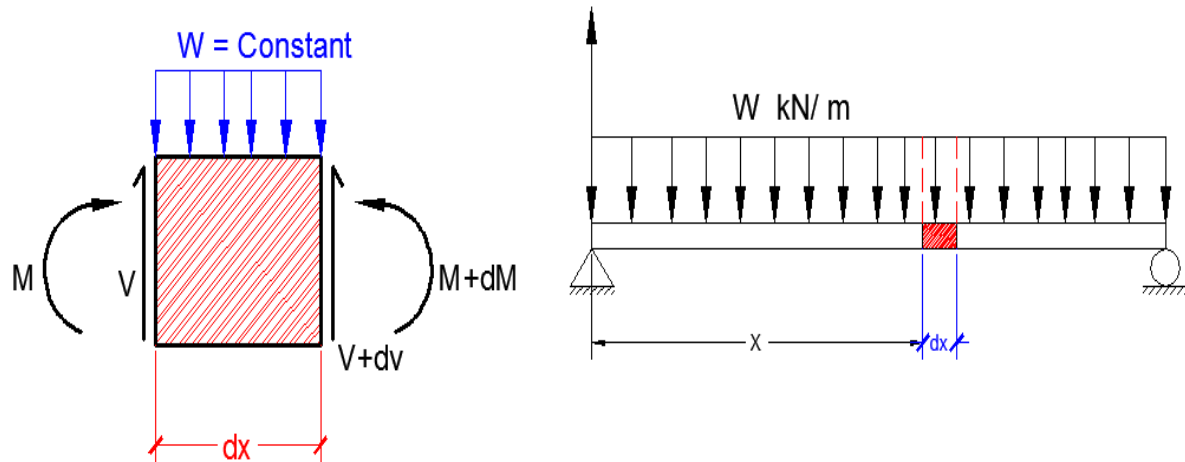## Shear force, Bending Moment and Axial Force Diagrams.

For the strip of width (dx) infinitesimal element

$$\sum f_y = 0$$

V + dv = v + w *d$_x$

W = $\dfrac{d_v}{d_x}$ ------------- 1

Slope of shearing force diagram of any section equal to the intensiting the load at that section.

By integration:-

$$\int_{v_1}^{v_2} dv = \int_{x_1}^{x_2} w(x).\,dx$$

$V]_{v_1}^{v_2} = \int_{x_1}^{x_2} w(x).\,dx$

V$_2$- V$_1$ = $\int_{x_1}^{x_2} w(x).\,dx$

$\Delta V = \int_{x_1}^{x_2} w(x).\,dx$ -------------2

Changing in shearing force between x$_1$&x$_2$ section equals the total applied load between x$_1$ &x$_2$.

Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College

$$\sum M_O = 0$$

$$M + dM = M + Vdx + \frac{W(x)\,dx^2}{2}$$

$\dfrac{W(x)\,dx^2}{2}$, neglect because is small magnitude

dM = v . dx

$$Vx = \frac{dM}{dX} \text{ ---------------- } 3$$

Slope of bending moment diagram at any section equals the shear force at that section.

By integration:-

$$\int_{M_1}^{M_2} dM = \int_{x_1}^{x_2} V(x).\,dx$$

$$M]_{M_1}^{M_2} = \int_{x_1}^{x_2} V(x).\,dx$$

$$M_2 - M_1 = \int_{x_1}^{x_2} V(x).\,dx$$

$$\Delta M = \int_{x_1}^{x_2} V(x).\,dx \text{ -------------- } 4$$

Change in bending moment diagram between section x_1 & x_2 equal to the area under (v – diagram) between x_1 & x_2 plus any additional concentrated moment applied between x_1 & x_2.

Differentiate Equation:-

With respect to x

$$\frac{dv}{dx} = \frac{d^2 M}{dx^2}, Then\ sub\ into\ equ.\,1$$

$$W = \frac{d^2 M}{dx^2} \text{ ------------ } 5$$

Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College

Sign Convention:-

1- Internal force:-

   a) Axial: tensile   +ve

      Comp.   –ve

   b) Shear: on left segment

      Down word +

      Up word -

   c) Bending Moment:
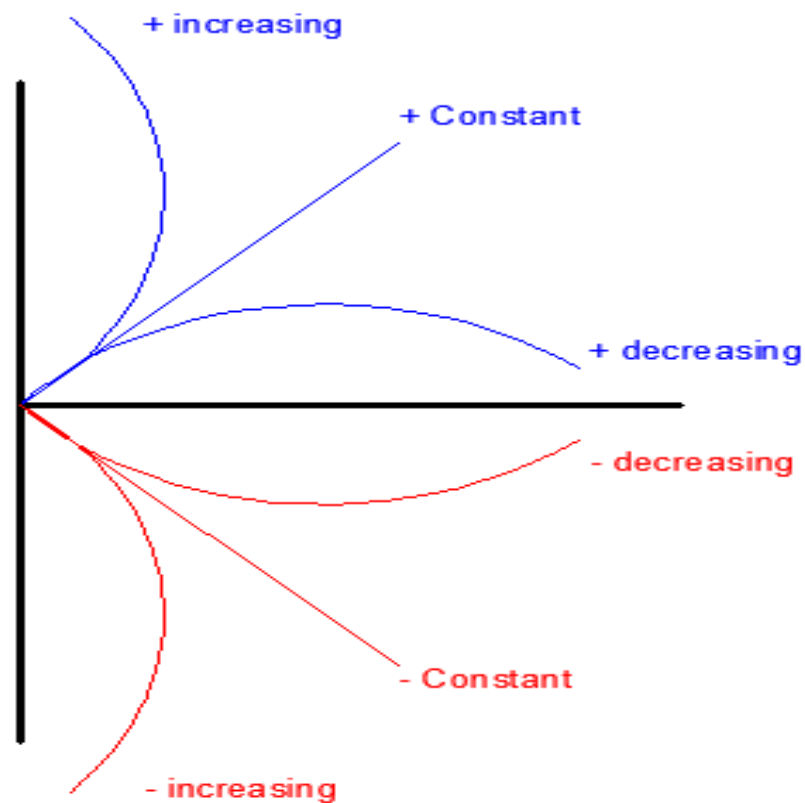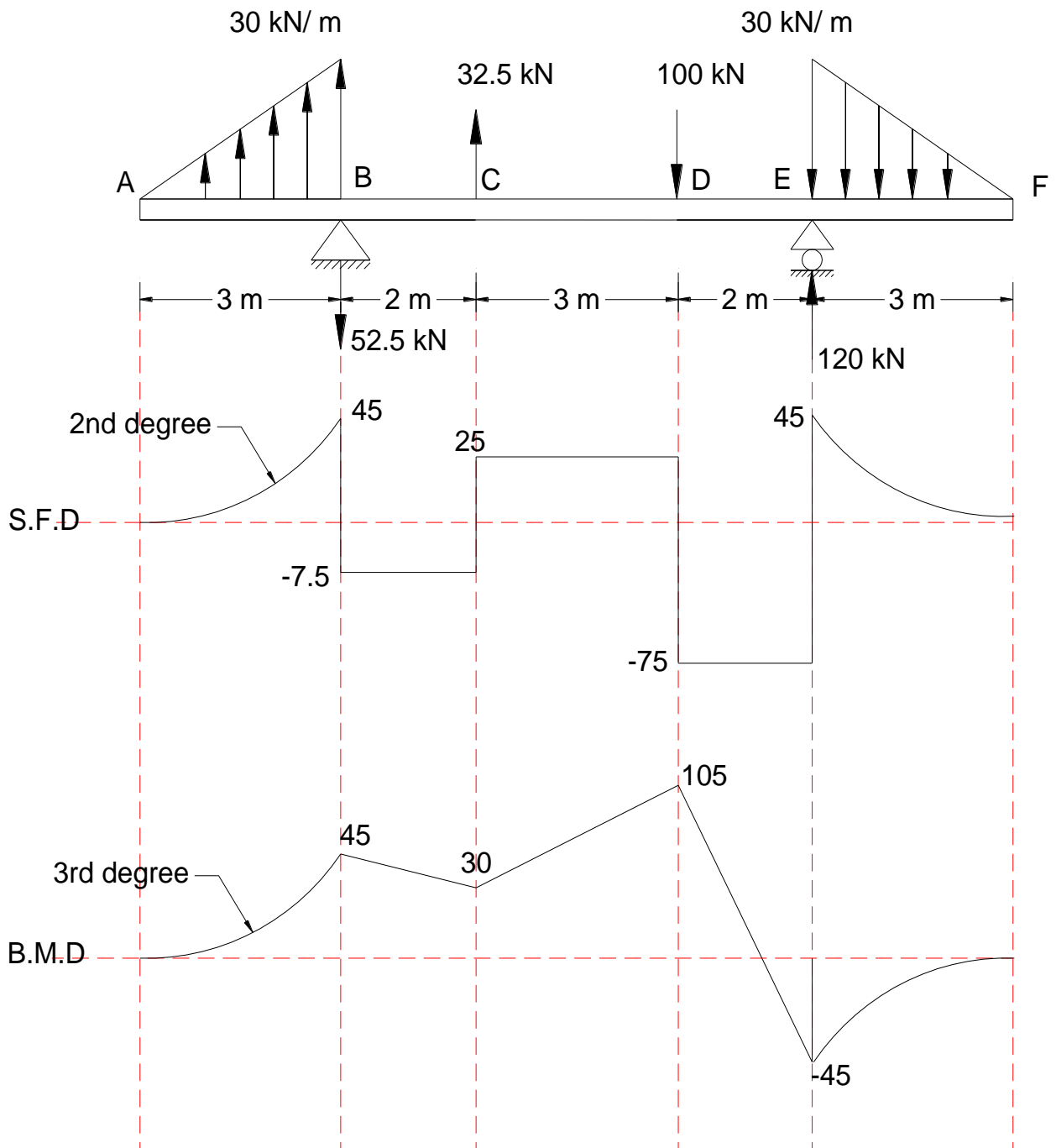
2- External force:-

   a- Up word force +ve
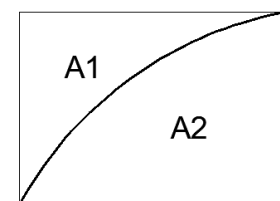   b- Clockwise moment +ve
   c- Axial force to the left +ve

3- Curves:-

Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College
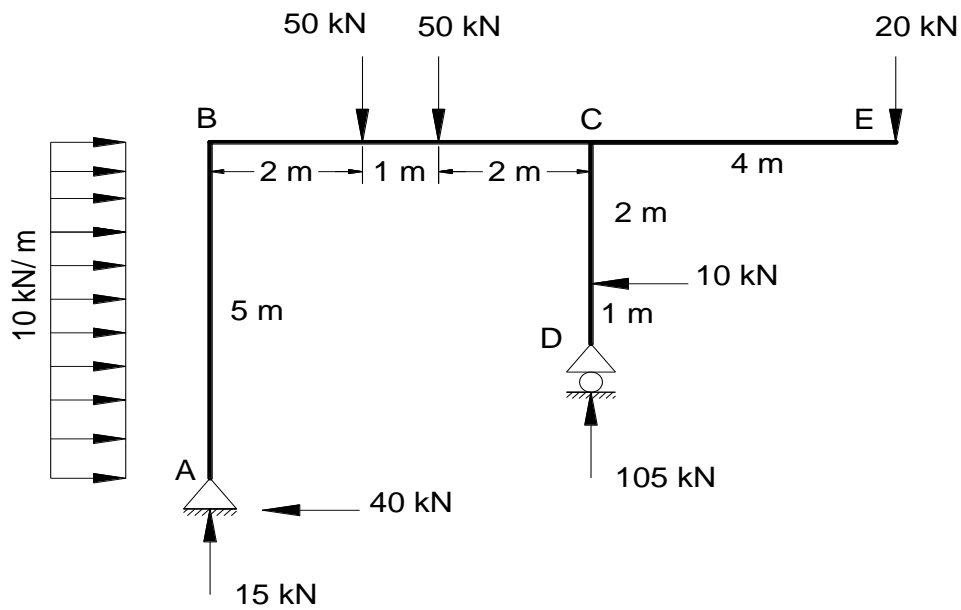
EX1:- Draw the axial force; shear force and bending moment diagrams.

$A_1 = 1/(n+1)$, الكيرف مقعر $A_2 = n/(n+1)$, الكيرف محدب

Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College
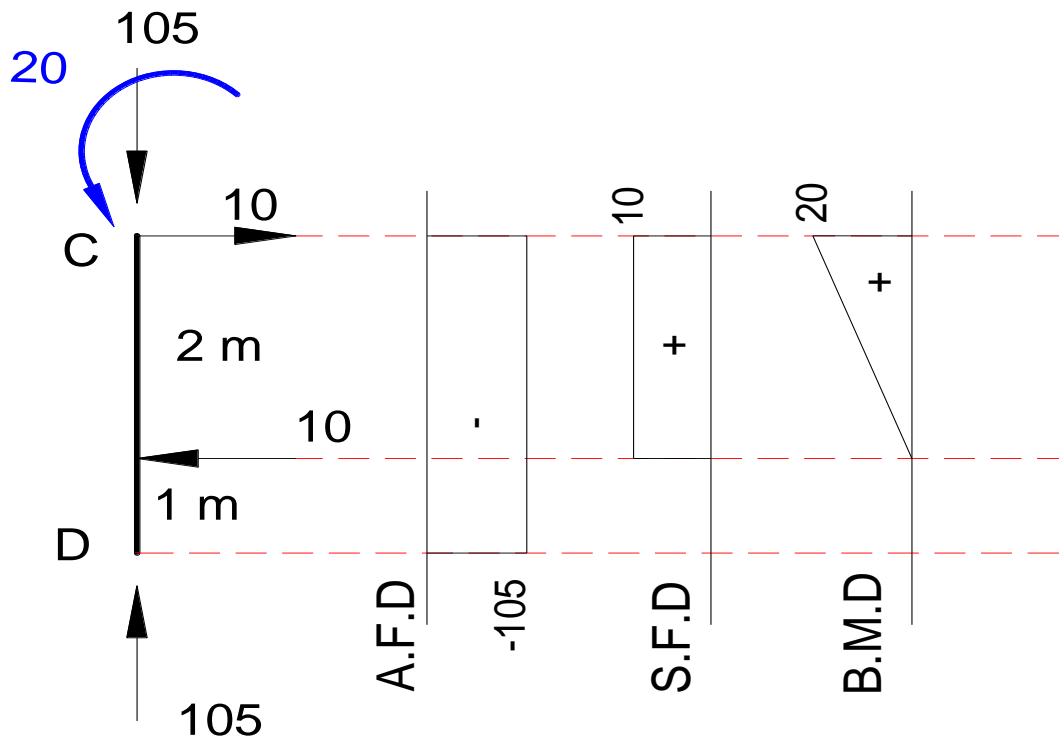
EX2: Draw the axial force; shear force and bending moment diagrams for the frame shown below.

50 kN   50 kN   20 kN
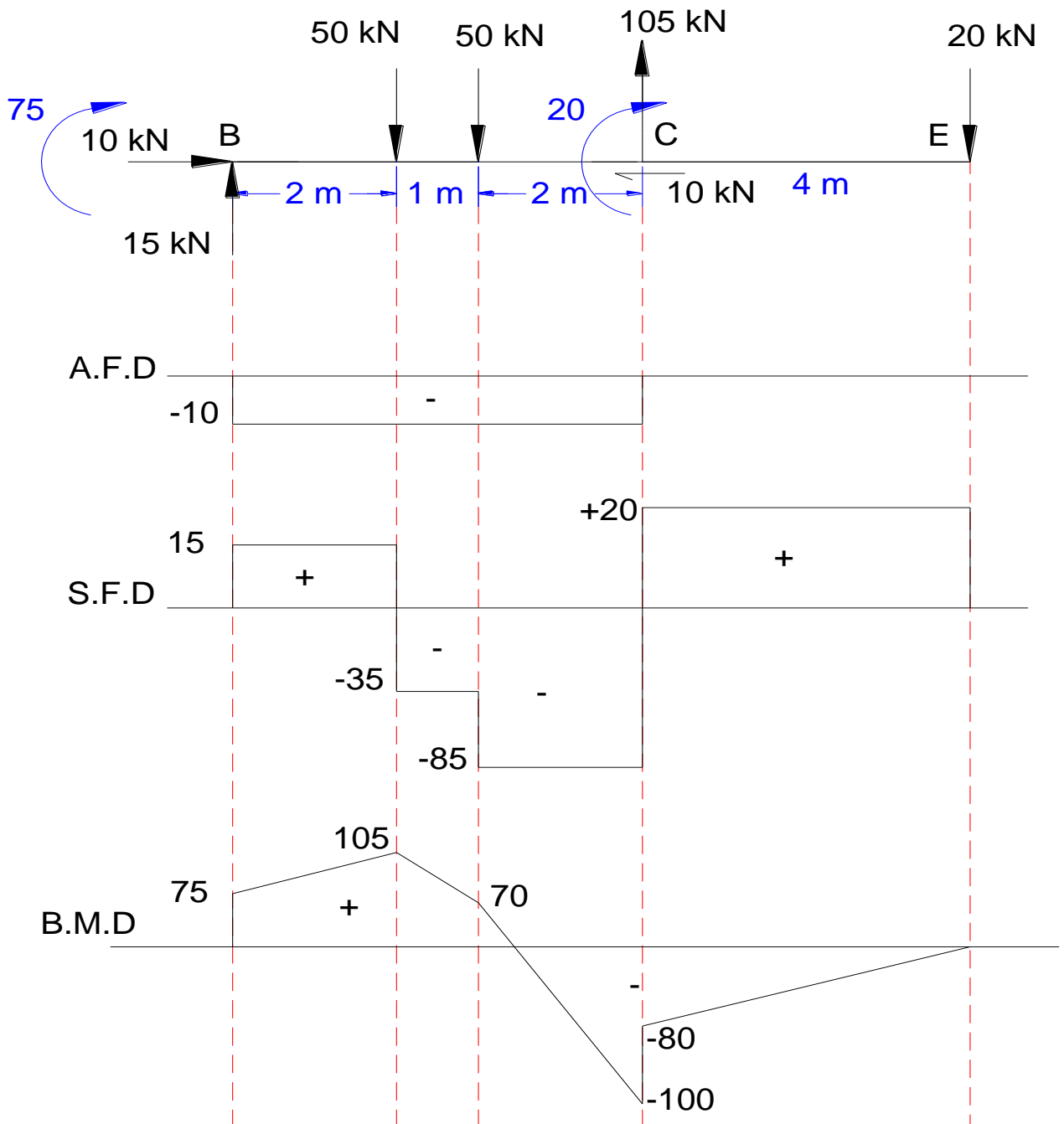
B   C   E

2 m — 1 m — 2 m —   4 m

2 m

10 kN/m   10 kN

5 m   1 m

D

A

40 kN   105 kN

15 kN

Sol: - SEGMENT (AB)

15

75

10

B

10 kN/m

5 m

A

40

15

A.F.D   -15   40   S.F.D   4 m   1 m   -10   80   75   B.M.D

Segment (DC)

Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College
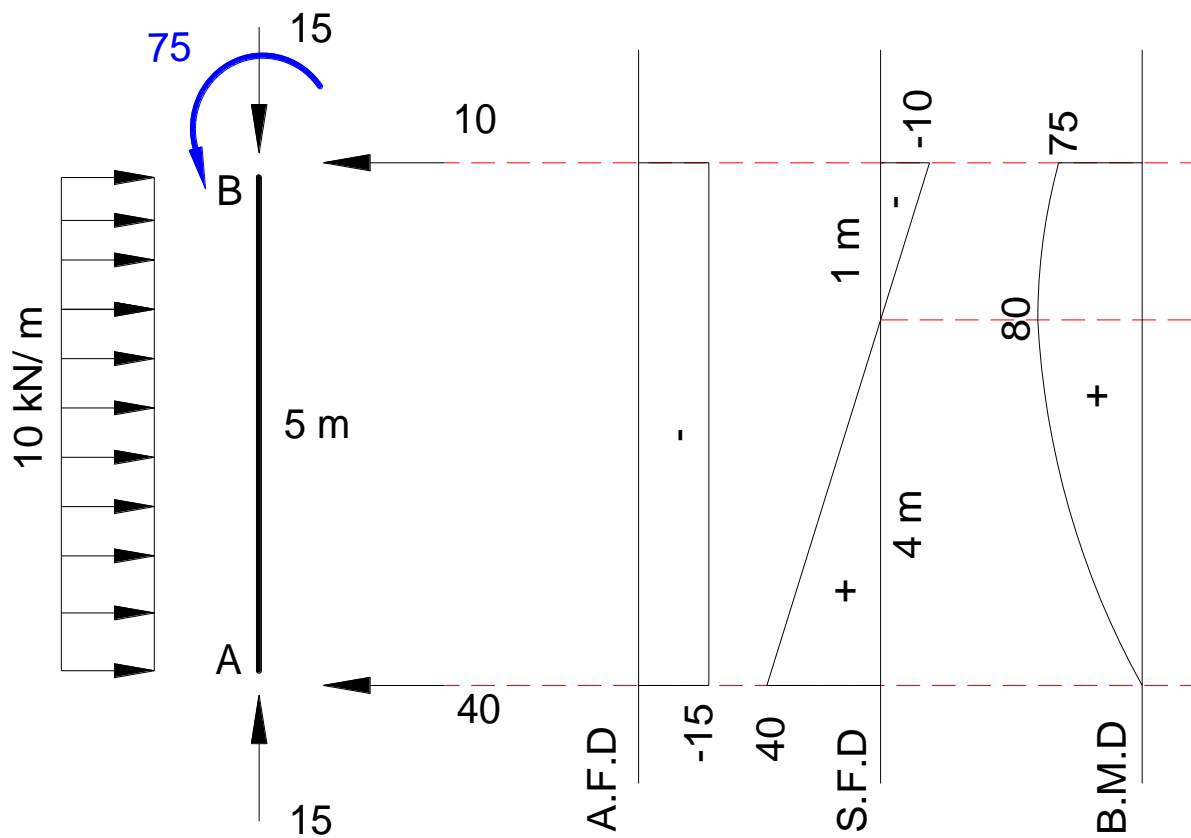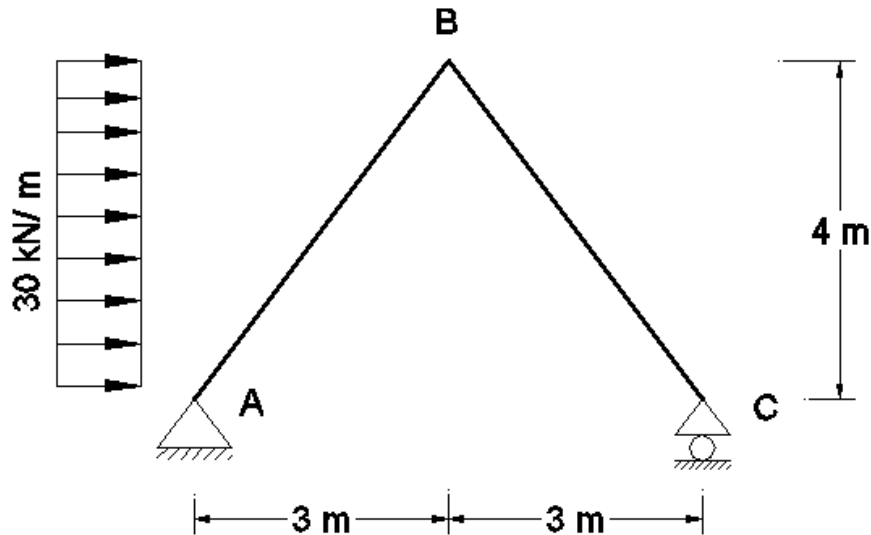
Segment BCE

EX3:- Draw the axial force; shear force and bending moment diagrams for the frame as shown below.

Sol:

$\sum M_A = 0$

Cy*6 – (120*2) =0, Cy= 40 kN ↑

↑$\sum F_y = 0$, 40 + Ay = 0, Ay= -40 kN = 40 kN ↓

$\sum F_X = 0$, 120 – Ax = 0, Ax = 120 kN ←

For segment. AB

↑$\sum F_y = 0$, -40 + By = 0, By = 40 kN ↑

→ $\sum F_X = 0$, 120 – 120 – Bx = 0, Bx = 0

$\sum M_B = 0$, M$_B$ + 120*2 + 40*3 -120*4 =0

M$_B$= 120 kN.m

Segment AB.

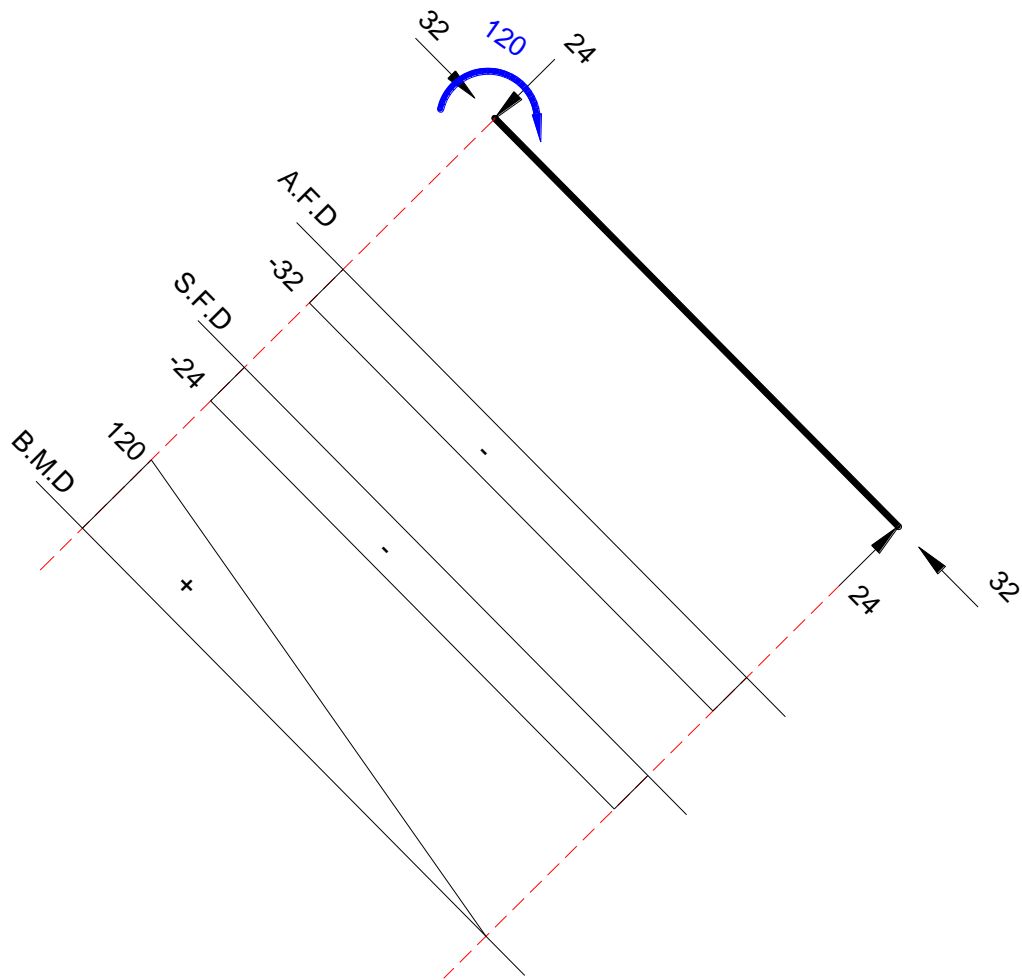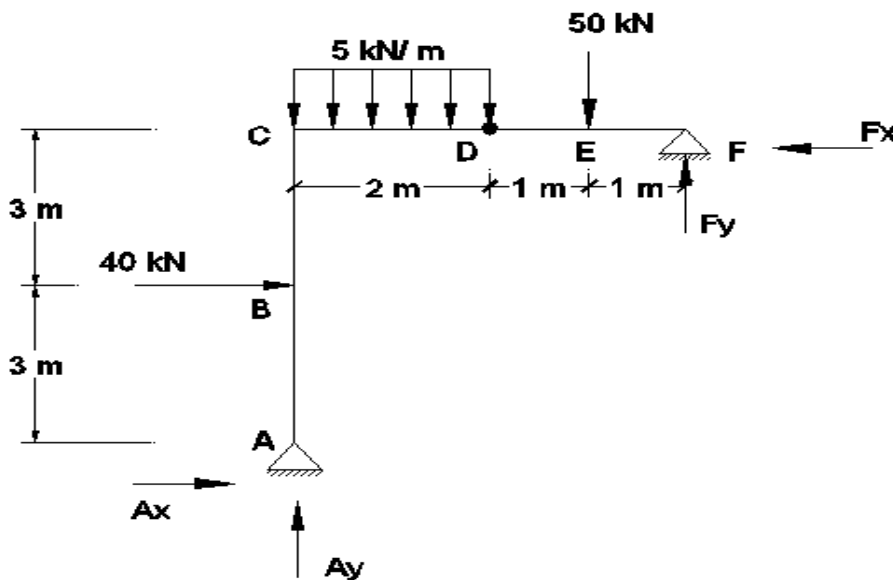Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College
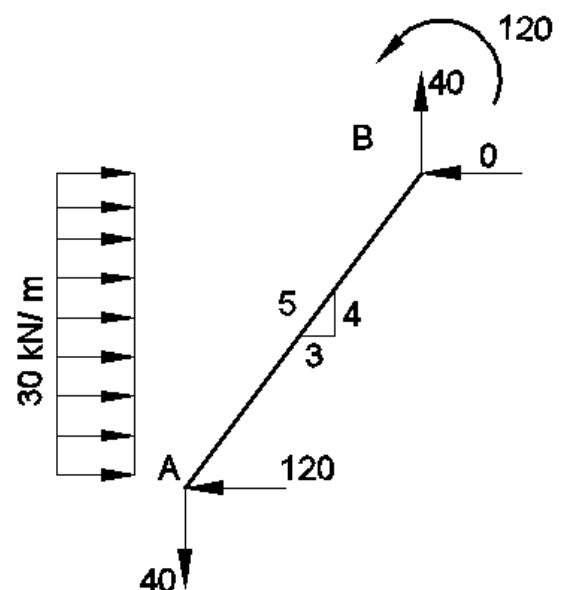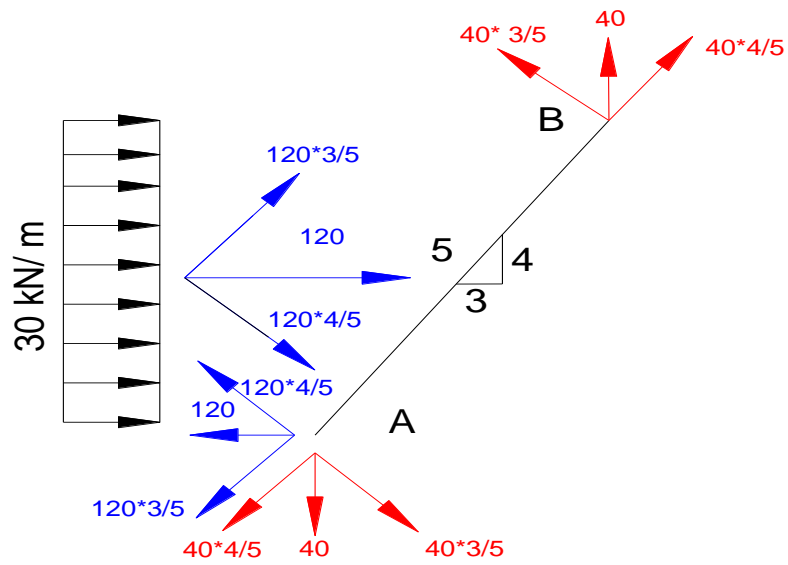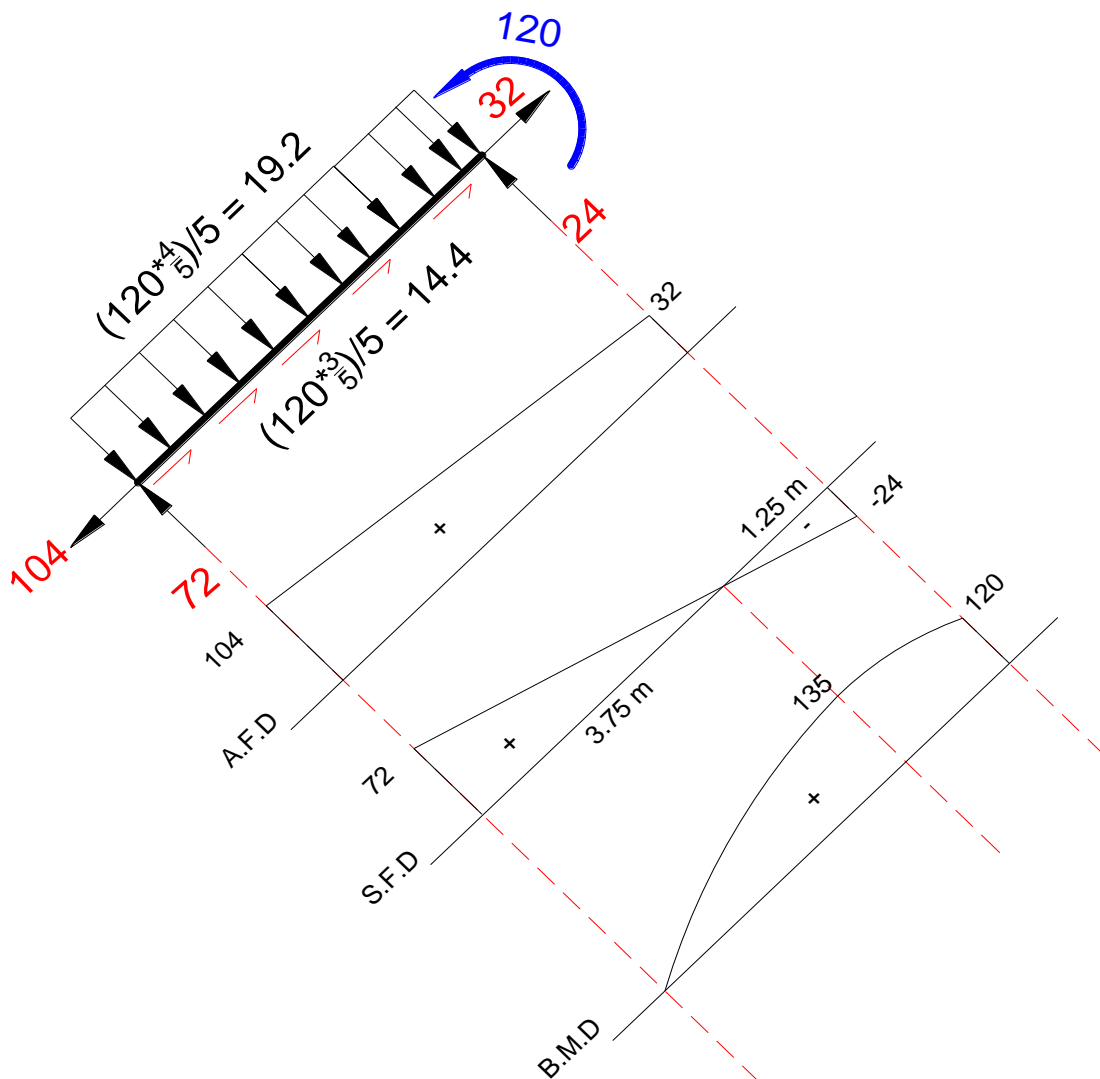
Segment BC

EX4: Draw the axial force; shear force and bending moment diagrams for the frame shown below.
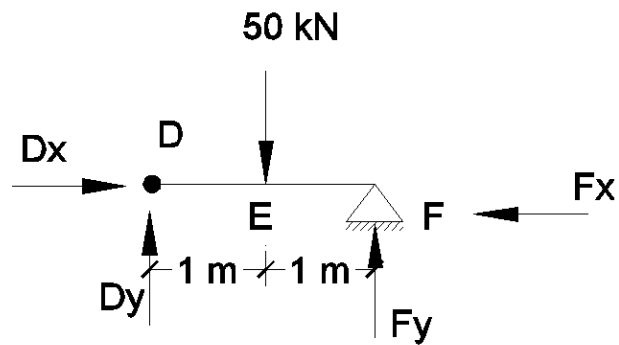
Sol:

For segment (DEF)

$\sum M_D = 0,$

Fy * 2 – (50*1) =0

Fy= 25 kN ↑

For whole structure

$\sum M_A = 0 \Rightarrow$ Fx*6 + (25*4) – (50*3) – (10 *1) – (40*3) =0

Fx = 30 kN ←

$\uparrow \sum F_y = 0,$

Ay +25 -50- (5*2) = 0, => Ay= 35 kN ↑

$\rightarrow \sum F_x = 0, \Rightarrow$ -Ax – 30 + 40 = 0, => Ax= 10 kN ←

Segment ABC

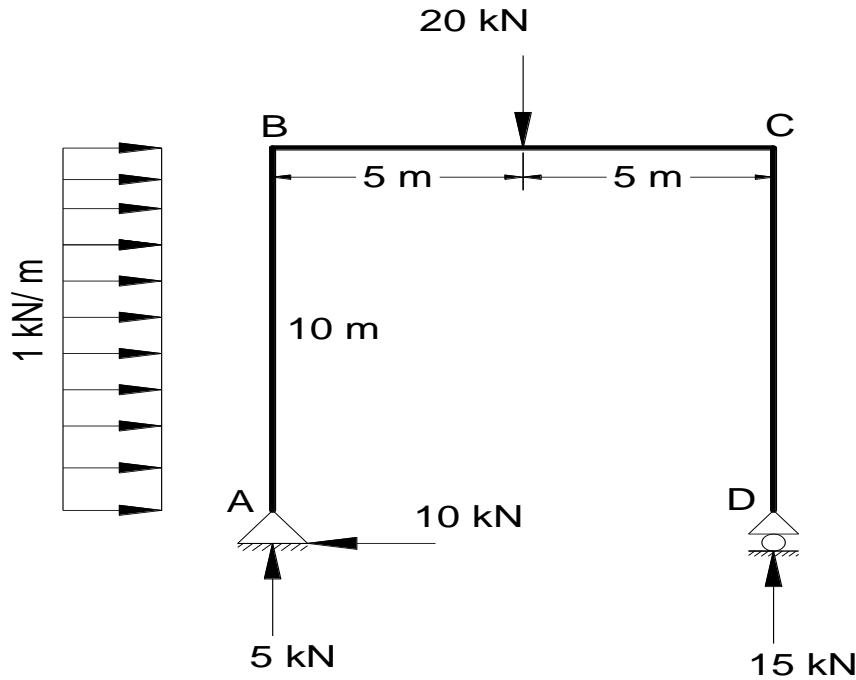Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College

Segment CDEF

EX5: Draw the axial force; shear force and bending moment diagrams for the frame shown below.

20 kN

B   C

5 m   5 m

1 kN/ m

10 m

A   10 kN   D

5 kN   15 kN

Sol:-

For Member AB:

5 kN

50

1 kN/ m

B

10 m

10 kN

A

5 kN

A.F.D   -5

10   S.F.D

50   B.M.D

For Member CD:

For Member BC:

20 kN

50

B   C

5 kN   5 kN

A.F.D   Zero
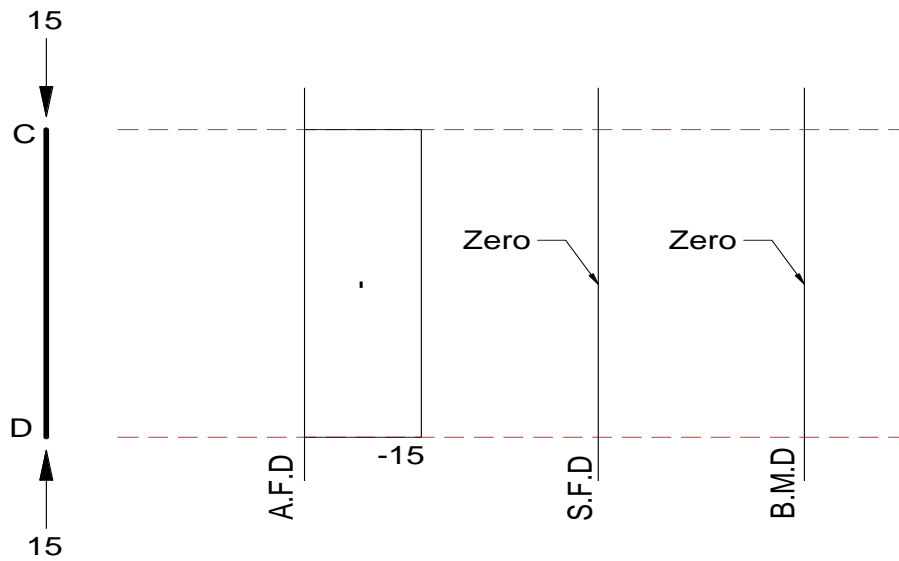
S.F.D   5
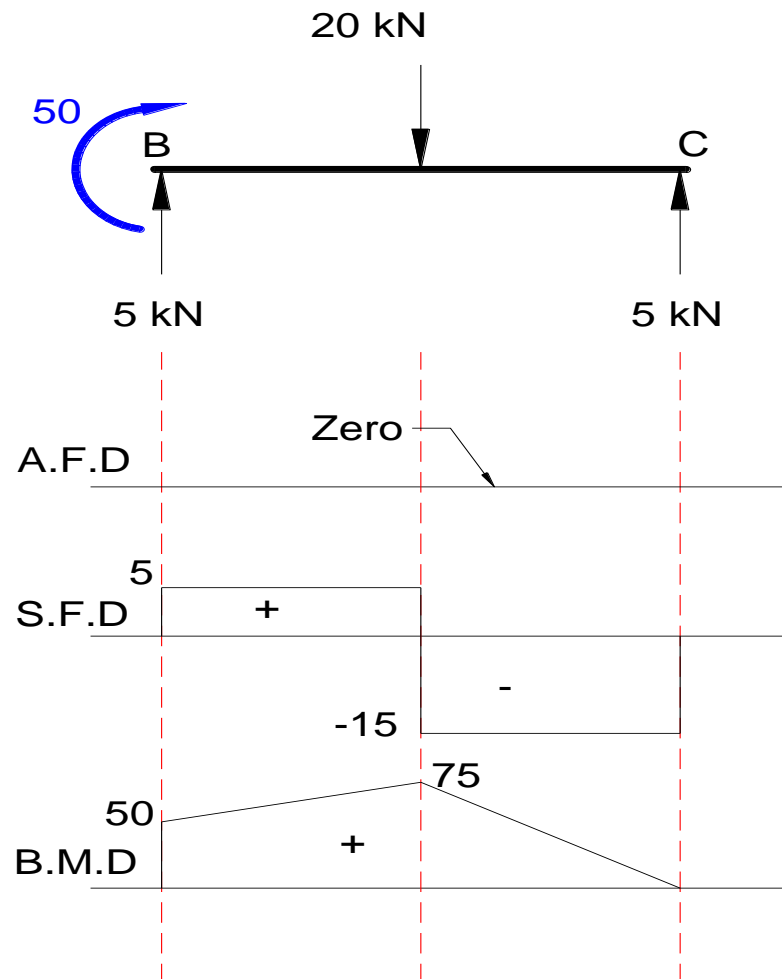   +

              -15   -

              75

B.M.D   50   +

H.W: Draw the axial force; shear force and bending moment diagrams for the
       frame shown below.

Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College

Asist Lecturer: Alaa Hussein Ali   AL- Mustaqbal University College