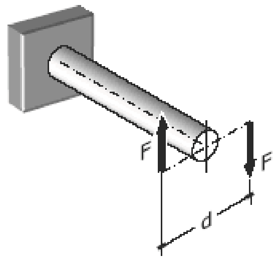
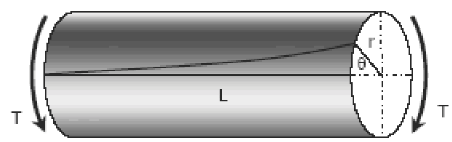
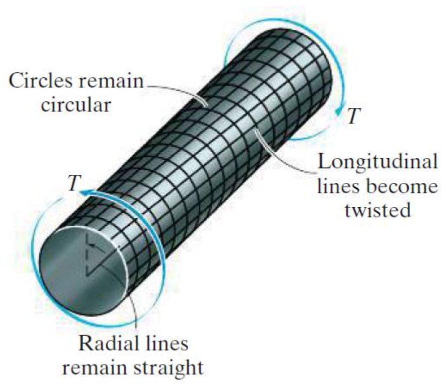
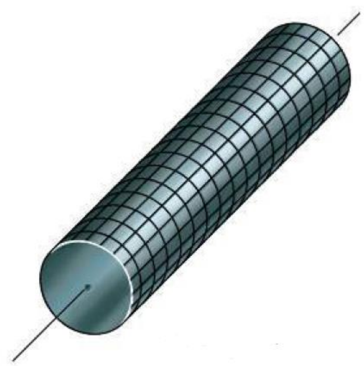
Torsion

Consider a bar to be rigidly attached at one end and twisted at the other end by a torque or twisting moment T equivalent to F × d, which is applied perpendicular to the axis of the bar, as shown in the figure. Such a bar is said to be in torsion.



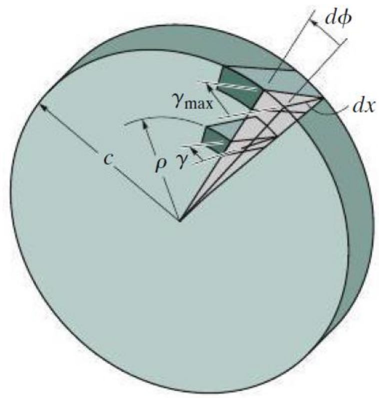






Torsional Shearing Stress, τ

For a solid or hollow circular shaft subject to a twisting moment T, the torsional shearing stress τ at a distance ρ from the center of the shaft is



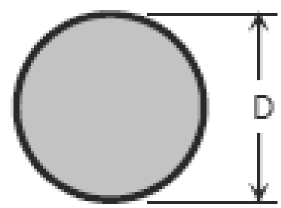
$$\tau = \frac{T\rho}{J} \text{ and } \tau_{max} = \frac{Tr}{J}$$

where J is the polar moment of inertia of the section and r is the outer radius.

- For solid cylindrical shaft:

$$J = \frac{\pi}{32}D^4$$

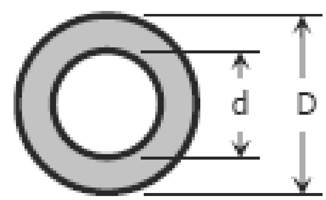
$$\tau_{max} = \frac{16T}{\pi D^3}$$



- For hollow cylindrical shaft:

$$J = \frac{\pi}{32}(D^4 - d^4)$$

$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)}$$



Angle of Twist

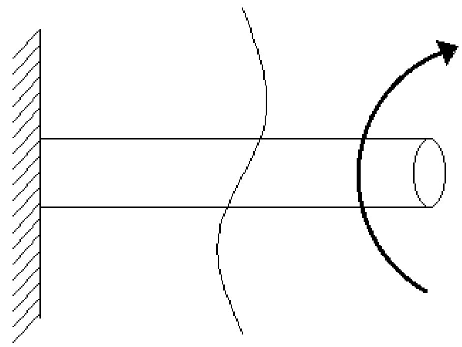
The angle θ through which the bar length L will twist is

$$\theta = \frac{TL}{JG} \text{ in radians}$$

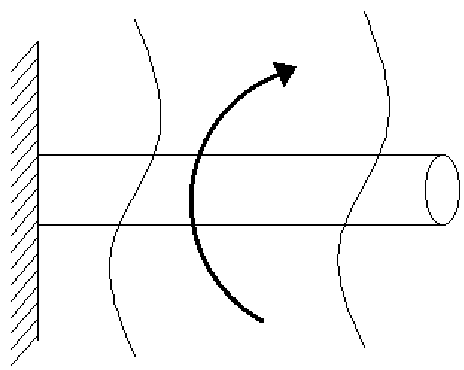
where T is the torque in N·mm, L is the length of shaft in mm, G is shear modulus in MPa, J is the polar moment of inertia in mm4, D and d are diameter in mm, and r is the radius in mm.

ملاحظات /

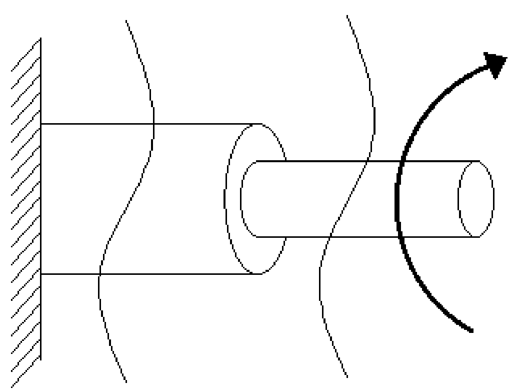
1- اذا كان العزم في طرف الشكل فيأخذ مقطع واحد فقط في الجسم (اذا كان المقطع ثابت)

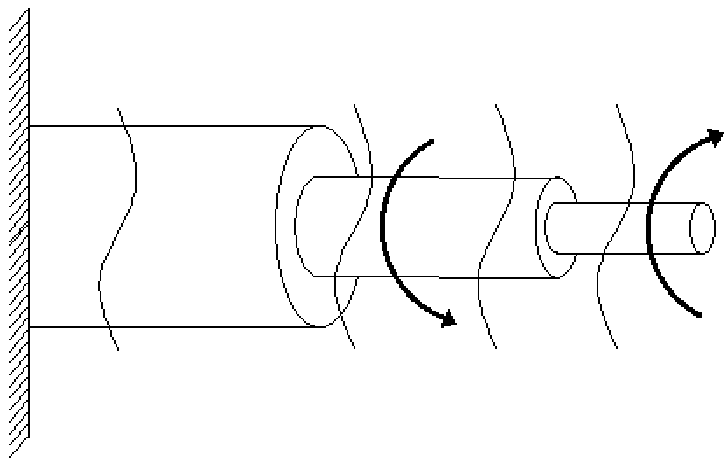


2- في حال كان العزم على مسافة من طرف الشكل فيجب اخذ مقطع قبل ومقطع بعد العزم

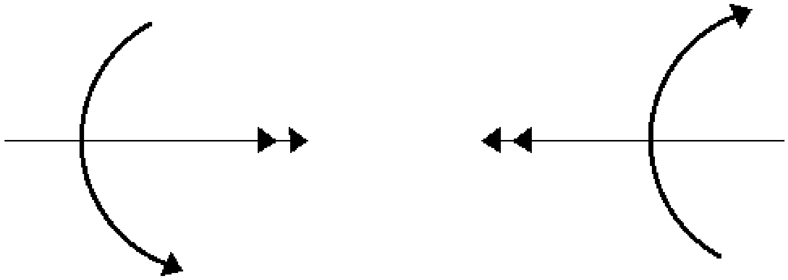


3- في حال تغيير مقطع الشكل فيأخذ مقطع في كل تغير

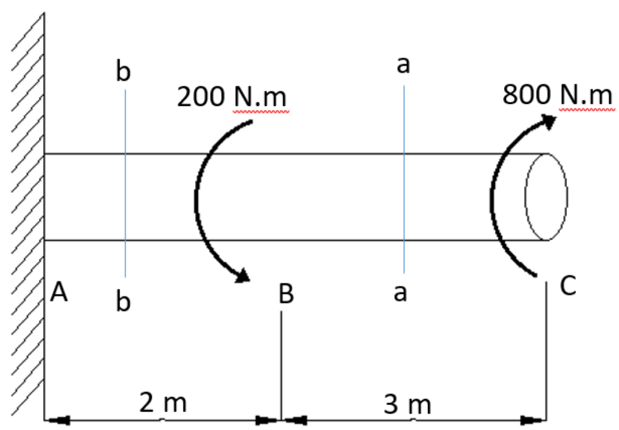




4- نستخدم قاعدة الكف اليمنى لمعرفة اتجاه عزم اللي



Ex; a shaft of diameter 25mm is subjected to two torques as shown in figure. Determine the shearing stress (τ) in each segment and the angle of rotation of the free end. Use G=80 GPa



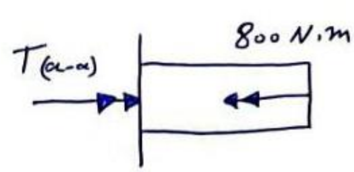
Sol :-

1) Section $(a-a)$

$$\Sigma T = 0$$

$$0 = -800 + T_{(a-a)}$$

$$\therefore T_{(a-a)} = 800 \text{ N.m}$$

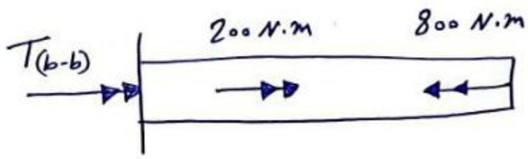


$$\tau = \frac{T * \frac{D}{2}}{J} = \frac{800 * 10^3 * \frac{25}{2}}{\frac{\pi}{32} * D^4}$$

$$= 260.89 \text{ MPa}$$

2) Section (b-b)

$\Sigma T = 0$



$0 = -800 + 200 + T_{(b-b)}$

$\therefore T_{(b-b)} = 600 \text{ N.m}$

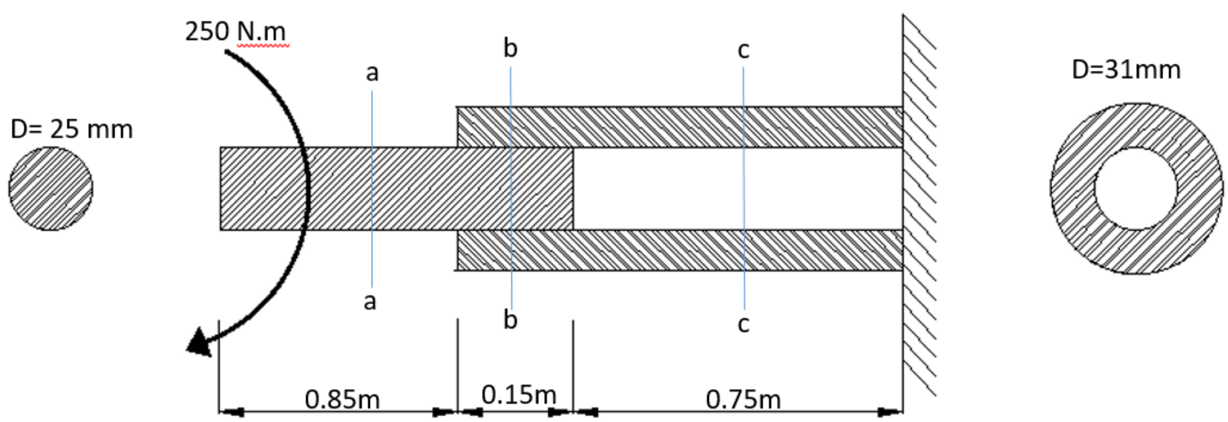
$$\therefore \tau = \frac{600 * 10^3 * \frac{25}{2}}{\frac{\pi}{32} * (25)^4} = 195.66 \text{ MPa}$$

$$\phi_{BC} = \frac{T * L}{G * J} = \frac{800 * 10^3 * 3 * 10^3}{80 * 10^3 * \frac{\pi}{32} * 25^4} = 0.782 \text{ rad}$$

$$\phi_{AB} = \frac{T * L}{G * J} = \frac{600 * 10^3 * 2 * 10^3}{80 * 10^3 * \frac{\pi}{32} * 25^4} = 0.391 \text{ rad}$$

$$\therefore \text{Total } \phi = \phi_{BC} + \phi_{AB} = 0.782 + 0.391 = 1.173 \text{ rad}$$

Ex; for the shaft shown in figure determine the greatest shear stress and the angular rotation of its end when the torque applied is 250 N.m , let G=79.6 GPa



Sol:-

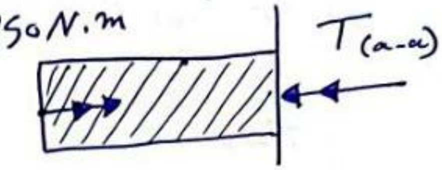
1) section (a-a) (solid)

$\Sigma T = 0$

$0 = 250 - T_{(a+a)}$

$0 = 250 - T_{(a-a)}$

$\therefore T_{(a-a)} = 250 \ N.m$



$J_{(solid)} = \frac{\pi}{32} * (D)^4$

$$\therefore \tau = \frac{T * \frac{D}{2}}{J_{(solid)}}$$

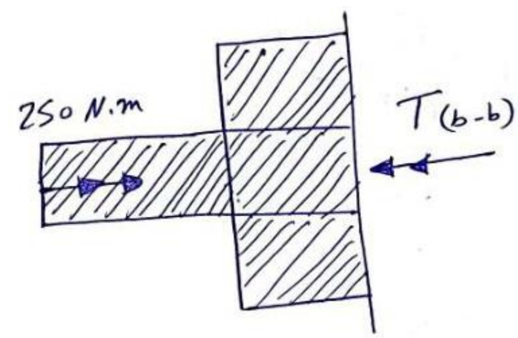
$$= \frac{250 * 10^3 * \frac{25}{2}}{\frac{\pi}{32} * (25)^4} = 81.52 \ MPa$$

2) section (b-b) (solid)

$\Sigma T = 0$

$0 = 250 - T_{(b-b)}$

$T_{(b-b)} = 250 \ N.m$



$J_{(solid)} = \frac{\pi}{32} * (31)^4$

$$\therefore \tau = \frac{T * \frac{31}{2}}{\frac{\pi}{32} * (31)^4} = \frac{250 * 10^3 * \frac{31}{2}}{\frac{\pi}{32} * (31)^4}$$

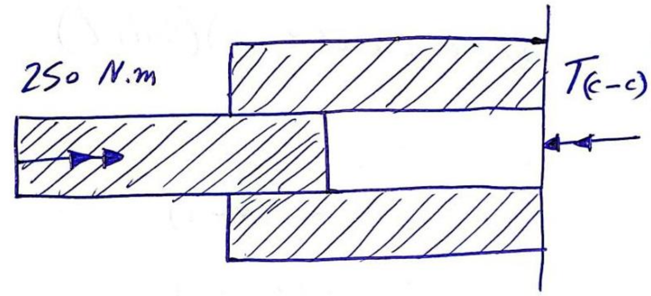
$$= 42.76 \ MPa$$

3) section (c-c) (hollow)

$\Sigma T = 0$

$0 = 250 - T_{(c-c)}$

$\therefore T_{(c-c)} = 250 \ N.m$



$J_{(hollow)} = \frac{\pi}{32} * (31^4 - 25^4)$

$$\therefore \tau = \frac{250 * 10^3 * \frac{31}{2}}{\frac{\pi}{32} * (31^4 - 25^4)} = 74.1 \ MPa$$

- Rotation

$$\phi_1 = \frac{T_{(a-a)} * L}{G * J}$$

$$= \frac{250 * 10^3 * 0{,}85 * 10^3}{79{,}6 * 10^3 * \frac{\pi}{32} * 25^4} = 0{,}0696 \text{ rad}$$

$$\phi_2 = \frac{T_{(b-b)} * L}{G * J}$$

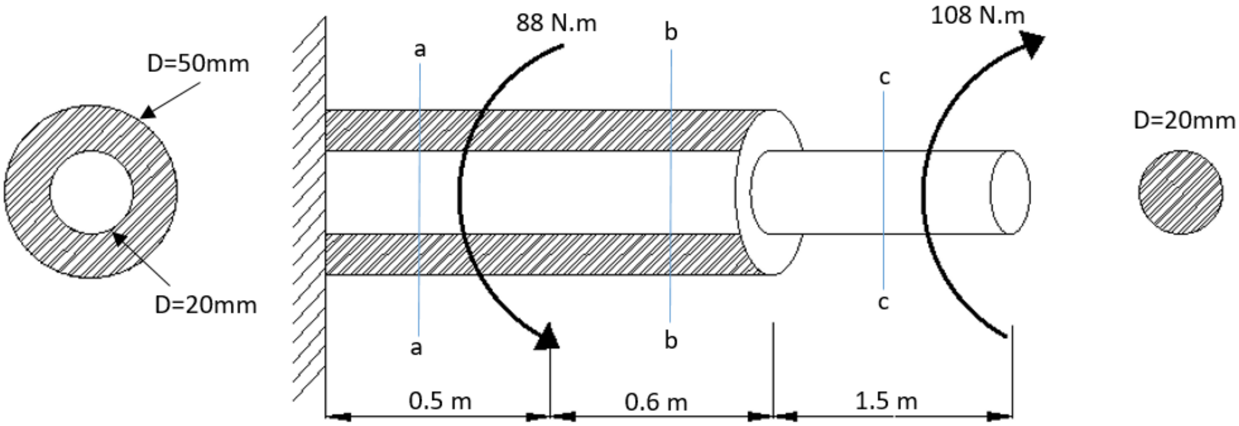
$$= \frac{250 * 10^3 * 0{,}15 * 10^3}{79{,}6 * 10^3 * \frac{\pi}{32} * 31^4} = 0{,}00519 \text{ rad}$$

$$\phi_3 = \frac{T_{(c-c)} * L}{G * J}$$

$$= \frac{250 * 10^3 * 0{,}75 * 10^3}{79{,}6 * 10^3 * \frac{\pi}{32} * (31^4 - 25^4)} = 0{,}045 \text{ rad}$$

$$\therefore \text{Total } \phi = 0{,}0696 + 0{,}00519 + 0{,}045 = 0{,}119 \text{ rad}$$

Ex; a shaft shown in figure have two segments one of them is a solid section of diameter 20mm and the second segment of a hollow section with inner diameter of 20mm and outer diameter 50mm. Determine the shearing stress (τ) in each segment and the angle of rotation of the free end. Use G=80 GPa



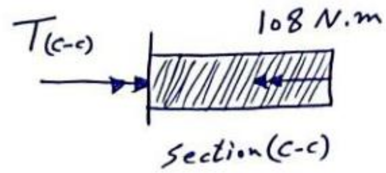
Sol.:-

1) Section (c-c)

$\Sigma T = 0$

$0 = -108 + T_{(c-c)}$

$\therefore T_{(c-c)} = 108 \ N.m$



$J_{(c-c)} = \frac{\pi}{32} * D^4 = \frac{\pi}{32} * (20)^4$

$= 15707 \ mm^4$

$\therefore \tau_{(a-a)} = \frac{T * \frac{D}{2}}{J}$

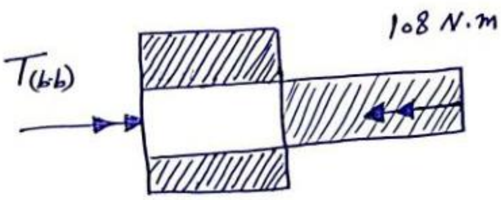
$= \frac{108 * 10^3 * \frac{20}{2}}{15707} = 68.75 \ MPa$

2) Section (b-b)

$\Sigma T = 0$

$0 = -108 + T_{(b-b)}$

$\therefore T_{(b-b)} = 108 \ N.m$



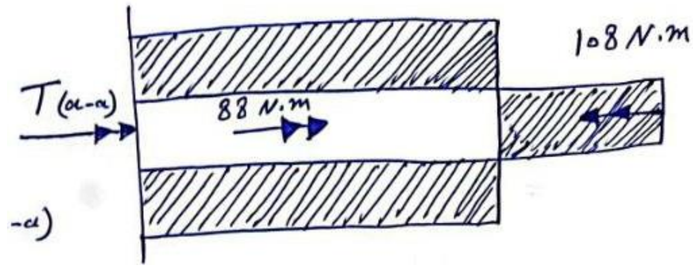
$J_{(b-b)} = \frac{\pi}{32} * (50^4 - 20^4)$

$= 597581.8 \ mm^4$

$$\therefore \tau_{(b-b)} = \frac{T * \frac{D}{2}}{J}$$

$$= \frac{108 * 10^3 * \frac{50}{2}}{597581.8} = 4.518 \text{ MPa}$$

3) section (a-a)



$$\sum T = 0$$

$$0 = -108 + 88 + T_{(a-a)}$$

$$\therefore T_{(a-a)} = 20 \text{ N.m}$$

$$\therefore \tau = \frac{T_{(a-a)} * \frac{D}{2}}{J}$$

$$= \frac{20 * 10^3 * \frac{50}{2}}{597581.8} = 0.83 \text{ MPa}$$

H.W

1- find the angle of rotation for the free end

2- if the torque (88 N.m) changed with torque equal to (150 N.m) in the same direction find (τ) and ($\emptyset$)

Power Transmitted by The Shaft

A shaft rotating with a constant angular velocity ω (in radians per second) is being acted by a twisting moment (T). The power transmitted by the shaft is

$$P = T\omega = 2\pi T f$$

where T is the torque in N·m, f is the number of revolutions per second, and P is the power in watts. one horse power is defined as almost 746 Watt in standard calculations. When the number of revolutions per minutes and P is the power in (hp) then the equation becomes

$$hp = \frac{T * 2 * \pi * n}{746 * 60}$$

Ex; a shaft of diameter (8.75 cm) makes (45) r.p.m . the maximum shearing stress is limited to 31 MN/m^2. determine the horse power that can be transmitted by this shaft.

Sol:-

$D = 8,75\ cm = 87,5\ mm$

$\tau_{max} = 31\ MN/m^2 = 31\ Mpa$

$n = 45\ r.p.m$

$$\therefore \tau = \frac{T * \frac{D}{2}}{J}$$

$$31 = \frac{T * \frac{87,5}{2}}{\frac{\pi}{32} * (87,5)^4}$$

$\therefore T = 4075637,2\ N.mm = 4075,63\ N.m$

$$hp = \frac{T * 2 * \pi * n}{746 * 60}$$

$$hp = \frac{4075,63 * 2\pi * 45}{746 * 60} = 25,73\ \frac{N.m}{s}$$

Ex; the inside diameter of hollow shaft is to be (3/4) of the outside diameter. The shaft transmits 36 hp at 1200 r.p.m with a maximum shear stress of 68948 kPa. Compute the inside and outside diameter

$$hp = \frac{T * 2\pi * n}{746 * 60}$$

$$36 = \frac{T * 2\pi * 1200}{746 * 60}$$

$T = 213,8\ N.m$

$$\tau = \frac{T * \frac{D_2}{2}}{J}$$

$$J = \frac{\pi}{32} * (D_2^4 - D_1^4)$$

$$J = \frac{\pi}{32} * \left(D_2^4 - \left(\frac{3}{4} D_2\right)^4\right) = \frac{\pi}{32} * \left(1 - \frac{81}{256}\right) D_2^4$$

$$\boxed{= 0{,}067 D_2^4}$$

$$\tau = \frac{213{,}8 * 10^3 * \frac{D_2}{2}}{0{,}067 D_2^4} \qquad 68{,}948 = \frac{213{,}8 * 10^3 * \cancel{D_2}}{0{,}067 D_2^{\cancel{4}\,3} * 2}$$

$$D_2^3 = 23140{,}95$$

$$\boxed{\therefore D_2 = 28{,}49 \text{ mm}} \qquad \boxed{\therefore D_1 = 21{,}37 \text{ mm}}$$