Vectors

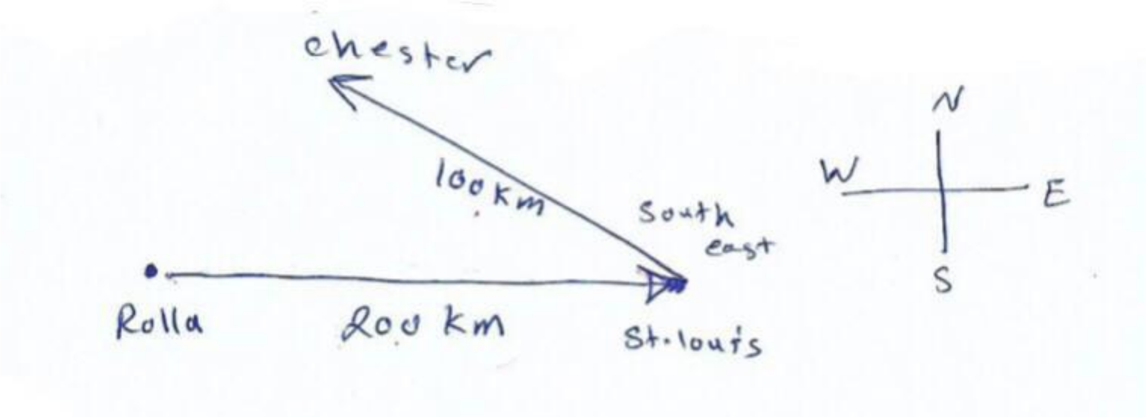
Scalars: a quantity that states only an amount.

For example, temperature is 5°C, 12 eggs.

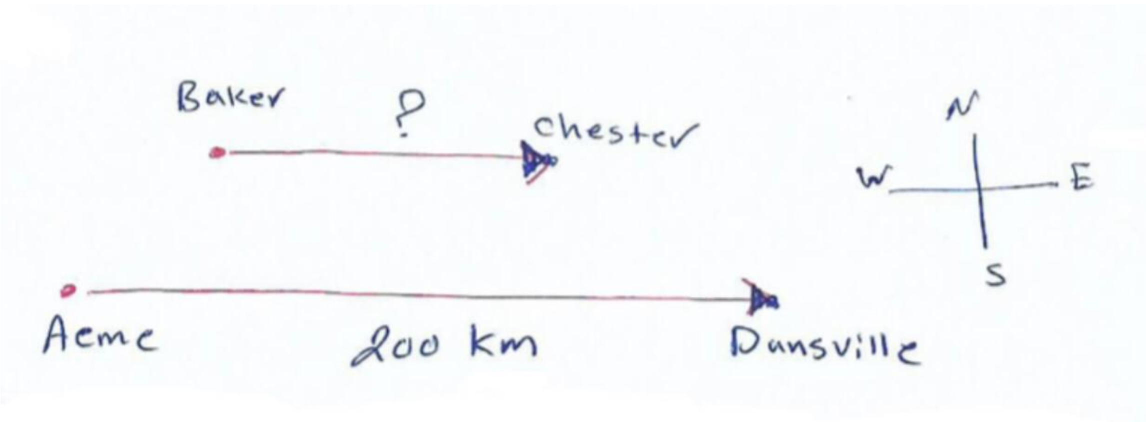
Vectors: a quantity specified by both magnitude and direction.

For example, vectors can be used to supply travelling instructions. If a pilot is told “fly 20 kilometers due south“, he is being given a displacement vector to follow, its magnitude is 20 kilometers and its direction is south.

Vectors: magnitude and direction. Vectors represented by arrows. Length of arrows is proportional to its magnitude.



<u>Example:</u>



If it is half as far from baker to chester as from aeme to dunsville. Describe the displacement vector from baker to chester?

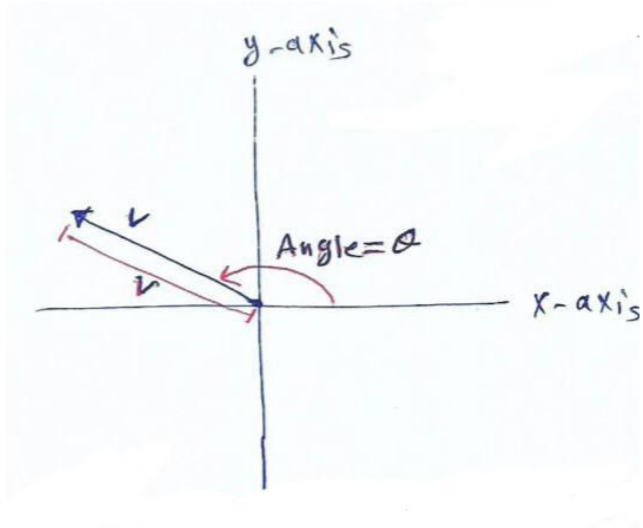
Sol.: displacement: 100 Km, east.

Polar notation: defining a vector by its angle and magnitude.

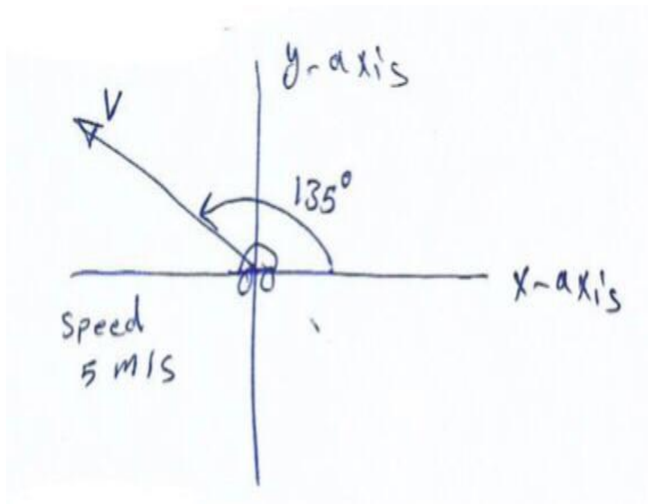
v: magnitude.

θ: vector.

Written as V = (v, θ)



Example: write the velocity vector of car in polar notation for the following figure.



Solution:

Since V = (v, θ)

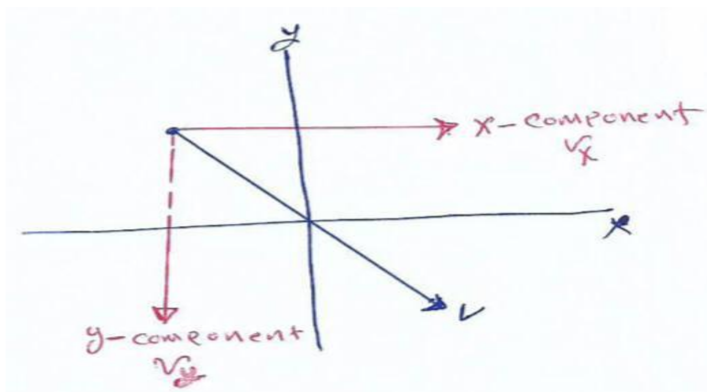
Hence V = (5m/s, 135°)

Rectangular notation:

v_x: horizontal component

v_y: vertical component

Written as V = (v_x,v_y)

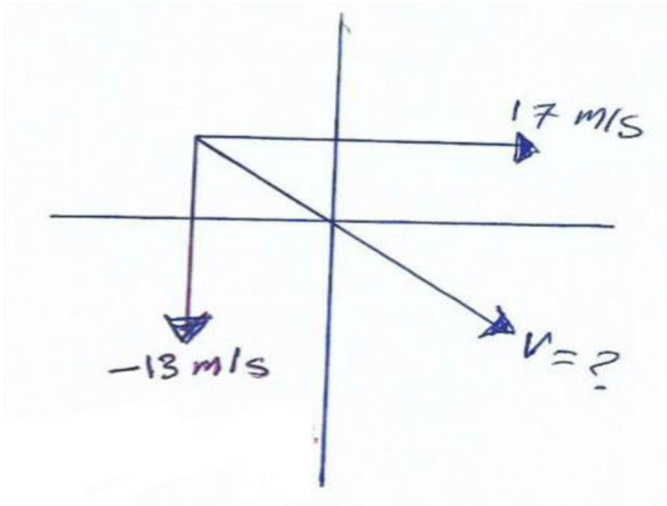


Example: what is the velocity vector in rectangular notation for the figure below?

Solution:

$V = (v_x, v_y)$

$V = (17,\ -13)$ m/s.



Adding and subtracting vectors by components:

- Add (or subtract) each component separately.

$A + B = (A_x + B_x\ ,\ A_y + B_y)$

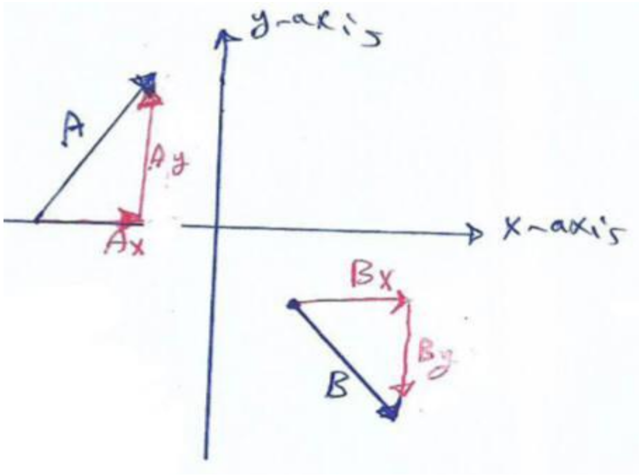
$A - B = (A_x - B_x\ ,\ A_y - B_y)$

Where:

A, B: vectors.

A_x, A_y: A components.

B_x, B_y: B components.



Example: the boat has the velocity A in still water. Calculate its velocity as the sum of A and the velocity B of the river's current?

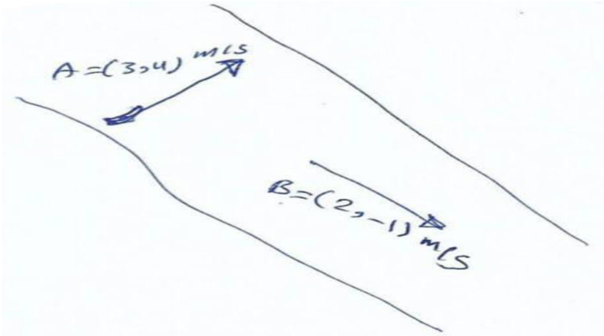
Solution:

$V = A + B$

$V = (3,4)$ m/s $+ (2,-1)$ m/s

$V = (3+2\ ,\ 4+(-1))$ m/s

$V = (5,3)$ m/s.



Multiplying rectangular vectors by a scalar:

- Multiply each component by scalar.
- Positive scalar does not affect direction.

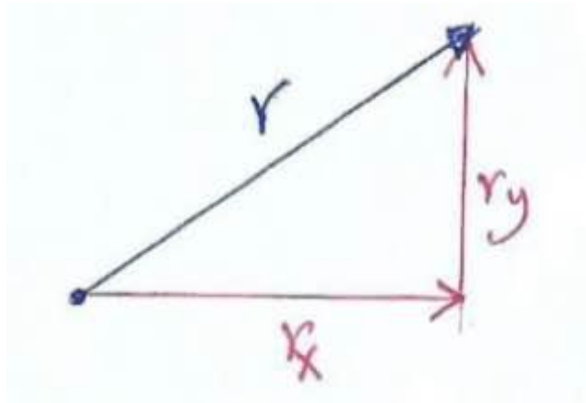
$Sr = (Sr_x , Sr_y)$

Where:

S: scalar.

r: vector.

r_x,r_y: r components.



Example: what is the displacement of the car after 5.0 seconds for the figure below?

Solution:

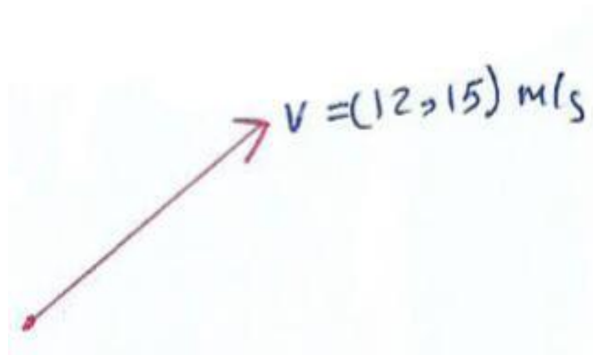
Since $V = \frac{\Delta X}{t}$

Hence $\Delta X = t.V$

$= (5.0\ s)(12,15)\ m/s$

$= (5.0)(12) , (5.0)(15)$

$= (60,75)\ m$



Multiplying polar vectors by a scalar:

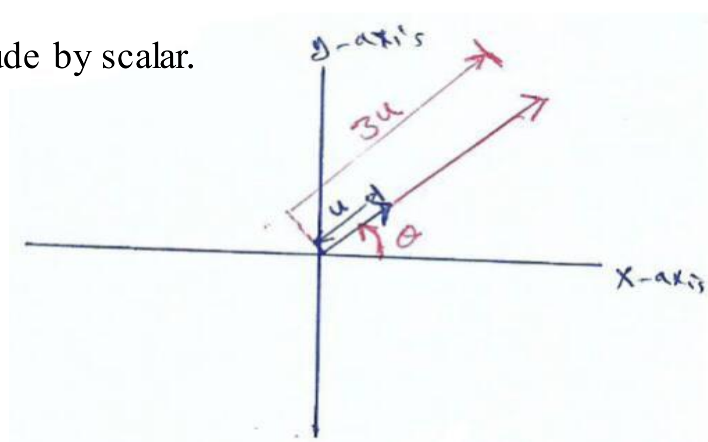
- Multiplying polar vector by positive scalar:

-Multiplying vector's magnitude by scalar.

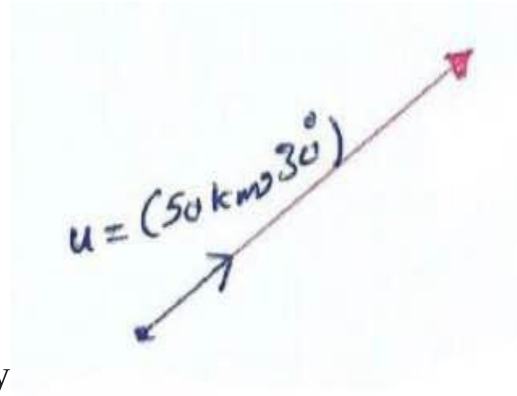
-Angle unchanged.

SV = S(v, θ)

SV = (Sv, θ), if S positive.



Example: what is the displacement vector if the car travels three times as far as the displacement in the figure below?



Solution:

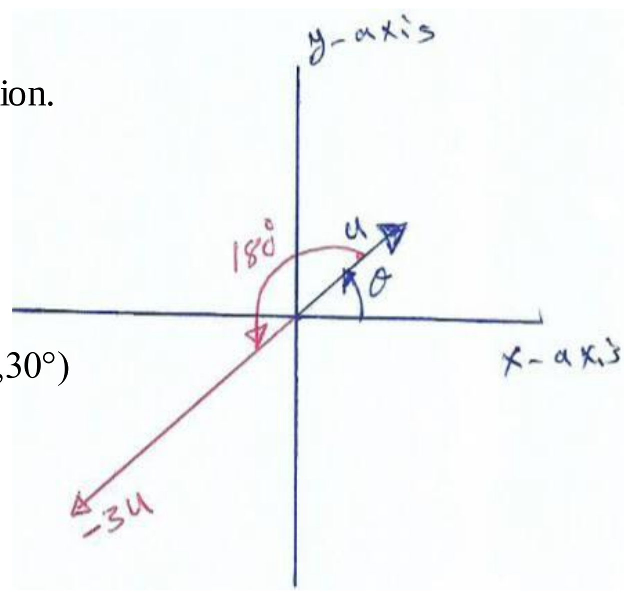
Since SV = (Sv,θ)

Hence 3u = (3*50,30°)

3u = (150 km,30°)

Notice that the direction still the same only the magnitude has changed.

- Multiplying polar vector by negative scalar:



Use absolute value and reverse direction.

SV = S(v, θ)

SV = (|S| v, θ+180), if S negative.

Example: what is -3u if u equals (3m,30°)

Solution:

Since SV = S(v, θ)

SV = (|S| v, θ+180)

Hence -3u = (|-3|*3,30+180)

-3u = (9m,210°)

Converting vectors from polar to rectangular notation:

$(r, \theta) \xrightarrow{\text{convert}} (r_x, r_y)$

$r_x = r\cos\theta$

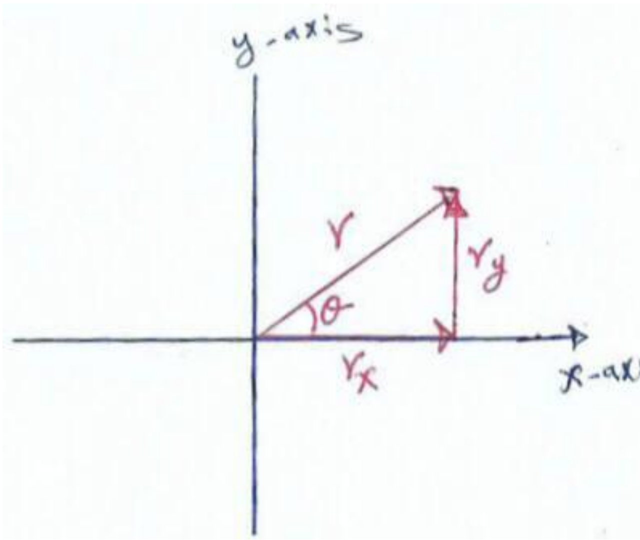
$r_y = r\sin\theta$

where

r: magnitude.

θ: angle.

$r_x + r_y$:components of vector.



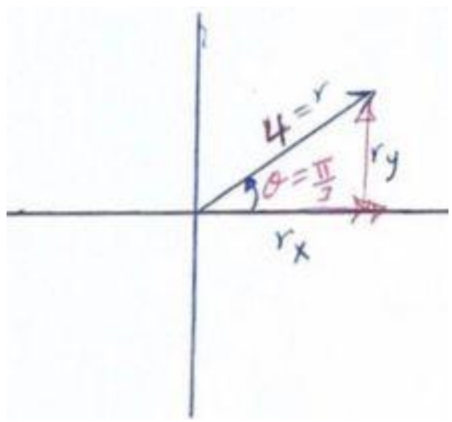
Example: convert the following polar coordinates into rectangular coordinates, $(4, \frac{\pi}{3})$?

Solution:

$r_x = r\cos\theta = 4\cos\frac{\pi}{3} = 4(\frac{1}{2}) = 2$

$r_y = r\sin\theta = 4\sin\frac{\pi}{3} = 4(\frac{\sqrt{3}}{2}) = 2\sqrt{3}$

hence $(r_x, r_y) = (2, 2\sqrt{3})$

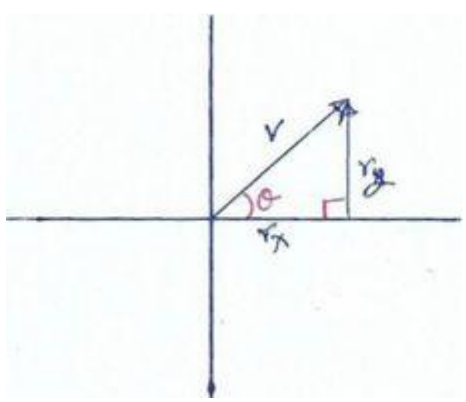


Converting vectors from rectangular to polar notation:

$(r_x, r_y) \xrightarrow{\text{convert}} (r, \theta)$

$r = \sqrt{(r_x^2 + r_y^2)}$

$\theta = \tan^{-1}(\frac{ry}{rx})$



<u>Example:</u> convert the following rectangular coordinates to polar coordinates, (2,2)?

Solution:

$(r_x,r_y) = (2,2)$

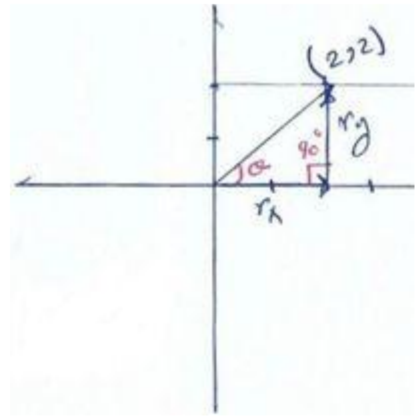
$r = \sqrt{(r_x^2+r_y^2)}$

$r = \sqrt{(2^2+2^2)}$

$r = \sqrt{8} = 2.83$

$\theta = \tan^{-1}(\frac{ry}{rx}) = \tan^{-1}(\frac{2}{2}) = \tan^{-1}(1) = 45^\circ$

hence (2,2) = (2.83,45°)



<u>Example:</u> for the figure below, what is the car's displacement (r) in polar notation?

Solution:

$r = \sqrt{(r_x^2+r_y^2)}$

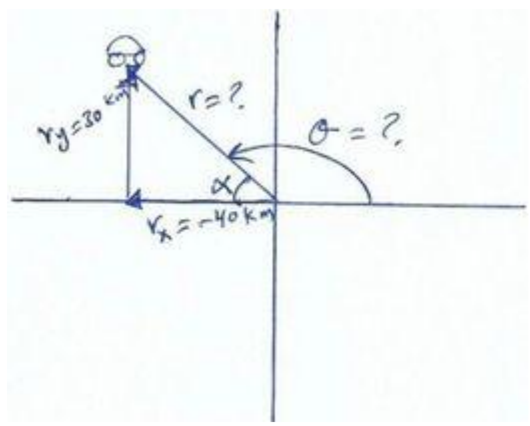
$r = \sqrt{(-40^2+30^2)} = \sqrt{(2500)} = 50Km$

$\alpha = \tan^{-1}(\frac{ry}{rx}) = \tan^{-1}(\frac{30}{-40}) = -36.9^\circ$

$\theta = -36.9^\circ+180 = 143^\circ$

hence

$(-40,30) \longrightarrow (50Km,143^\circ) = r(r, \theta)$



Example: you are told to drive 3.5 Km at 42.0° then drive as directed by a vector of (4,-3) Km. what is your resulting displacement in rectangular coordinates? In polar notation?

Solution:

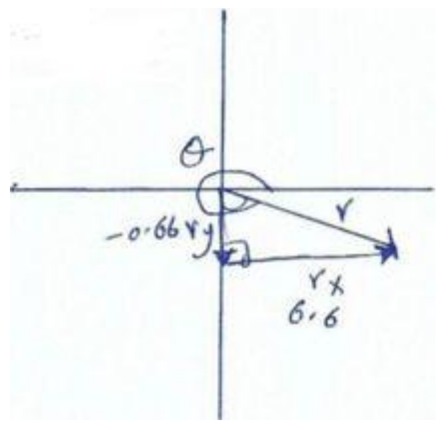
First we need to convert (3.5,42°) to rectangular coordinates

$A_x = A\cos\theta$

$A_x = 3.5\cos 42° = 2.6$ Km

$A_y = A\sin\theta$

$A_y = 3.5\sin 42° = 2.34$ Km



Hence

$A = (A_x, A_y) = (2.6, 2.34)$ Km

Since

$B = (B_x, B_y) = (4,-3)$ Km

$C = A+B = (A_x+B_x\ ,\ A_y+B_y)$

$= (2.6+4\ ,\ 2.34+(-3)) = (6.6,-0.66)$Km

Now we need to convert rectangular coordinates (6.6,0.66)Km into polar coordinates

$r = \sqrt{(r_x^2+r_y^2)}$

$r = \sqrt{(6.6^2+(-0.66)^2)} = \sqrt{(43.446)} = 6.6$

$\alpha = \tan^{-1}\left(\frac{ry}{rx}\right) = \tan^{-1}\left(\frac{-0.66}{6.6}\right)$

$\alpha = -5.71°$

$\theta = \alpha + 360 = -5.71+360 = 354.3°$

$C = (r,\ \theta) = (6.6\text{ Km}, 354.3°)$