Chapter Three: Columns or Compression Members

AISC manual / Chapter B & E

3.1 General

Compression members (Columns) are structural elements that subjected to compression forces. These forces applied along longitudinal axis through centered of the cross section.

The axial stress should be calculated using the following expression:

$$\boldsymbol{F = \frac{P}{A_g}}$$

Where:

P: Applied compression force, kips

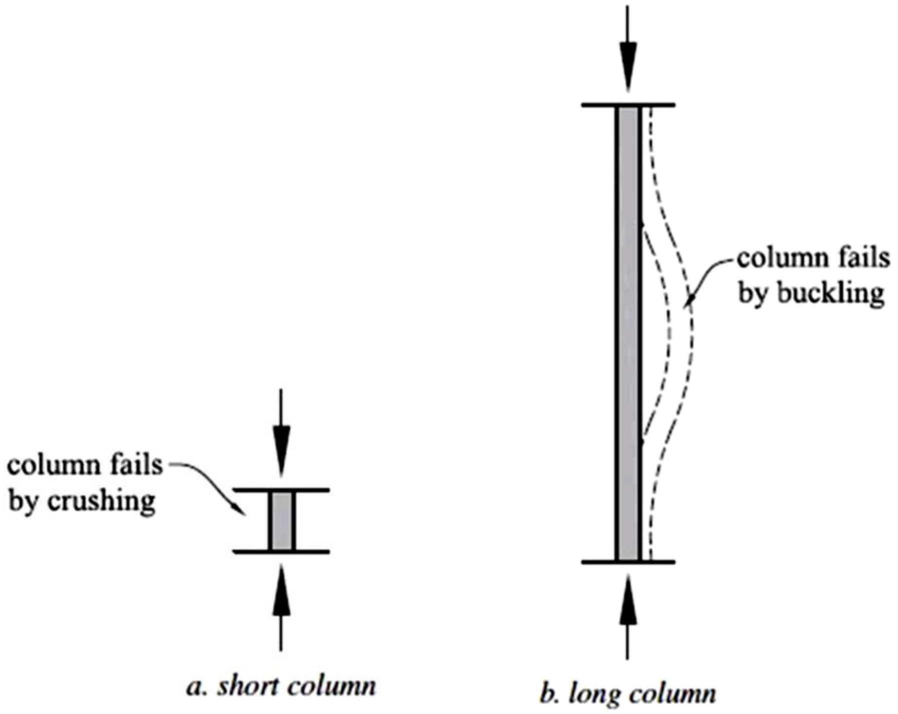
A_g: the gross sectional area, in^2

3.2 Type of columns:

a) **Short column** failed when the stress exceeds yielding stress is called **Crashing failure**.

b) **Long column** which failed by buckling before yielding is called **Buckling failure**.

c) **Intermediate columns** which failed yielding and buckling at the same time is called **Crashing-Buckling failure**.



Buckling is a slight bent around one or more side of column. In our current study the buckling is around $\boldsymbol{x}$ or $\boldsymbol{y} - \mathbf{axes}$ of cross section.

3.3 Euler Buckling of Columns:

For a pure compression member, the axial load at which the column begins to bent outward is called the Euler Critical Buckling Load. The Euler critical buckling load for an elastic column with pinned ends is:

$$\boldsymbol{P_e} = \frac{\boldsymbol{n^2 \pi^2 EI}}{\boldsymbol{L^2}} \ldots\ldots\ldots\ldots\ldots (\mathbf{1})$$

Where:

$\boldsymbol{P_e}$: Elastic critical buckling load, Kips,

E: Modulus of elasticity, 29000 ksi,

I: Moment of inertia, in^4, and

L: Length of the column, in.

It is useful to express load in terms of stresses in steel design then:

$$\therefore F_e = \frac{P_e}{A} = \frac{n^2\pi^2 EI}{L^2 \cdot A} \qquad but \quad r = \sqrt{\frac{I}{A}} \qquad \rightarrow \qquad F_e = \frac{n^2\pi^2 E\, r^2}{L^2}$$

$$\therefore \boldsymbol{F_e} = \frac{\boldsymbol{n^2\pi^2 E}}{\left(\frac{\boldsymbol{L}}{\boldsymbol{r}}\right)^2} \ldots\ldots\ldots\ldots\ldots (\mathbf{2})$$

Where:

$\boldsymbol{F_e}$: Elastic critical buckling stress, ksi,

$\boldsymbol{L/r}$: The slenderness ratio.

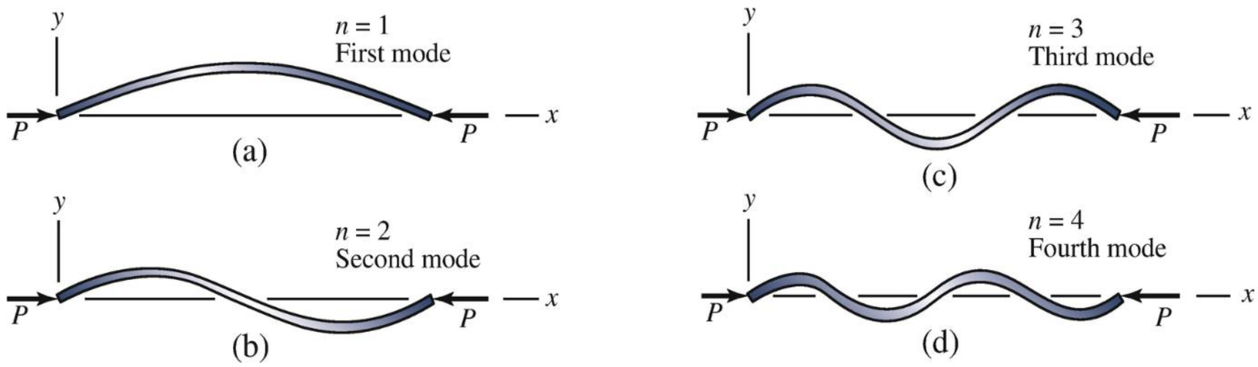
Here $\boldsymbol{n}$ is an positive number (n= 1, 2, 3, ……) and it is called **the mode shape number**:

- $n = 1$ first mode

$$F_e = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$

- $n = 2$ second mode

$$F_e = \frac{4\pi^2 E}{\left(\frac{L}{r}\right)^2}$$



3.4 Effect of Boundary Conditions

The Euler equations above assumed that the ends of the column are pinned. For other end conditions, effective length factor, ***K***, is applied to the column length. The effective length of a column is defined as ***KL***, where K is the effective length factor as defined by AISC manual in Table C-C2.2.

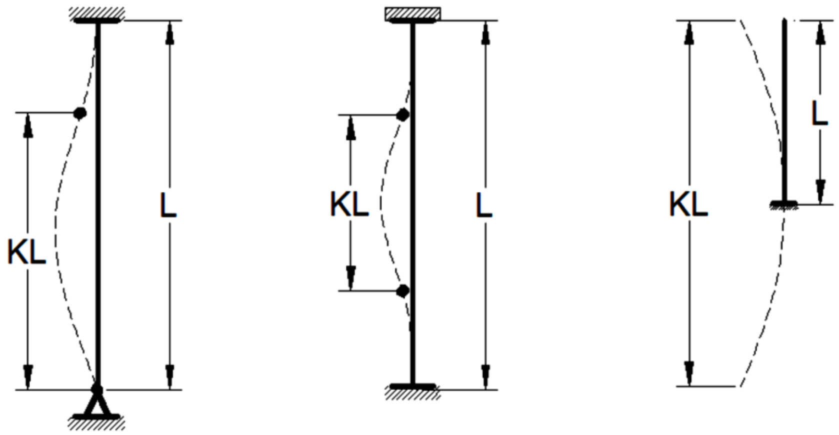
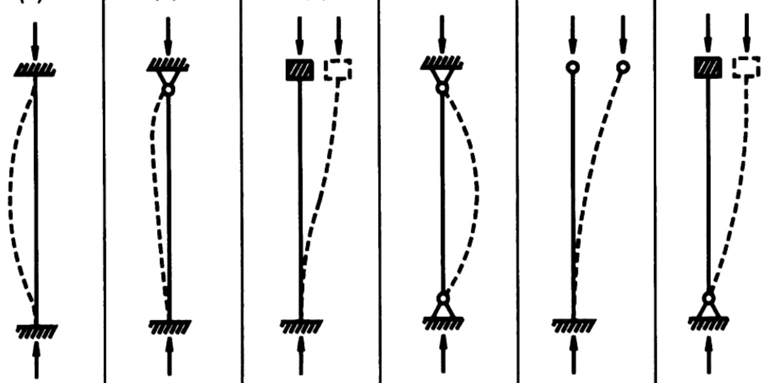


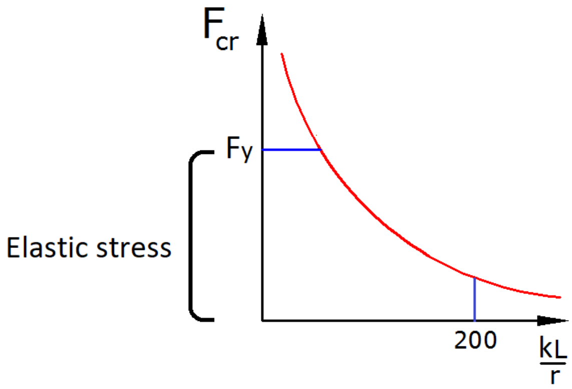
TABLE C-C2.2
Approximate Values of Effective Length Factor, *K*

Buckled shape of column is shown by dashed line.	(a)	(b)	(c)	(d)	(e)	(f)
						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.10	2.0
End condition code	 *Rotation fixed and translation fixed* *Rotation free and translation fixed* *Rotation fixed and translation free* *Rotation free and translation free*					

When the column end conditions are other than pinned, equations (1) and (2) are modified as follows:

$$\boldsymbol{P_e} = \frac{\boldsymbol{\pi^2 EI}}{(\boldsymbol{KL})^2} \quad \dots\dots\dots\dots (3)$$

$$\boldsymbol{F_e} = \frac{\boldsymbol{\pi^2 E}}{\left(\frac{\boldsymbol{KL}}{\boldsymbol{r}}\right)^2} \quad \dots\dots\dots\dots\dots (4)$$



- Equation for $\boldsymbol{P_e}$ is valid only when the material of column in the elastic region. If the material goes inelastic then this Equation becomes useless and cannot be used. While $\boldsymbol{F_e}$ is the critical buckling stress (elastic or inelastic).

- The term $\boldsymbol{KL/r}$ is called the slenderness ratio, and the AISC Specification recommends limiting the column slenderness ratio such that:

$$\frac{\boldsymbol{KL}}{\boldsymbol{r}} \leq \mathbf{200}$$

- The columns are classified as long, short and intermediate columns according to slenderness ratio $\boldsymbol{KL/r}$:

1) Long Columns:

$$\frac{\boldsymbol{KL}}{\boldsymbol{r}} \geq \mathbf{150}$$

2) Short Columns:

$$\frac{\boldsymbol{KL}}{\boldsymbol{r}} \leq \mathbf{40}$$

3) Intermediate Columns:

$$\mathbf{40} < \frac{\boldsymbol{KL}}{\boldsymbol{r}} < \mathbf{150}$$

3.5 AISC Specifications for Compression Members

The **LRFD** design strength (ultimate strength) and **ASD** allowable strength of a column may be determined as follows:

$$\boldsymbol{Pn = F_{cr}\,A_g}$$

ASD Compression Strength $\boldsymbol{\Omega} = 1.67$	LRFD Compression Strength $\emptyset = 0.9$
$P_a = \frac{P_n}{\Omega} = \frac{F_{cr}\,A_g}{\Omega} = 0.6\,F_{cr}\,A_g$	$P_u = \emptyset P_n = \emptyset\,F_{cr}\,A_g = 0.9\,F_{cr}\,A_g$

The flexural buckling stress, $(\boldsymbol{F_{cr}})$ is depends on slenderness ratio $\left(\boldsymbol{\frac{KL}{r}}\right)$ and determined as follows:

a) If $\frac{KL}{r} \leq 4.71\sqrt{\frac{E}{Fy}}$ $\;or\;\; F_e \geq 0.44\,Fy,$ then:

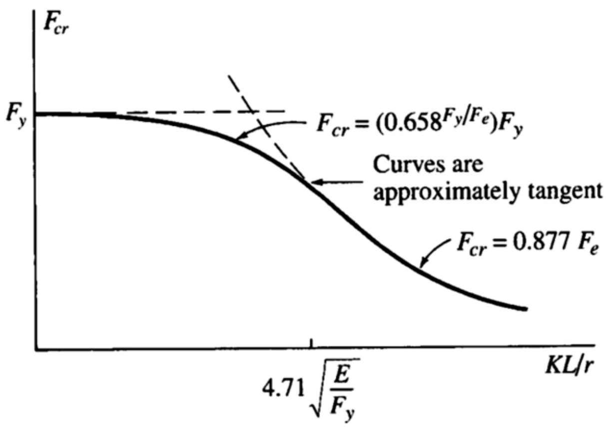
$$\boldsymbol{F_{cr} = \left[0.658^{\frac{Fy}{F_e}}\right] Fy} \;\ldots\ldots\ldots\ldots \text{AISC Equation E3} - 2$$

b) If $\frac{KL}{r} > 4.71\sqrt{\frac{E}{Fy}}$ $\;or\;\; F_e < 0.44\,Fy,$ then:

$$\boldsymbol{F_{cr} = 0.877\,F_e} \;\ldots\ldots\ldots\ldots \text{AISC Equation E3} - 3$$

Where:

$$\boldsymbol{F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}} \;\ldots\ldots\ldots\ldots\; \text{AISC Equation E3} - 4$$



Example No. 1: Compute the design (ultimate) strength $\boldsymbol{Pu}$ can be subjected to a $\boldsymbol{W12 \times 96}$ column which has buckling length $\boldsymbol{KL = 15\,ft}$ and made from $\boldsymbol{A36}$ steel.

Solve:

Steel and Section Properties:

$Fy = 36\ ksi, \quad Fu = 58\ ksi$

From section $W12 \times 96$: ($A_g = 28.2\ in^2, r_x = 5.44\ in\ , r_y = 3.09\ in$)

$r_{min} = r_y = 3.09\ in$

To find design strength:

$$\frac{KL}{r} = \frac{15 \times 12}{3.09} = 58.25 < 4.71\sqrt{\frac{E}{Fy}} = 4.71\sqrt{\frac{29000}{36}} = 133.68$$

then:

$$F_{cr} = \left[0.658^{\frac{Fy}{F_e}}\right] Fy$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000}{(58.25)^2} = 84.27\ ksi$$

$$F_{cr} = \left[0.658^{\frac{36}{84.27}}\right] \times 36 = 30.11\ ksi$$

$$P_u = \emptyset P_n = \emptyset\ F_{cr}\ A_g = 0.9 \times 30.11 \times 28.2 = 764.19\ kips$$

Example No. 2: if the previous column in **Ex1** has bracing condition in $x - axis$ as pin ended, while in $y - axis$ as Fixed-Fixed ends. What will be it compressive strength. Use $\boldsymbol{L = 15\,ft}$ and $\boldsymbol{Fy = 36\,ksi}$.

Solve:

$$\left(\frac{KL}{r}\right)_x = \frac{1 \times 15 \times 12}{5.44} = 33.09, \qquad \left(\frac{KL}{r}\right)_y = \frac{0.65 \times 15 \times 12}{3.09} = \mathbf{37.86}$$

$$\frac{KL}{r} = \mathbf{37.86} < 4.71\sqrt{\frac{E}{Fy}} = 133.68$$

then:

$$F_{cr} = \left[0.658^{\frac{Fy}{F_e}}\right] Fy$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000}{(37.86)^2} = 199.68\ ksi$$

$$F_{cr} = \left[0.658^{\frac{36}{199.68}}\right] \times 36 = 33.38\ ksi$$

$$P_u = \emptyset\ F_{cr}\ A_g = 0.9 \times 33.38 \times 28.2 = 847\ kips$$

Tables for Compression Members

Table 4-22 in Part 4 used for evaluating both Fcr/Ω_c for the (ASD) method and $\phi_c Fcr$ for the (LRFD) method. Depending on the value of maximum slenderness ratio (kL/r).

Table 4–22
Available Critical Stress for Compression Members

$F_y = 35$ksi			$F_y = 36$ksi			$F_y = 42$ksi			$F_y = 46$ksi			$F_y = 50$ksi		
Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD	Kl/r	F_{cr}/Ω_c ksi ASD	$\phi_c F_{cr}$ ksi LRFD
1	**21.0**	31.5	1	**21.6**	32.4	1	**25.1**	37.8	1	**27.5**	41.4	1	**29.9**	45.0
2	**21.0**	31.5	2	**21.6**	32.4	2	**25.1**	37.8	2	**27.5**	41.4	2	**29.9**	45.0
3	**20.9**	31.5	3	**21.5**	32.4	3	**25.1**	37.8	3	**27.5**	41.4	3	**29.9**	45.0
4	**20.9**	31.5	4	**21.5**	32.4	4	**25.1**	37.8	4	**27.5**	41.4	4	**29.9**	44.9
5	**20.9**	31.5	5	**21.5**	32.4	5	**25.1**	37.7	5	**27.5**	41.3	5	**29.9**	44.9
6	**20.9**	31.4	6	**21.5**	32.3	6	**25.1**	37.7	6	**27.5**	41.3	6	**29.9**	44.9
7	**20.9**	31.4	7	**21.5**	32.3	7	**25.1**	37.7	7	**27.5**	41.3	7	**29.8**	44.8
8	**20.9**	31.4	8	**21.5**	32.3	8	**25.1**	37.7	8	**27.4**	41.2	8	**29.8**	44.8
9	**20.9**	31.4	9	**21.5**	32.3	9	**25.0**	37.6	9	**27.4**	41.2	9	**29.8**	44.7
10	**20.9**	31.3	10	**21.4**	32.2	10	**25.0**	37.6	10	**27.4**	41.1	10	**29.7**	44.7
11	**20.8**	31.3	11	**21.4**	32.2	11	**25.0**	37.5	11	**27.3**	41.1	11	**29.7**	44.6
12	**20.8**	31.3	12	**21.4**	32.2	12	**24.9**	37.5	12	**27.3**	41.0	12	**29.6**	44.5
13	**20.8**	31.2	13	**21.4**	32.1	13	**24.9**	37.4	13	**27.2**	40.9	13	**29.6**	44.4
14	**20.7**	31.2	14	**21.3**	32.1	14	**24.8**	37.3	14	**27.2**	40.9	14	**29.5**	44.4
15	**20.7**	31.1	15	**21.3**	32.0	15	**24.8**	37.3	15	**27.1**	40.8	15	**29.5**	44.3

Example No. 1: Compute the design strength $\boldsymbol{Pu}$ can be subjected to a $\boldsymbol{W12 \times 96}$ column which has buckling length $\boldsymbol{KL = 15\ ft}$ and made from $\boldsymbol{A36}$ steel.

Solve:

Steel and Section Properties:

$Fy = 36\ ksi, \quad Fu = 58\ ksi$

From section $W12 \times 96$: ($A_g = 28.2\ in^2, r_x = 5.44\ in\ , r_y = 3.09\ in$)

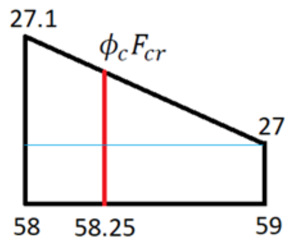
$r_{min} = r_y = 3.09\ in$

$\frac{kL}{r}$	$\phi_c F_{cr}$
58	27.1
59	27.0

To find design strength:

$$\frac{kL}{r} = \frac{15 \times 12}{3.09} = 58.25, \quad fy = 36\ ksi$$

$$\frac{27.1 - 27}{59 - 58} = \frac{\phi_c Fcr - 27}{59 - 58.25}$$



$$\phi_c Fcr = 0.075 + 27 = 27.075\ ksi$$

$$P_u = \emptyset P_n = \emptyset\ F_{cr}\ A_g = 27.075 \times 28.2 = 763.52\ kips$$

Example No. 2: A $\boldsymbol{W12 \times 96}$ column which has bracing condition in $x - axis$ as pin ended, while in $y - axis$ as Fixed-Fixed ends. What will be ultimate compressive strength. Use $\boldsymbol{L = 15\ ft}$ and $\boldsymbol{Fy = 36\ ksi}$.

Solve:

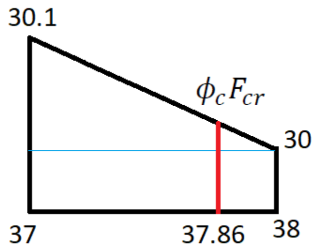
$$\left(\frac{KL}{r}\right)_x = \frac{1 \times 15 \times 12}{5.44} = 33.09$$

$$\left(\frac{KL}{r}\right)_y = \frac{0.65 \times 15 \times 12}{3.09} = \mathbf{37.86}$$

$\frac{kL}{r}$	$\phi_c F_{cr}$
37	30.1
38	30.0

$$\frac{kL}{r} = 37.86, \quad fy = 36\ ksi$$

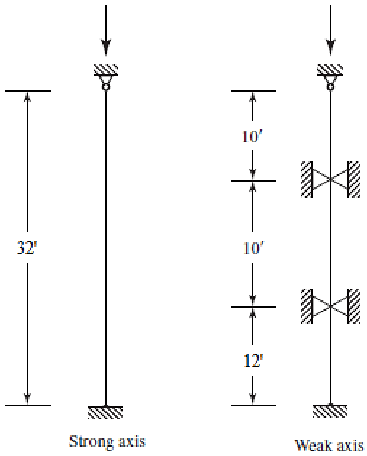
$$\frac{30.1 - 30}{38 - 37} = \frac{\phi_c Fcr - 30}{38 - 37.86}$$



$$\phi_c Fcr = 0.014 + 30 = 30.014\ ksi$$

$$P_u = \emptyset P_n = \emptyset\ F_{cr}\ A_g = 30.014 \times 28.2 = 846.395\ kips$$

Example No.3: Determine the **LRFD** design strength and the **ASD** allowable design strength for the $\boldsymbol{Fy} = \mathbf{50}\ \boldsymbol{ksi}$ axially loaded $\boldsymbol{W}\mathbf{14} \times \mathbf{90}$ shown in the Figure. (Note: solve by using Equations Method and Tables of Part 4 of the **AISC**).



Solve:

Steel and Section Properties:

$Fy = 50\ ksi$

From section $W14 \times 90$: ($A_g = 26.5\ in^2, r_x = 6.14\ in\ , r_y = 3.7\ in$)

To find design strength:

$(KL)_y = 1.0 \times 10 = 10',\ \ or \quad (KL)_y = 0.8 \times 12 = 9.6'$

$(KL)_x = 0.8 \times 32 = 25.6'$

$\left(\frac{KL}{r}\right)_x = \frac{25.6 \times 12}{6.14} = \mathbf{50.03}, \qquad \left(\frac{KL}{r}\right)_y = \frac{10 \times 12}{3.7} = 32.43$

By using Equations Method:

$$\frac{KL}{r} = \mathbf{50.03} < 4.71\sqrt{\frac{E}{Fy}} = 4.71\sqrt{\frac{29000}{50}} = 113.43$$

then:

$$F_{cr} = \left[0.658^{\frac{Fy}{F_e}}\right] Fy$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 \times 29000}{(50.03)^2} = 114.35\ ksi$$

$$F_{cr} = \left[0.658^{\frac{50}{114.35}}\right] \times 50 = 41.64\ ksi$$

$$P_u = \emptyset P_n = \emptyset\ F_{cr}\ A_g = 0.9 \times 41.64 \times 26.5 = 993.114\ kips$$

$$P_a = F_{cr}/\Omega_c\ A_g = 0.6\ F_{cr}\ A_g = 0.6 \times 41.64 \times 26.5 = 662.08\ kips$$

By using Table Method:

$\frac{KL}{r} = \mathbf{50.03} \approx 50, \quad Fy = 50\ ksi$

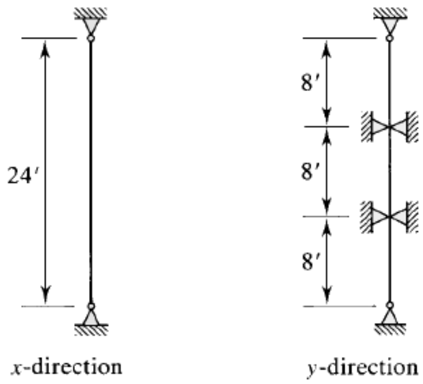
$\phi_c Fcr = 37.5\ ksi \ \ (LRFD), \qquad Fcr/\Omega_c = 24.9\ ksi \ \ (ASD)$

$P_u = \emptyset P_n = \emptyset\ F_{cr}\ A_g = 37.5 \times 26.5 = 993.75\ kips$

$P_a = P_n/\Omega_c = F_{cr}/\Omega_c\ A_g = 24.9 \times 26.5 = 659.85\ kips$

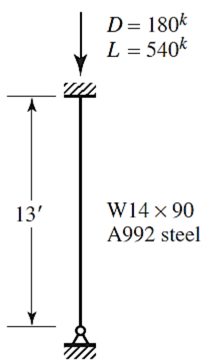
H.W: Solve by Equation and Table Method

1. A $\boldsymbol{W12 \times 58}$ is pinned at both ends and braced in the weak direction at the third points, as shown in Figure. **A992** steel and $\boldsymbol{L = 24\ ft}$ are used. Determine the available compressive strength.



Ans: $LRFD: Pu = 616\ kips, \quad ASD: Pa = 410\ kips$

2. Does the column shown in Figure have enough available strength to support the given service loads? use: **a.** Use LRFD. **b.** Use ASD.



Ans: $LRFD: Pu = 1080\ kips < 1100\ kips\ (OK), \quad ASD: Pa = 720 < 730\ kips\ (OK)$