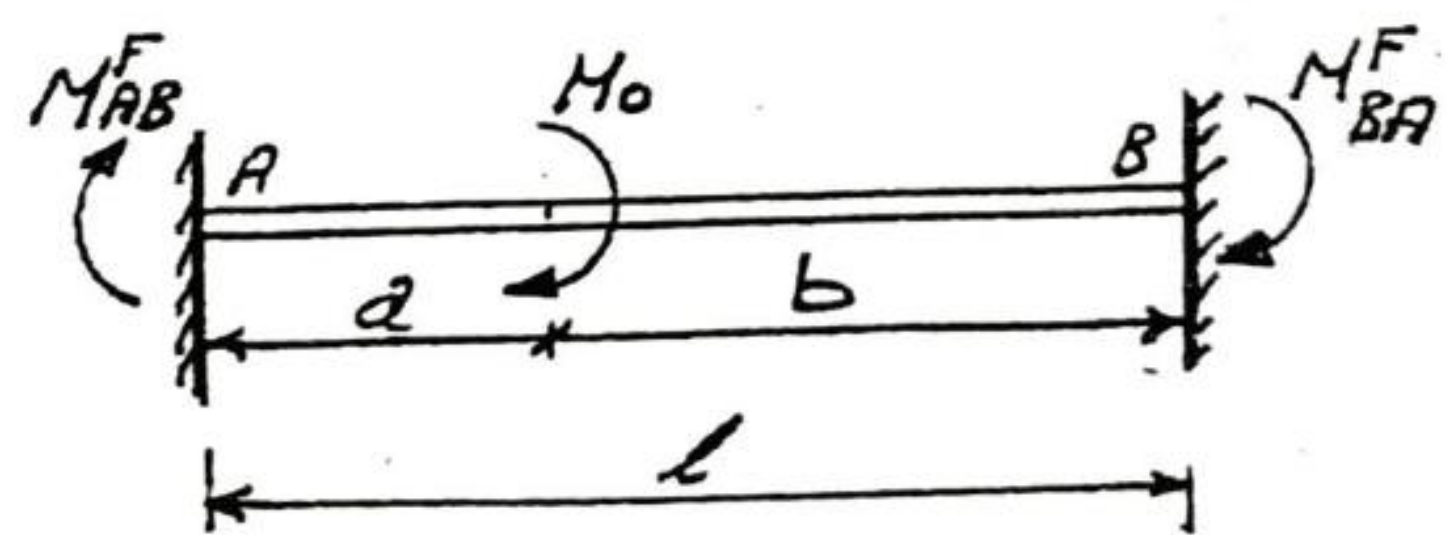
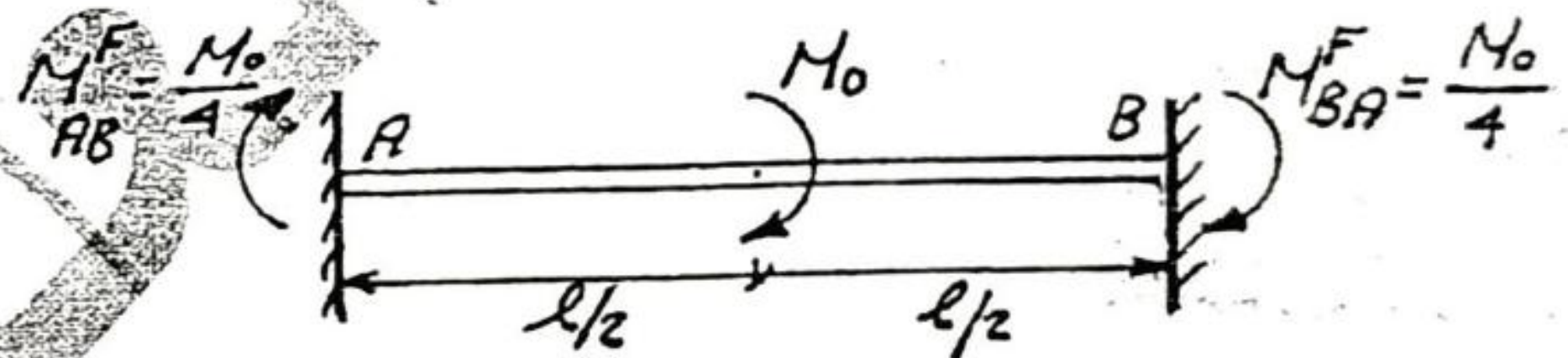
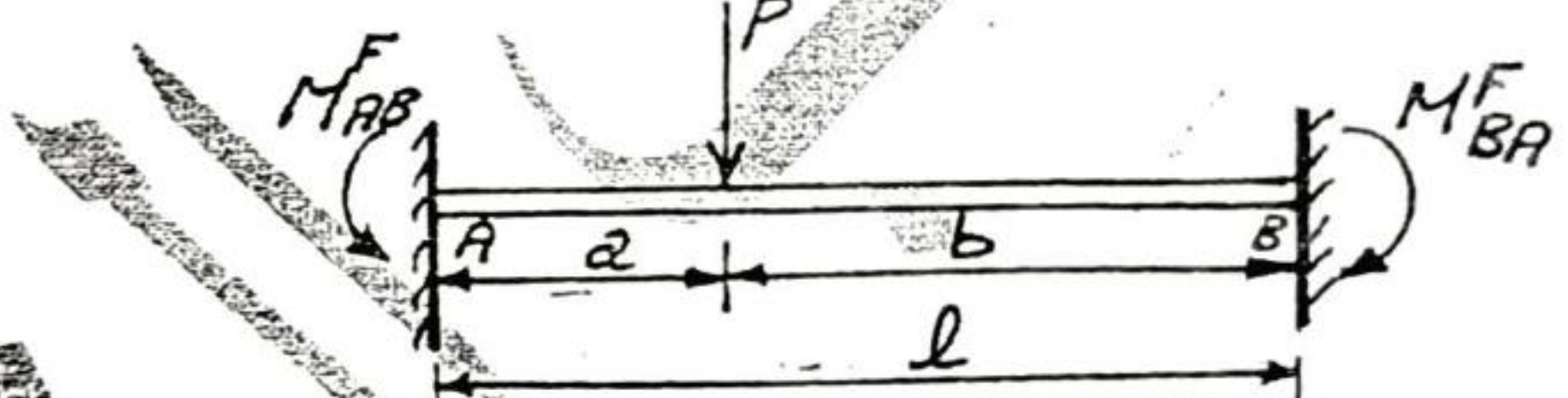
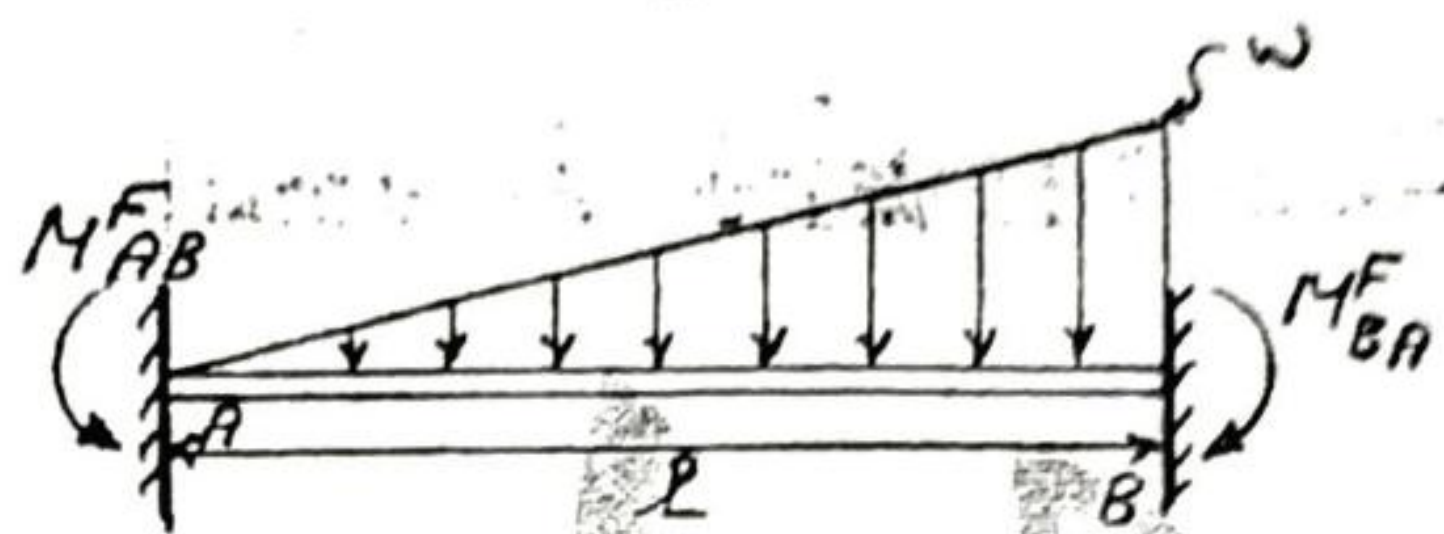
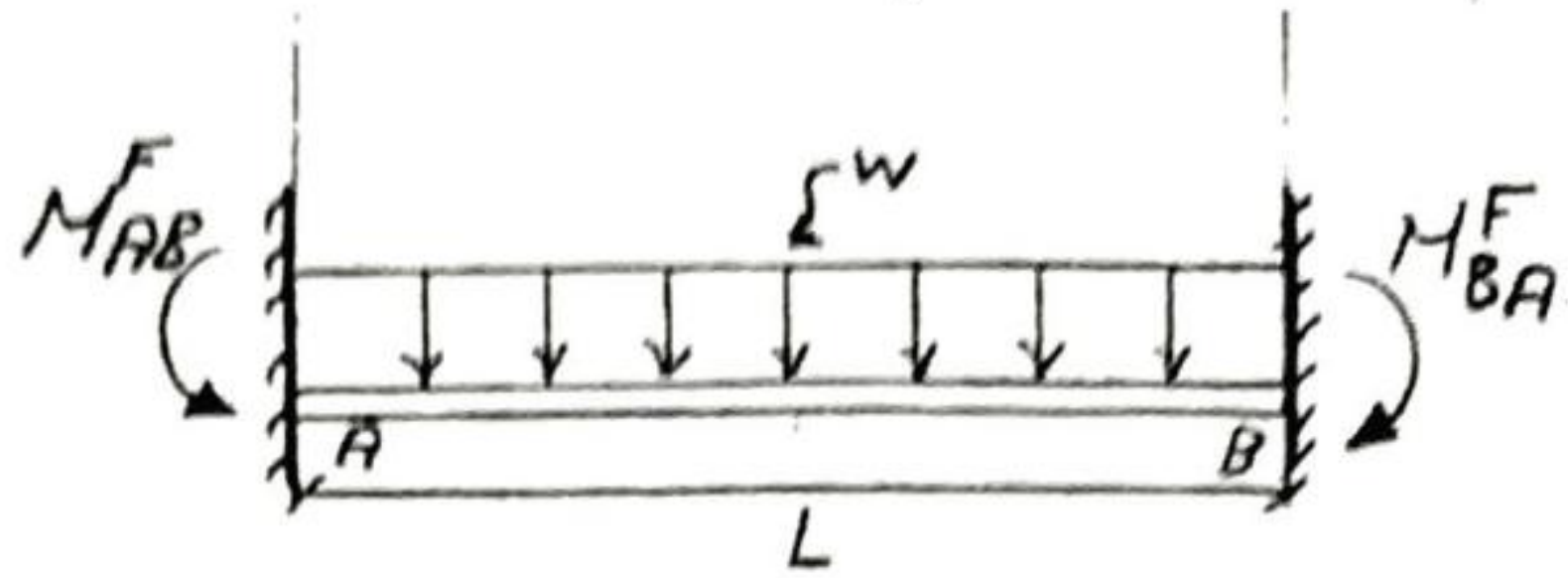
$$M_{BA}^F = + \frac{P \cdot b \cdot a^2}{l^2}$$

$$M_{AB}^F = + \frac{M_0}{4}$$

$$M_{BA}^F = + \frac{M_0}{4}$$

$$M_{AB}^F = + \frac{M_0 \cdot a \cdot (2b - a)}{l^2}$$

$$M_{BA}^F = + \frac{M_0 \cdot b \cdot (2a - b)}{l^2}$$

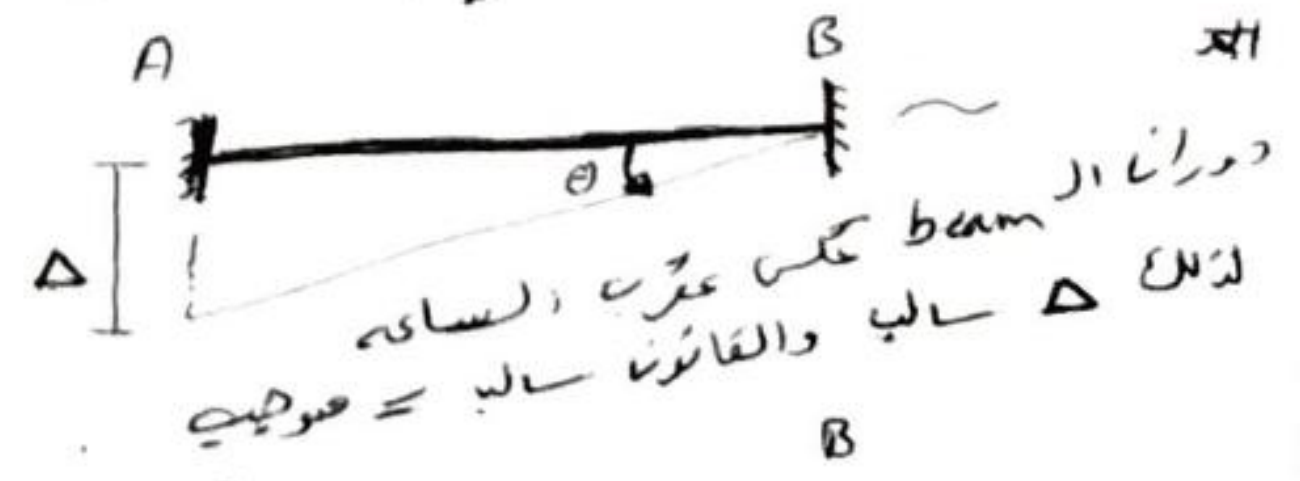


* دائماً (MF) يكون ضعف عدد العزادات ونقطة التقاطع هو المسافة بين مسنديه.

$$M = - \frac{6EI\Delta}{l^2}$$

$$M_{AB}^F = \frac{6EI\Delta}{l^2}$$

$$M_{BA}^F = \frac{6EI\Delta}{l^2}$$

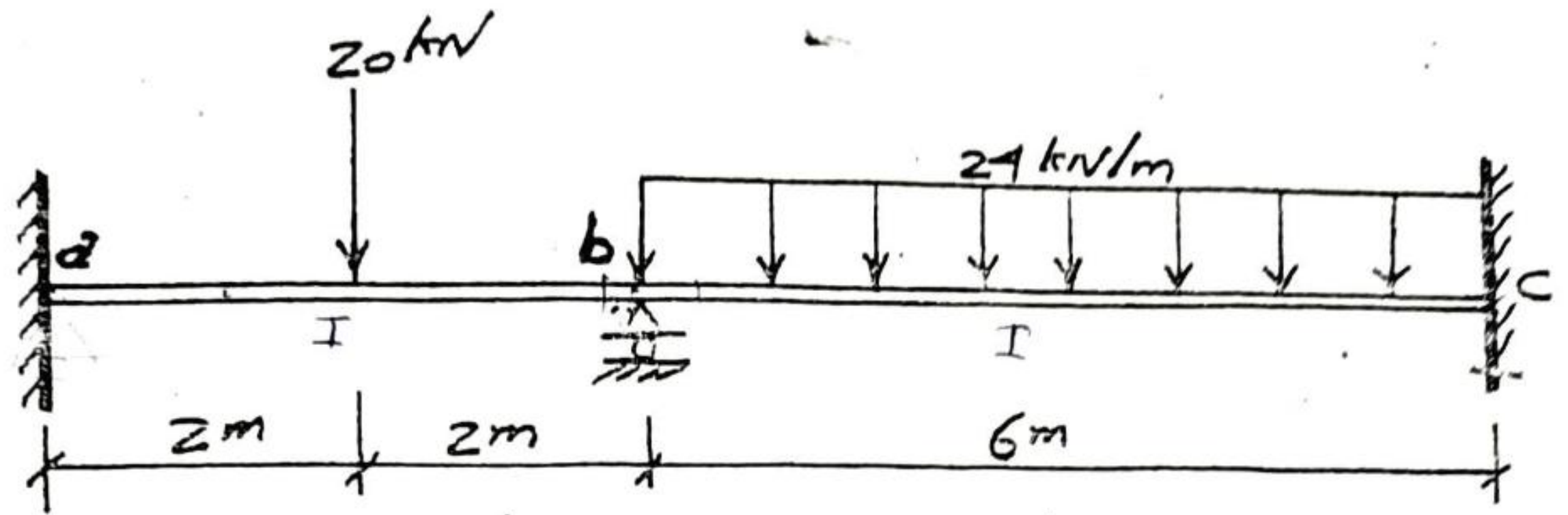


$$M_{AB}^F = - \frac{6EI\Delta}{l^2}$$

$$M_{BA}^F = - \frac{6EI\Delta}{l^2}$$

دوران beam بالاعتماد على المسافة لذلك
 Δ موجب والقانون سالب = موجب
 Δ سالب والقانون سالب = سالب

Ex:- Analyze the beam shown in fig. $EI = \text{constant}$.



Sol.

① - Find D.f:

Joint a : D.f) $ab = 0$

Joint c : D.f) $cb = 0$

Joint b:

$$k_{ba} = \frac{I}{4} ; \quad k_{bc} = \frac{I}{6}$$

$$\sum k = \frac{5}{12} I$$

$$D.f)_{ba} = \frac{\frac{I}{4}}{\frac{5}{12} I} = \frac{3}{5} ; \quad D.f)_{bc} = \frac{\frac{I}{6}}{\frac{5}{12} I} = \frac{2}{5}$$

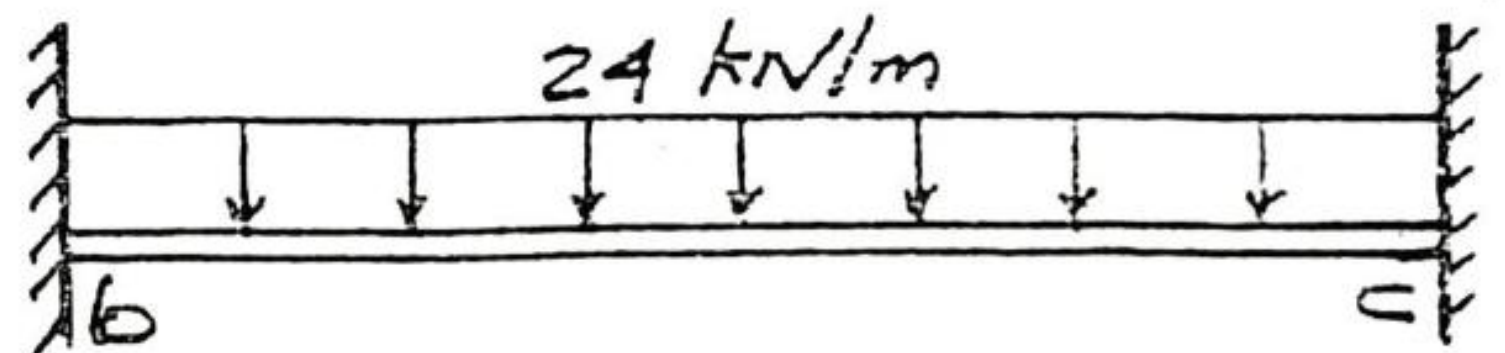
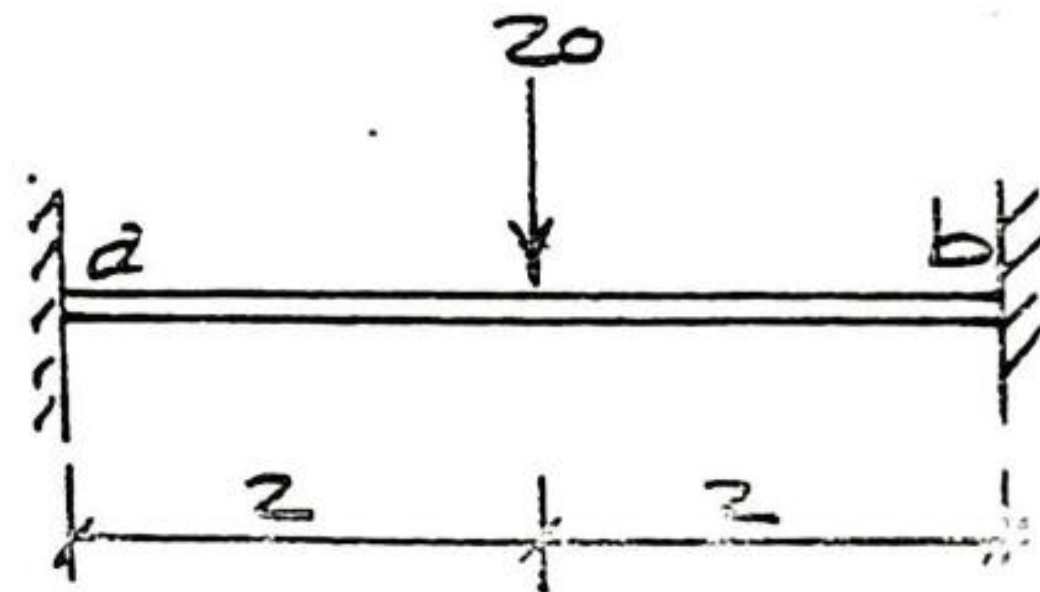
② - F.E.M

$$M_{ab}^F = - \frac{20(4)}{8} = -10 \text{ kN}\cdot\text{m}$$

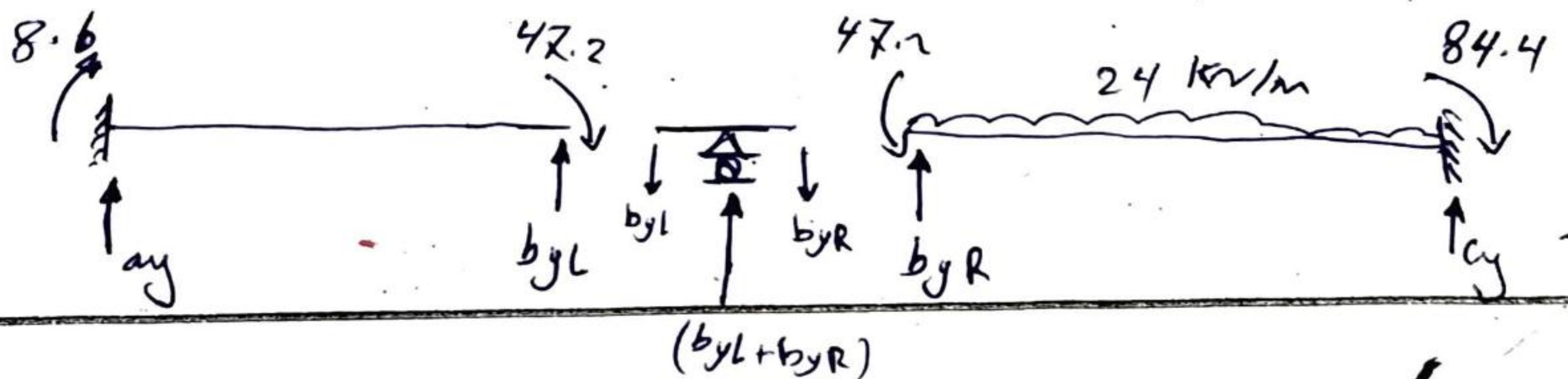
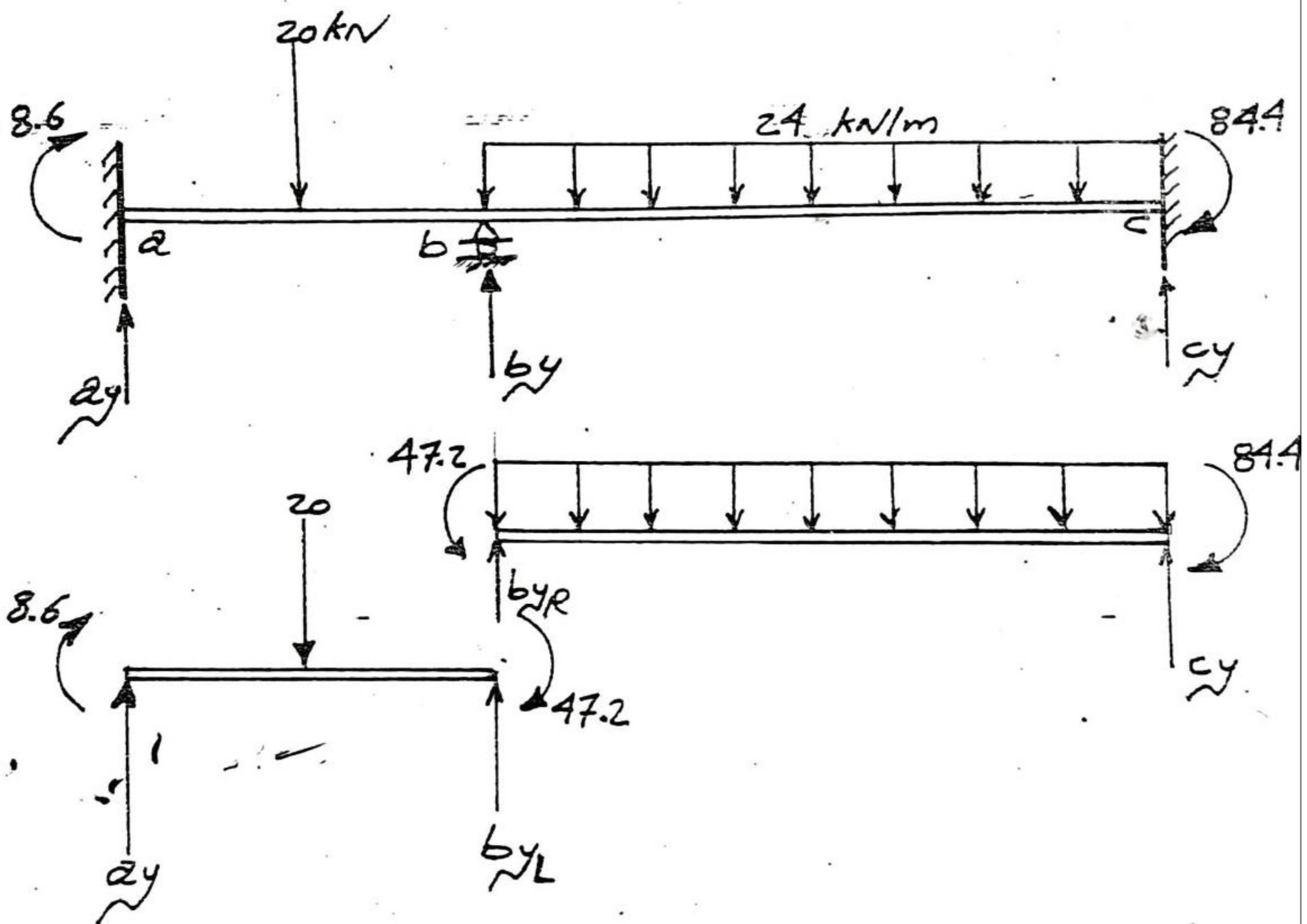
$$M_{ba}^F = + \frac{20(4)}{8} = +10 \text{ kN}\cdot\text{m}$$

$$M_{bc}^F = - \frac{24(6)^2}{12} = -72 \text{ kN}\cdot\text{m}$$

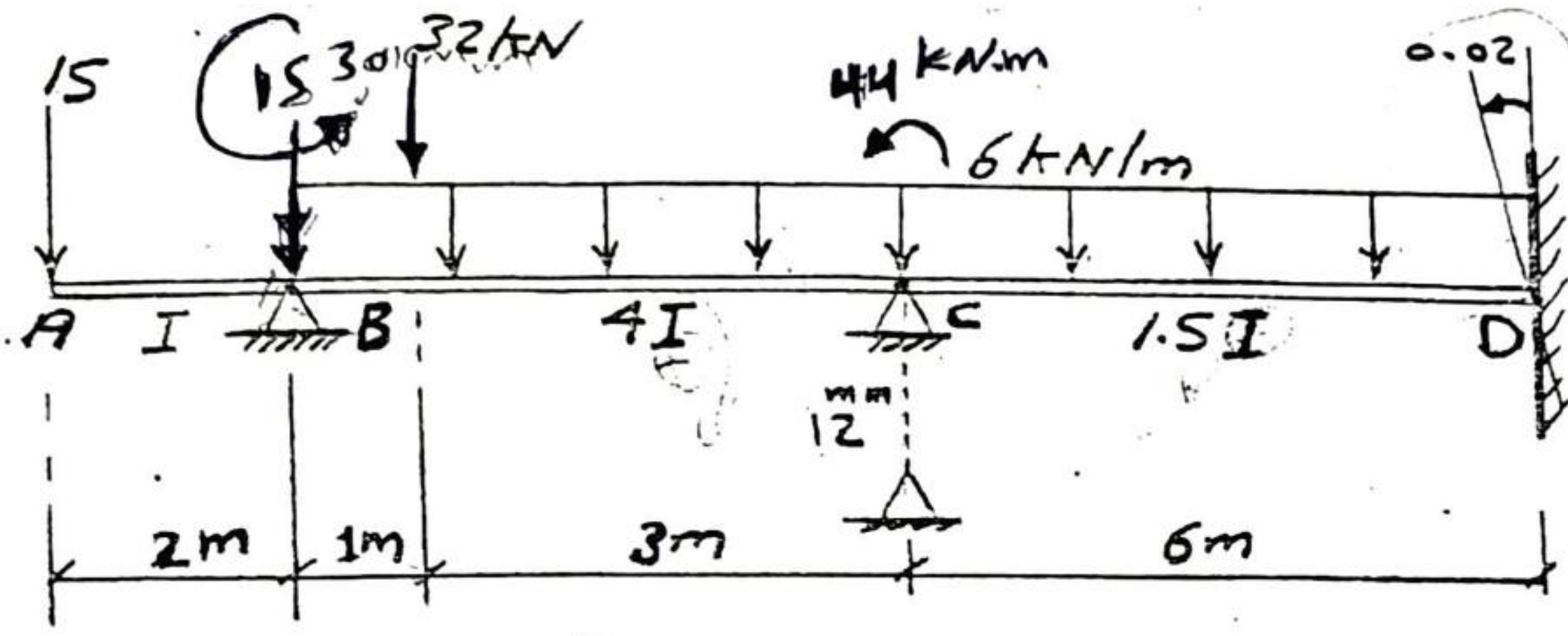
$$M_{cb}^F = + \frac{24(6)^2}{12} = +72 \text{ kN}\cdot\text{m}$$



Joint	a	b		c	cycle
mem.	ab	ba	bc	cb	
D.f	0	$\frac{3}{5}$	$\frac{2}{5}$	0	
F.E.M	-10	+10	-72	+72	
D.M	0	37.2	24.8	0	①
C.o.M	18.6	0	0	12.4	
D.M	0	0	0	0	②
ΣM	+8.6	47.2	-47.2	84.4	



Ex:- Analyze the beam shown in fig. $EI = 10^4 \text{ kN}\cdot\text{m}^2$.



Sol.

① - Find D.f:

Joint B: D.f) $B_C = 1$ (pin)

Joint D: D.f) $D_C = 0$ (Fixed)

Joint C:

$$k_{CB} = \frac{4I}{4} = I ; k_{CD} = \frac{1.5I}{6} = \frac{1}{4} I$$

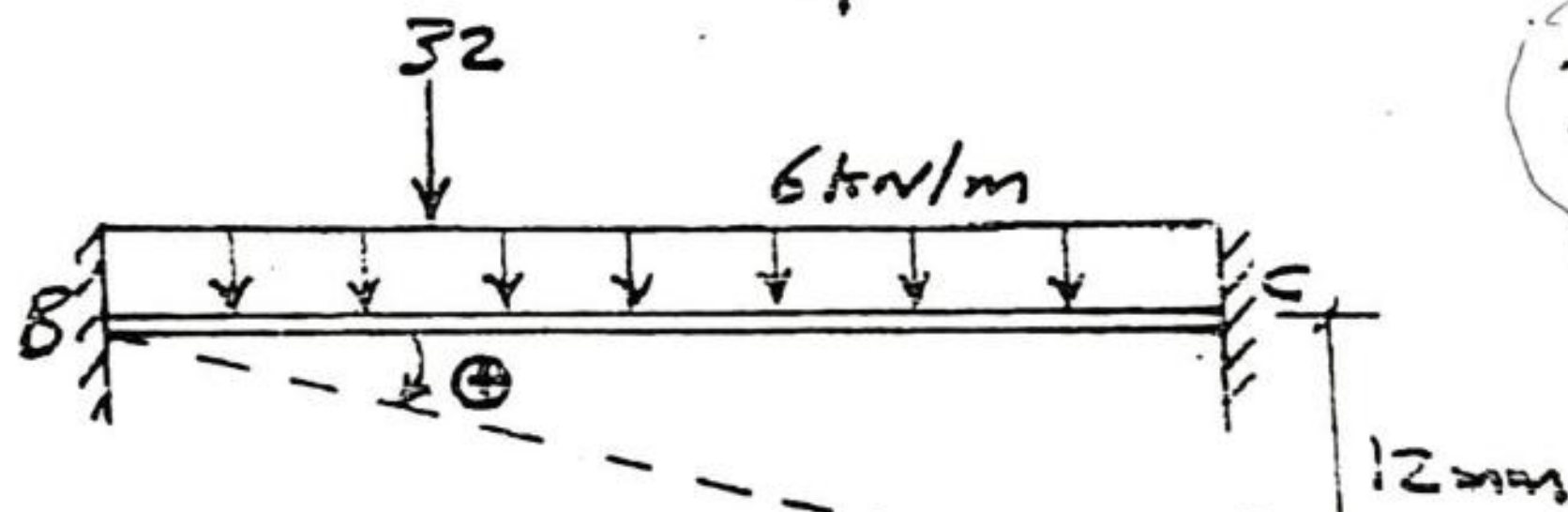
$$\Sigma k = I + \frac{1}{4} I = \frac{5}{4} I$$

$$D.f)_{CB} = \frac{I}{\frac{5}{4} I} = \frac{4}{5}, \quad D.f)_{CD} = \frac{\frac{1}{4} I}{\frac{5}{4} I} = \frac{1}{5}$$

$$D.f) = \frac{K}{\Sigma k}$$

Settlement Δ

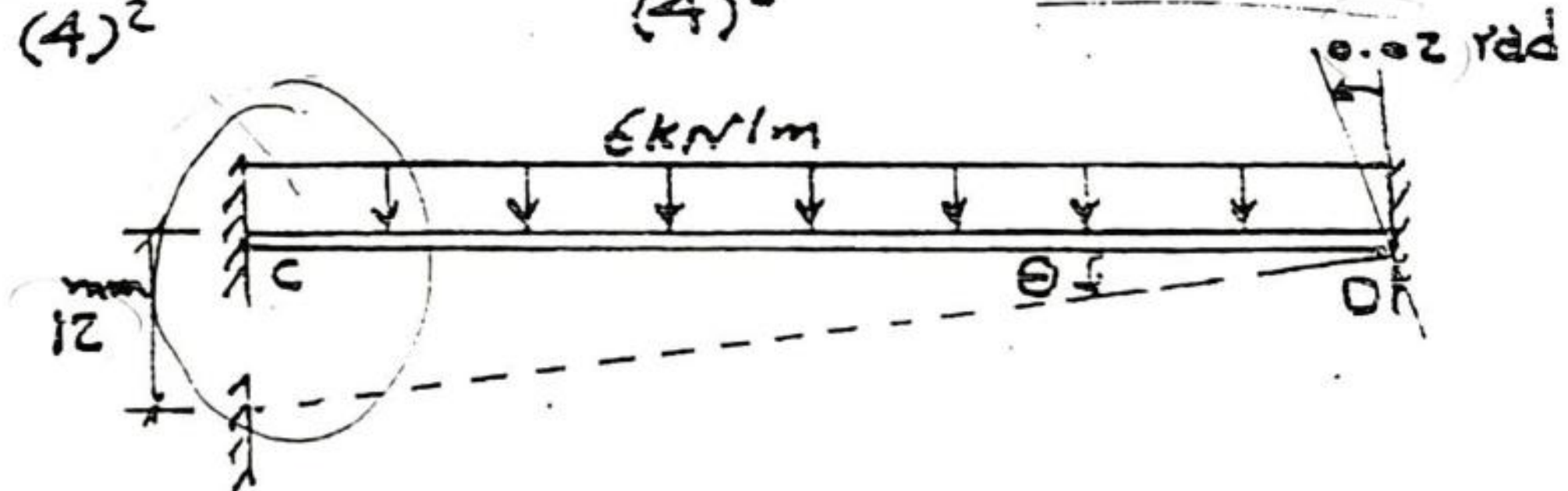
$$\frac{-6EI\Delta}{l^2}$$



② - F.E.M

$$M_{BC}^F = - \frac{6(4)^2}{12} - \frac{32(1)(3)^2}{(4)^2} - \frac{6(4 \times 10^4)(0.012)}{(4)^2} = -206 \text{ kN}\cdot\text{m}$$

$$M_{CB}^F = + \frac{6(4)^2}{12} + \frac{32(3)(1)^2}{(4)^2} - \frac{6(4 \times 10^4)(0.012)}{(4)^2} = -166 \text{ kN}\cdot\text{m}$$



$$M_{CD}^F = - \frac{6(6)^2}{12} + \frac{6(1.5 \times 10^4)(0.012)}{(6)^2} - \frac{2(1.5 \times 10^4)(0.02)}{6} = -88 \text{ kN}\cdot\text{m}$$

$$M_{DC}^F = + \frac{6(6)^2}{12} + \frac{6(1.5 \times 10^4)(0.012)}{(6)^2} - \frac{4(1.5 \times 10^4)(0.02)}{6} = -152 \text{ kN}\cdot\text{m}$$

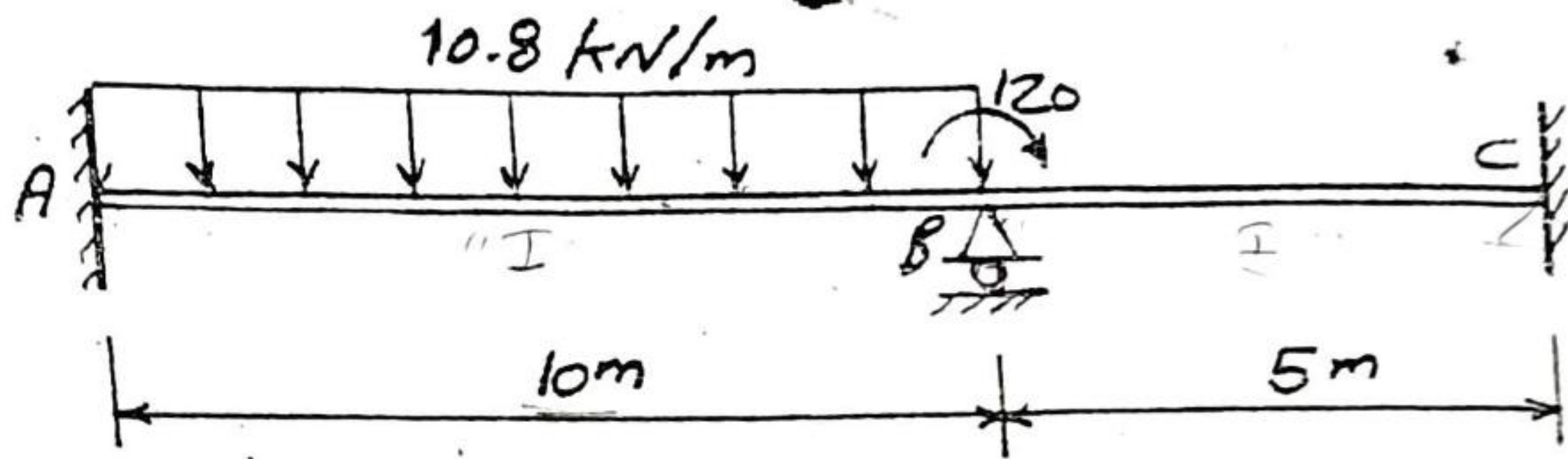
+30

+44

Cycle

Joint	B	C		D
mem.	Bc	cB	cD	Dc
$\bar{D}.f$	1	$\frac{4}{5}$	$\frac{1}{5}$	0
F.E.M	-206	-166	-88	-152
D.M	+176	168	42	0
C.o.M	84	88	0	21
D.M	-84	-70.4	-17.6	0
C.o.M	-35.2	-42	0	-8.8
D.M	35.2	33.6	8.4	0
C.o.M	16.8	17.6	0	4.2
D.M	-16.8	-14.08	-3.52	0
C.o.M	-7.04	-8.4	0	-1.76
D.M	+7.04	6.72	1.68	0
C.o.M	⋮	⋮	⋮	⋮

Ex:- Draw B.M.D for the beam shown. $EI = \text{Const}$



Sol.

① - Find D.F:

Joint A: D.F)_{AB} = 0

Joint C: D.F)_{CB} = 0

Joint B:

$$k_{BA} = \frac{I}{10}, \quad k_{BC} = \frac{I}{5}$$

$$\sum k = \frac{3}{10} I$$

$$D.F)_{BA} = \frac{k_{BA} \cdot \frac{I}{10}}{\sum k} = \frac{\frac{I}{10}}{\frac{3}{10} I} = \frac{1}{3}$$

$$D.F)_{BC} = \frac{k_{BC} \cdot \frac{I}{5}}{\sum k} = \frac{\frac{I}{5}}{\frac{3}{10} I} = \frac{2}{3}$$

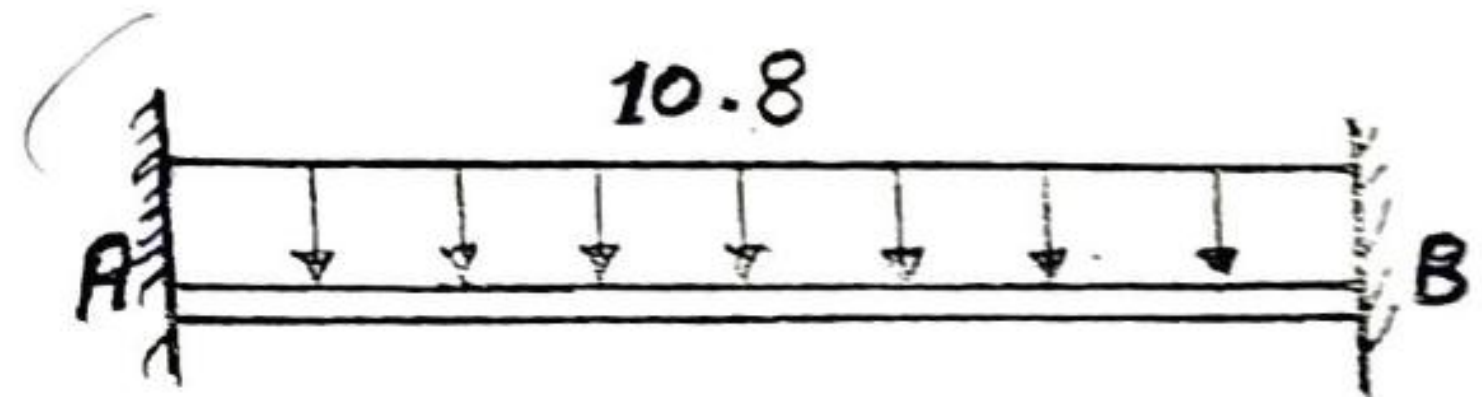
② - F.E.M

$$M_{AB}^F = \frac{-10.8(10)^2}{12} = -90 \text{ kN}\cdot\text{m}$$

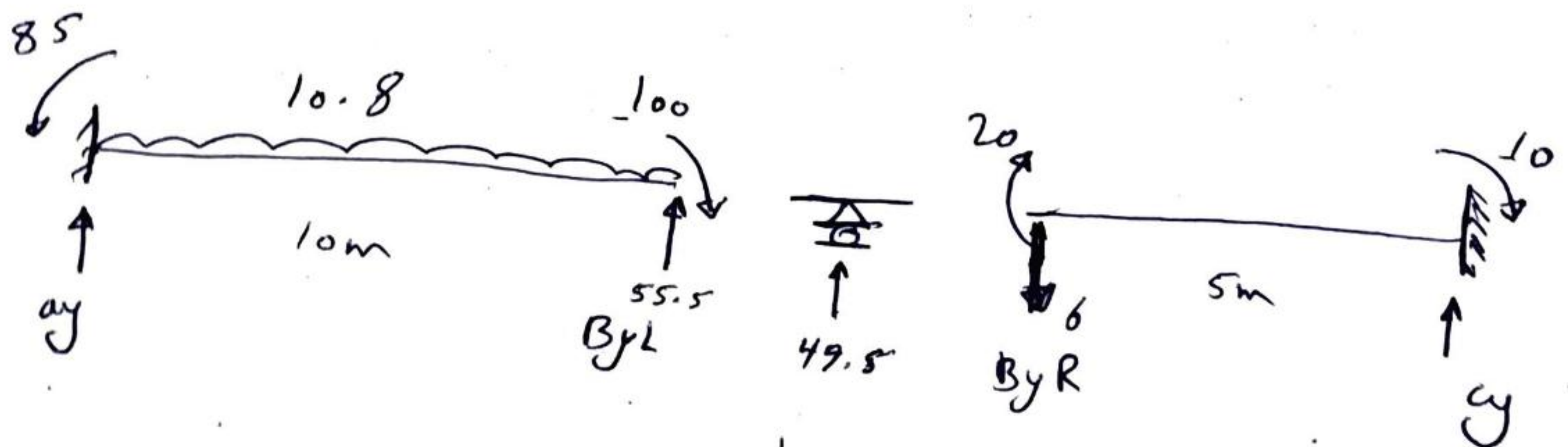
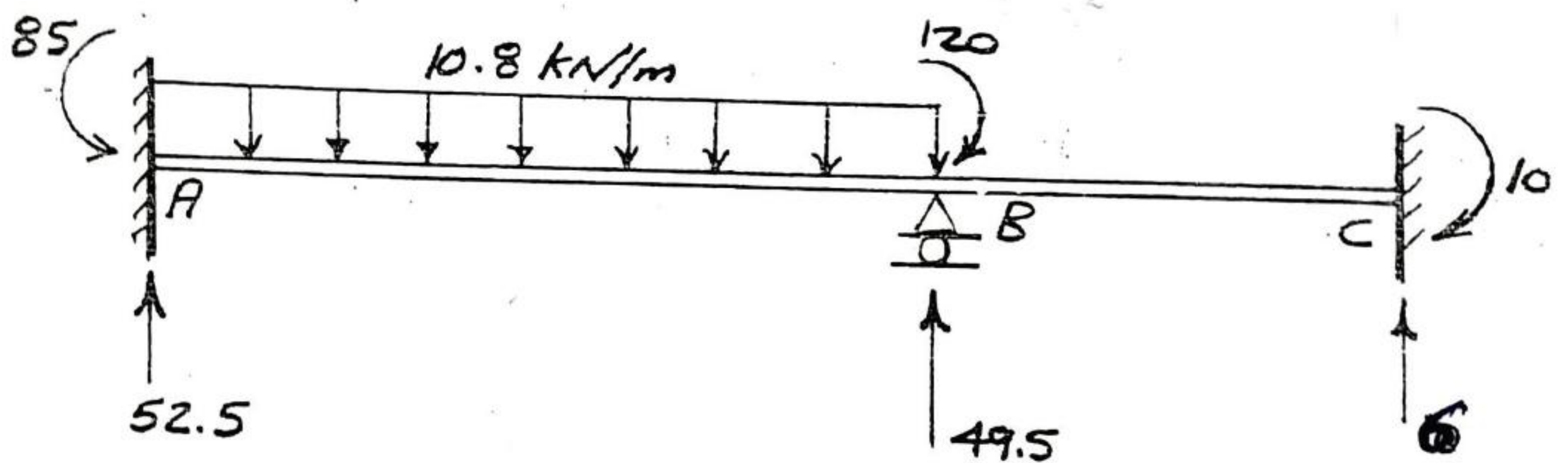
$$M_{BA}^F = \frac{+10.8(10)^2}{12} = +90 \text{ kN}\cdot\text{m}$$

$$M_{BC}^F = 0$$

$$M_{CB}^F = 0$$



		-120		
Joint	A	B		C
mem.	AB	BA	Bc	CB
D.f	0	$\frac{1}{3}$	$\frac{2}{3}$	0
F.E.M	-90	+90	0	0
D.M	0	+10	+20	0
C.o.M	5	0	0	10
D.M	0	0	0	0
ΣM	-85	+100	+20	+10

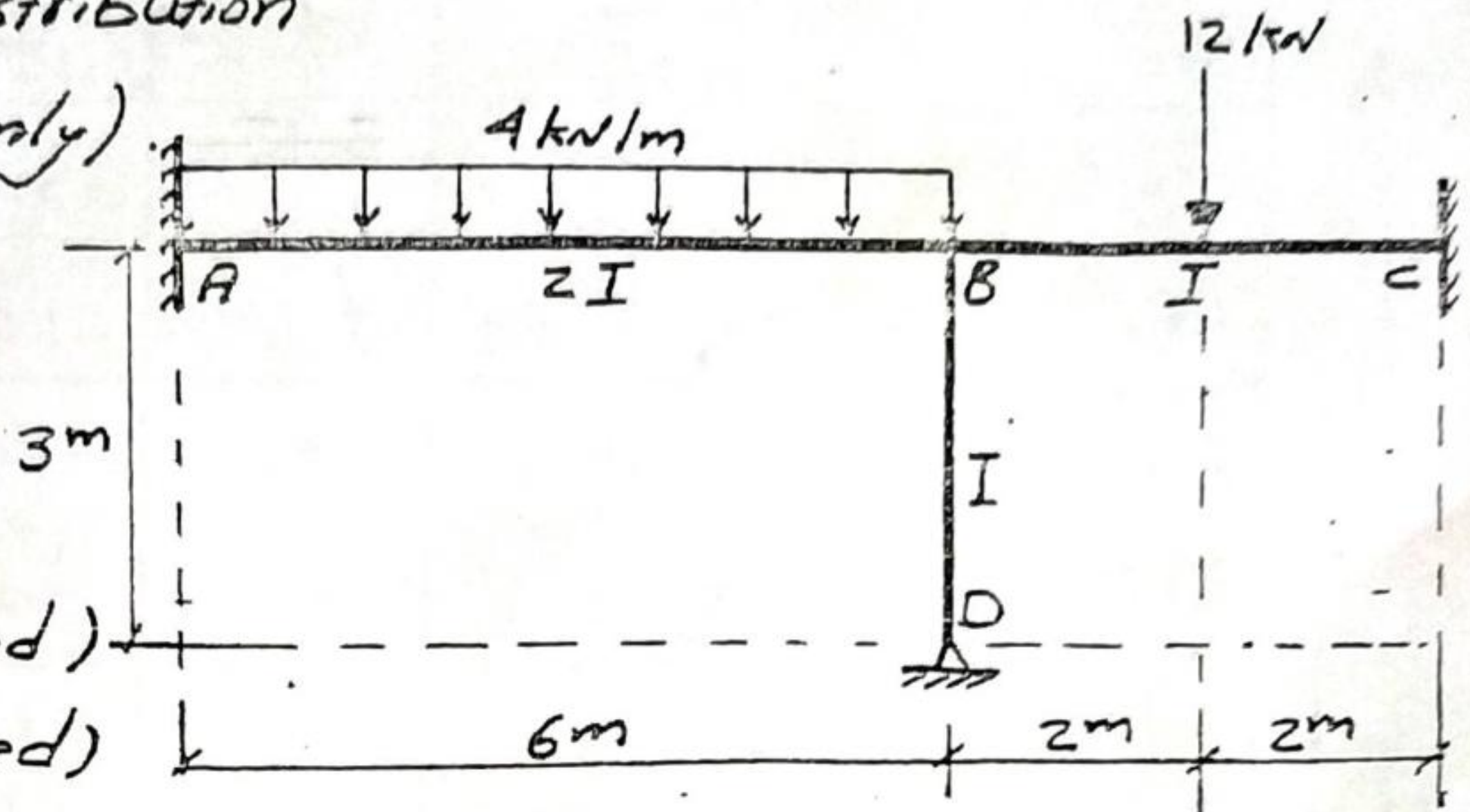


$$\begin{aligned} \sum M_A = 0 \\ -85 + 100 + (10.8 \times \frac{10^2}{2}) - B_{yL} \times 10 = 0 \\ B_{yL} = 55.5 \text{ kN} \uparrow \\ \sum F_y = 0 \\ a_y = 52.5 \text{ kN} \uparrow \end{aligned}$$

$$\begin{aligned} \sum M_C = 0 &\Rightarrow 20 + 10 + B_{yR} \times 5 \\ B_{yR} &= 6 \text{ kN} \downarrow \\ \sum F_y = 0 & \quad c_y = 6 \text{ kN} \uparrow \end{aligned}$$

Ex: Analyze the frame shown in fig.

Use the moment Distribution method. (3 cycles only)



Sol.

① - Find D.f:

Joint A: D.f)_{AB} = 0 (fixed)

Joint C: D.f)_{CB} = 0 (fixed)

Joint D: D.f)_{DB} = 1 (Pin) or roller = 1

Joint B:

$$k_{BA} = \frac{2I}{6} = \frac{I}{3}, \quad k_{BD} = \frac{I}{3}, \quad k_{BC} = \frac{I}{4}$$

$$\sum k = k_{BA} + k_{BD} + k_{BC} = \frac{I}{3} + \frac{I}{3} + \frac{I}{4} = \frac{11}{12} I$$

$$D.f)_{BA} = \frac{k_{BA}}{\sum k} = \frac{\frac{I}{3}}{\frac{11}{12} I} = \frac{4}{11}$$

$$D.f)_{BD} = \frac{\frac{I}{3}}{\frac{11}{12} I} = \frac{4}{11}$$

① Fixed D.f. = 0

$$D.f)_{BC} = \frac{\frac{I}{4}}{\frac{11}{12} I} = \frac{3}{11}$$

Sattlement
rotation

② - F.E.M

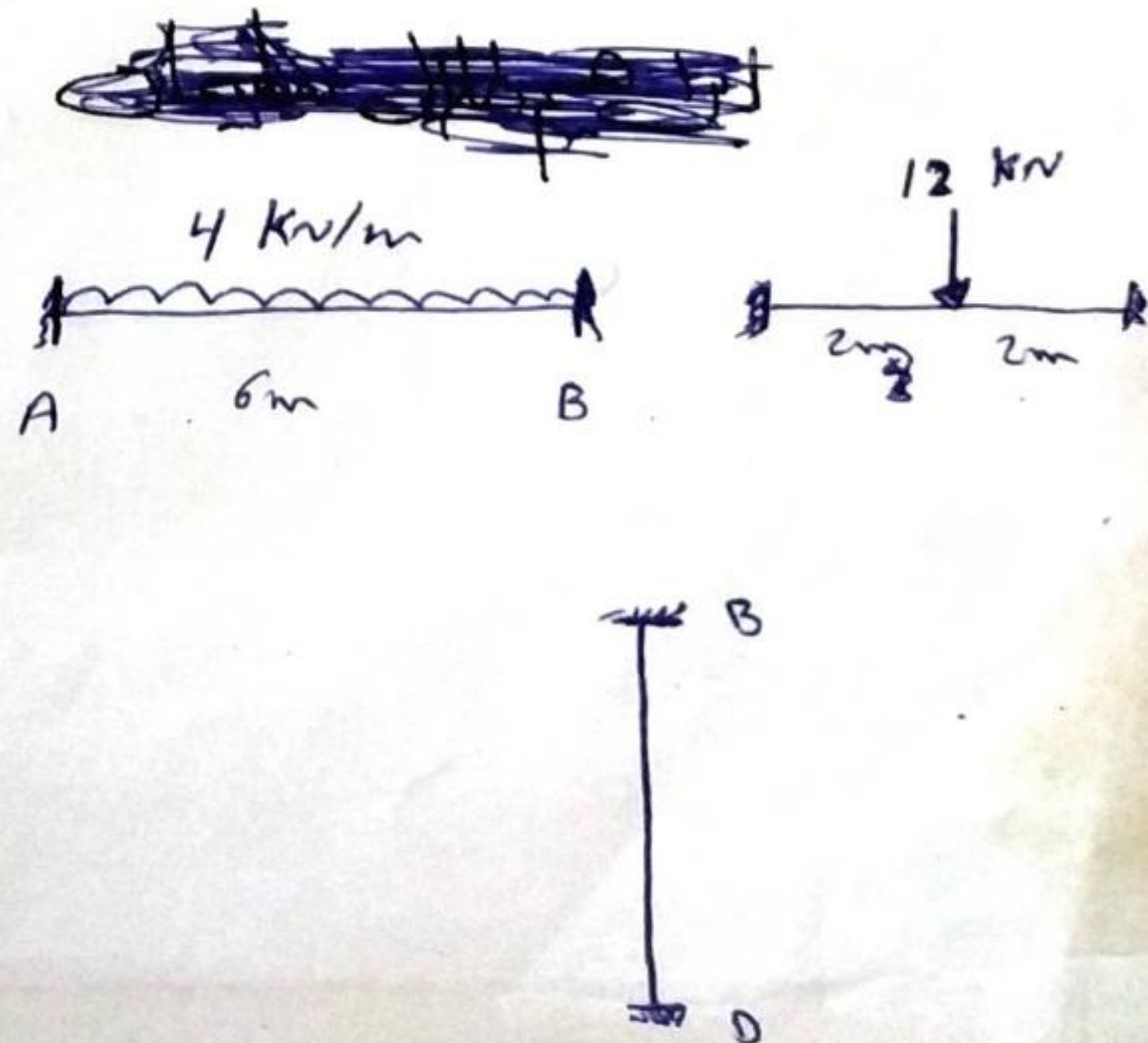
$$M_{BD}^F = M_{DB}^F = 0$$

$$M_{AB}^F = -\frac{4(6)^2}{12} = -12 \text{ kN.m}$$

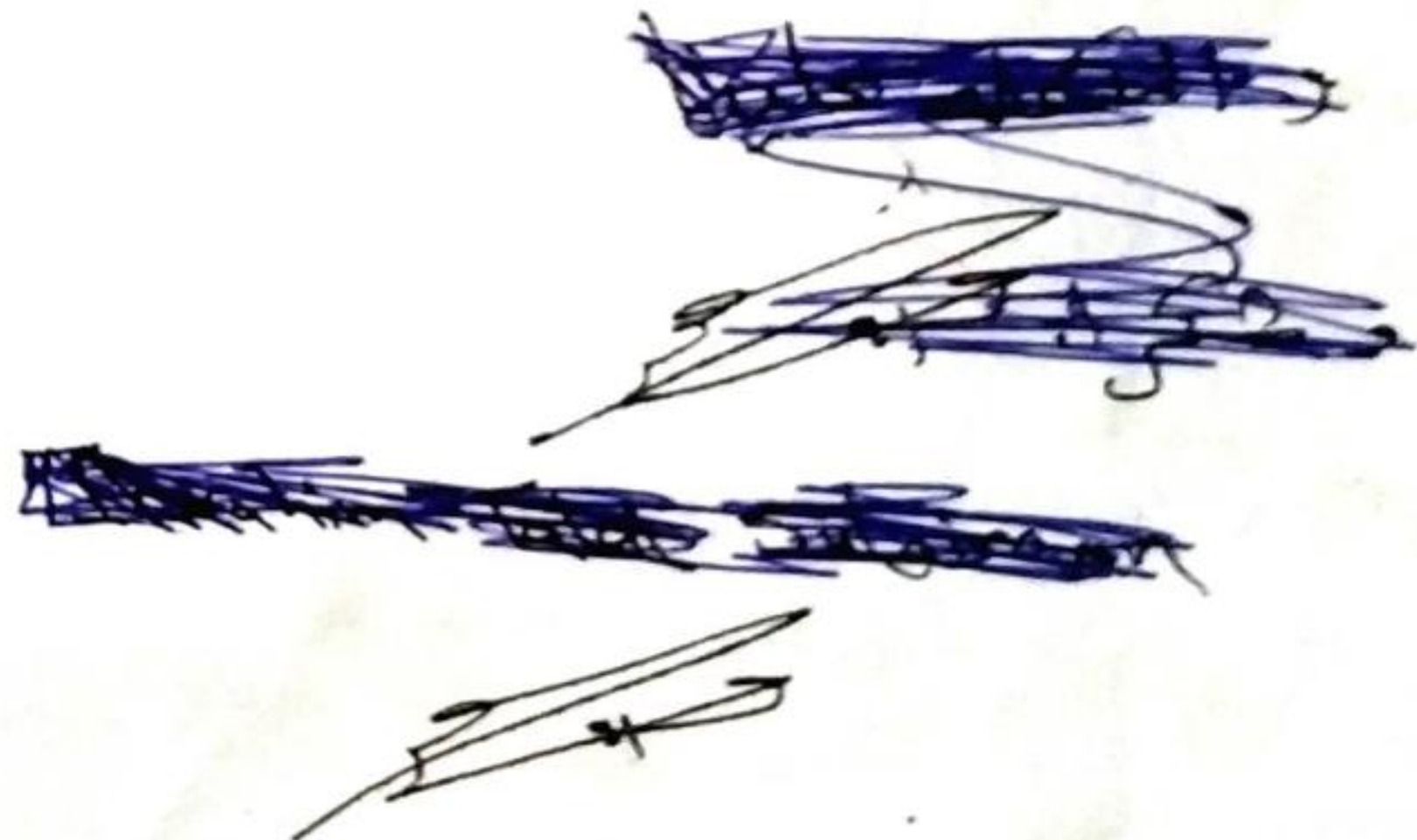
$$M_{BA}^F = +\frac{4(6)^2}{12} = 12 \text{ kN.m}$$

$$M_{BC}^F = -\frac{12(4)}{8} = -6 \text{ kN.m}$$

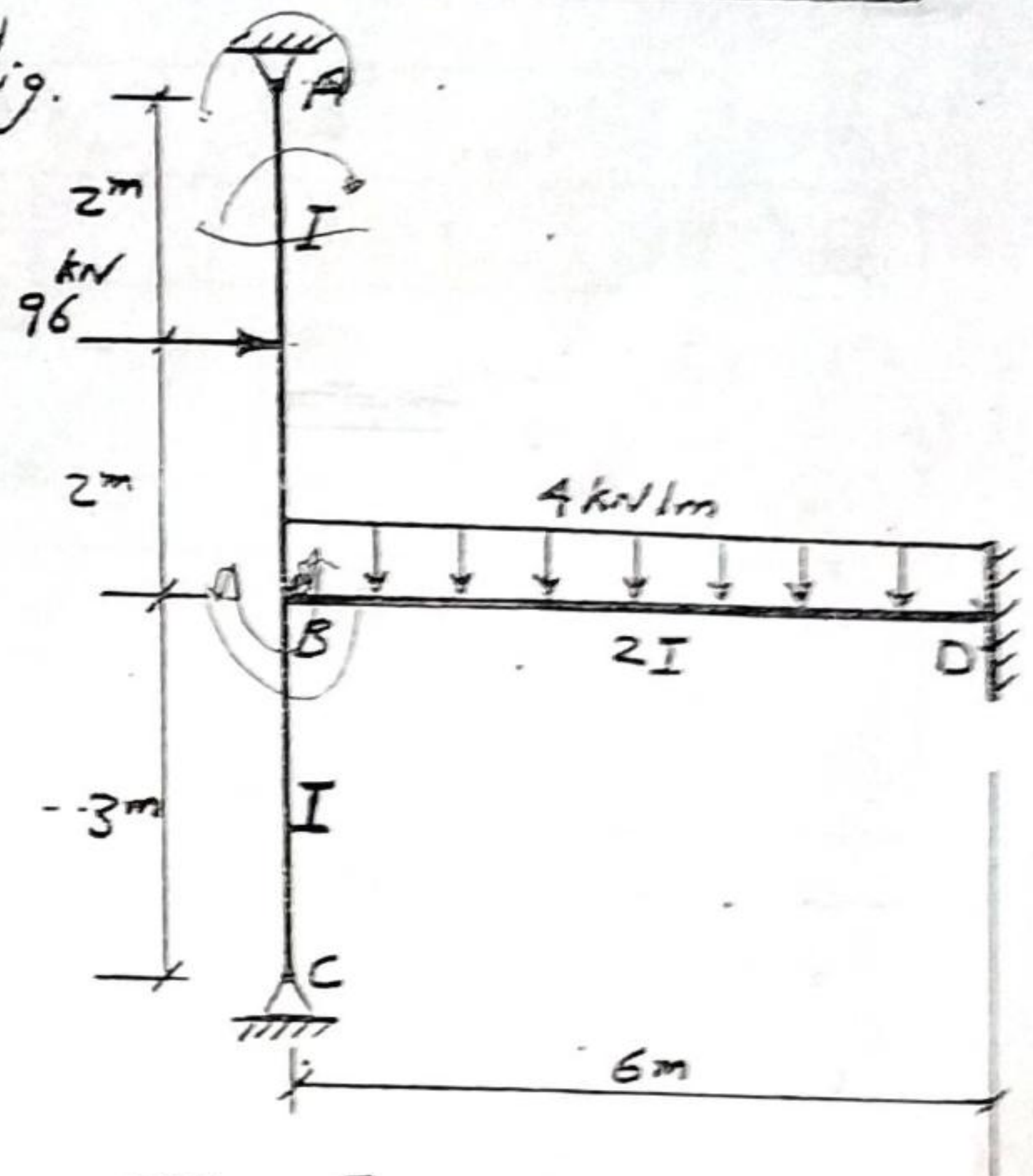
$$M_{CB}^F = +\frac{12(4)}{8} = 6 \text{ kN.m}$$



Joint	A	B			C	D	Cycle
mem.	AB	BA	BD	BC	CB	DB	
D.F	0	$\frac{4}{11}$	$\frac{4}{11}$	$\frac{3}{11}$	0	1	
F.E.M	-12	12	0	-6	6	0	①
D.M	0	-2.1818	-2.1818	-1.636	0	0	
C.o.M	-1.0909	0	0	0	-0.818	-1.0909	②
D.M	0	0	0	0	0	1.0909	
C.o.M	0	0	0.545	0	0	0	③
D.M	0	-0.198	-0.198	-0.148	0	0	
ΣM	-13.0909	9.62	-1.834	-7.784	5.182	0	



Ex:- For the frame shown in fig. Use moment distribution to find Reaction at D (RD).



Sol.

①. Find D.f:

Joint A: D.f) $AB = 1$ (pin)

Joint C: D.f) $CB = 1$ (pin)

Joint D: D.f) $DB = 0$ (Fixed)

Joint B:

$$k_{BA} = \frac{I}{4}, \quad k_{BC} = \frac{I}{3}, \quad k_{BD} = \frac{2I}{6} = \frac{I}{3}$$

$$\sum k = k_{BA} + k_{BC} + k_{BD} = \frac{I}{4} + \frac{I}{3} + \frac{I}{3} = \frac{11}{12} I$$

$$D.f)_{BA} = \frac{\frac{I}{4}}{\frac{11}{12} I} = \frac{3}{11}$$

$$D.f)_{BC} = \frac{\frac{I}{3}}{\frac{11}{12} I} = \frac{4}{11}$$

$$D.f)_{BD} = \frac{\frac{I}{3}}{\frac{11}{12} I} = \frac{4}{11}$$

② - F.E.M

$$M_{BC}^F = M_{CB}^F = 0$$

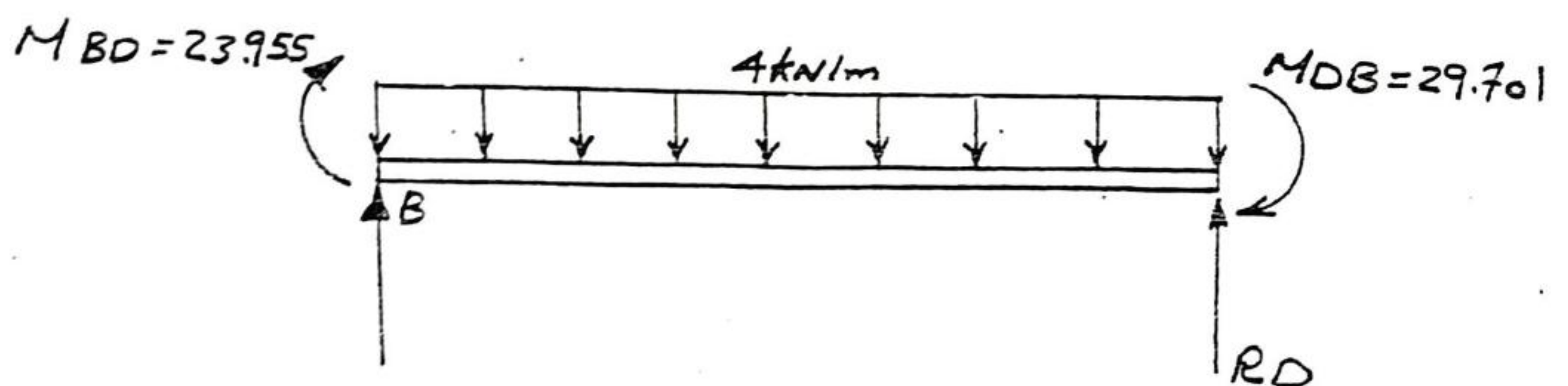
$$M_{AB}^F = + \frac{96(4)}{8} = +48$$

$$M_{BA}^F = - \frac{96(4)}{8} = -48$$

$$M_{BD}^F = - \frac{4(6)^2}{12} = -12$$

$$M_{DB}^F = + \frac{4(6)^2}{12} = +12$$

Joint	A		B		C	D	cycle
mem.	AB	BA	BD	Bc	cB	DB	
D.f	1	$\frac{3}{11}$	$\frac{4}{11}$	$\frac{4}{11}$	1	0	
F.E.M	48	-48	-12	0	0	+12	①
D.M	-48	16.363	21.818	21.818	0	0	
C.o.M	8.1815	-24	0	0	10.909	10.909	
D.M	-8.1815	6.545	8.727	8.727	-10.909	0	②
C.o.M	3.272	-4.09	0	-5.454	4.363	4.363	
D.M	-3.272	2.603	3.47	3.47	-4.363	0	③
C.o.M	1.3015	-1.636	0	-2.1815	1.735	1.735	
D.M	-1.3015	1.041	1.388	1.388	-1.735	0	④
C.o.M	0.5205	-0.6507	0	-0.8675	0.694	0.694	
D.M	-0.5205	0.414	0.552	0.552	-0.694	0	⑤
ΣM	0	-51.41	+23.955	+27.452	0	+29.701	



$$\Sigma M_B = 0 \quad \text{②}$$

$$23.955 + 29.701 + 4 \times 6 \times 3 - 6R_D = 0$$

$$R_D = 21 \text{ kN} \uparrow$$

Ex:- For the frame shown in figure. Use the moment-distribution method to find the reaction at the supports if the joint (b) settles (3cm) downward.

$$EI = 120 \times 10^5 \text{ kN}\cdot\text{mm}^2$$

$$\frac{(1000)^2}{(1000)^2} \rightarrow \boxed{\text{kN}\cdot\text{m}}$$

Sol.

① - Find D.f :

$$\text{Joint d : D.f)}_{db} = 0$$

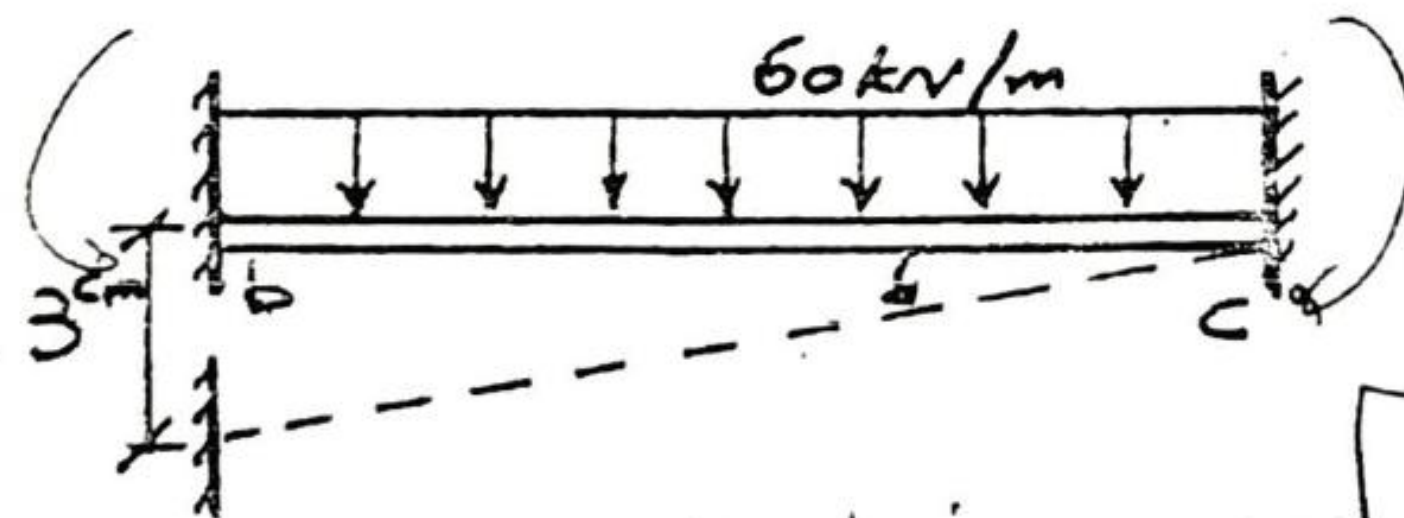
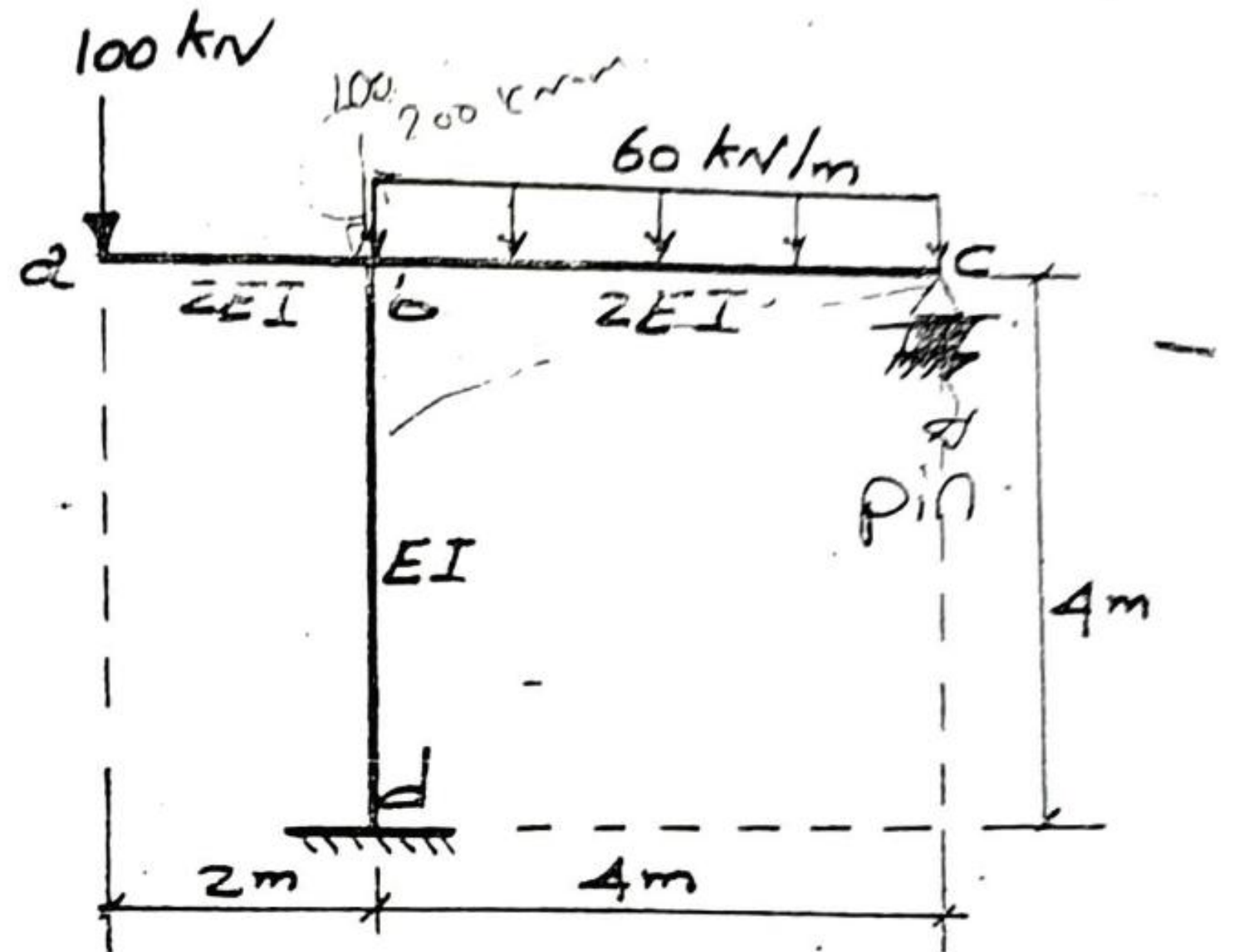
$$\text{Joint c : D.f)}_{cb} = 1$$

Joint b :

$$k_{bd} = \frac{I}{4}, \quad k_{bc} = \frac{2I}{4} = \frac{I}{2}$$

$$\Sigma k = \frac{I}{4} + \frac{I}{2} = \frac{3}{4} I$$

$$\text{D.f)}_{bd} = \frac{\frac{I}{4}}{\frac{3}{4} I} = \frac{1}{3}, \quad \text{D.f)}_{bc} = \frac{\frac{I}{2}}{\frac{3}{4} I} = \frac{2}{3}$$



Sahtemate \rightarrow usle

$$\boxed{\frac{-6EI\Delta}{l^2}}$$

② - F.E.M

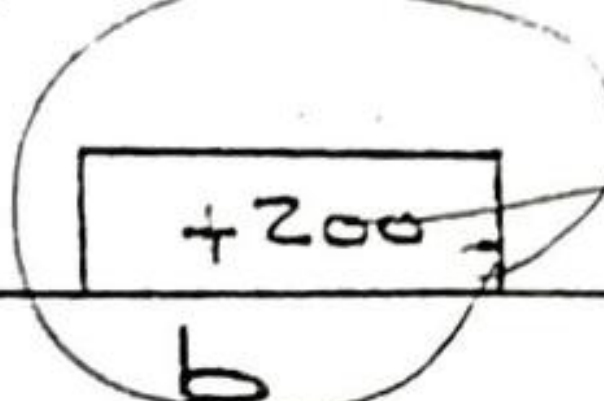
$$M_{db}^F = M_{bd}^F = 0$$

$\ominus \rightarrow \oplus$

$$M_{bc}^F = -\frac{60(4)^2}{12} + \frac{6(2 \times 12) \times 0.03}{(4)^2} = -79.73 \text{ kN}\cdot\text{m}$$

$$M_{cb}^F = +\frac{60(4)^2}{12} + \frac{6(2 \times 12) \times 0.03}{(4)^2} = 80.27 \text{ kN}\cdot\text{m}$$

در صورتی که در این صورت



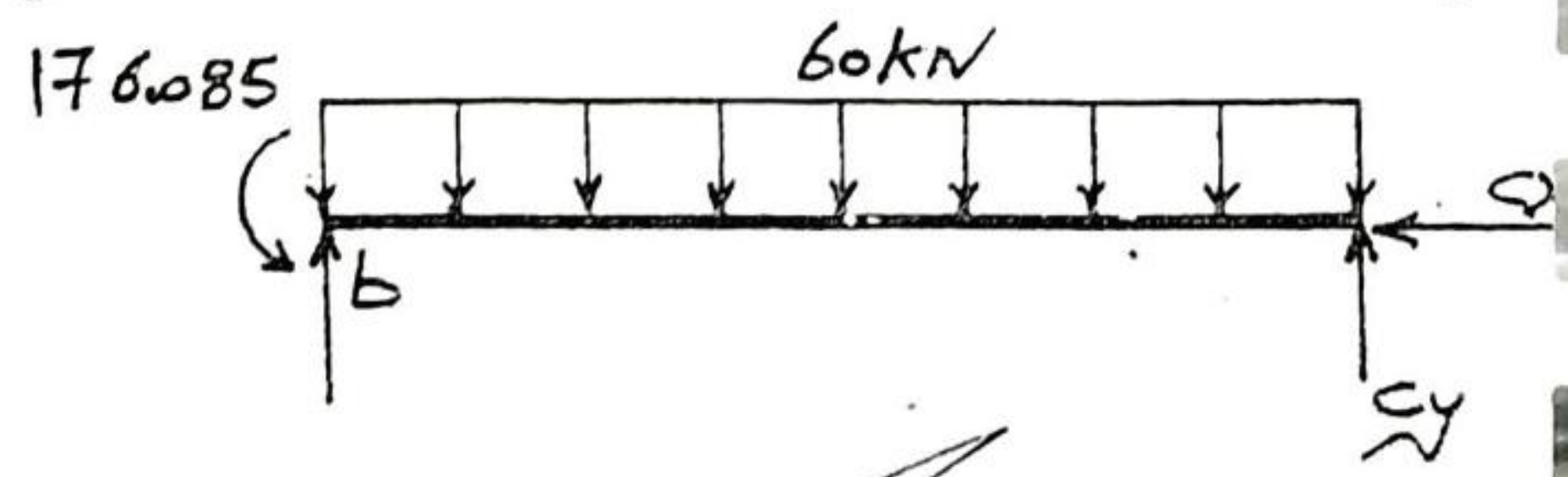
در صورتی که در این صورت $0 + 200 - 79.73$

Joint	d	b	c
mem.	db	bd	bc
D.f	0	$\frac{1}{3}$	$\frac{2}{3}$
F.E.M	0	0	-79.73
D.M	0	-40.09	-80.18
C.o.M	-20.045	0	-40.135
D.M	0	13.378	26.756
C.o.M	6.689	0	20.045
D.M	0	-6.68	-13.363
C.o.M	-3.34	0	-6.689
D.M	0	2.229	4.459
C.o.M	1.1145	0	3.34
D.M	0	-1.113	-2.226
C.o.M	-0.5565	0	-1.1145
D.M	0	0.3715	0.743
C.o.M	0.185	0	0.5565
D.M	0	0.1855	0.371
ΣM	-15.953	-31.719	-176.085

اذا حدك سنوي 5 طلقان ونگنه

من جمع شعول من داخل 200

$\Sigma Mb = 0 \Rightarrow c_y$
 From whole frame.
 $\Sigma Md = 0 \Rightarrow c_x = v$
 $\Sigma Fy = 0 \Rightarrow c_y = v$
 $\Sigma Fx = 0 \Rightarrow c_x = v$



~~Handwritten scribbles and notes.~~