Al-Mustaqbal University College
Department of Medical Instrumentation Techniques Engineering
Class: second stage
Subject: Mathematics II
Lecturer: Dr. Diyar Hussain Habbeb
Lecture: Lec7

Taylor's Series (Power Series about $x=a$)

Let $f(x)$ be a continuous function with derivatives of all orders are exists at $(x=a)$, then the Taylor Series generated by $f(x)$ @ $x=a$ is :

$$\left[\sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}(x-a)^{n} = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^{2} + \cdots + \frac{f^{n}(a)}{n!}(x-a)^{n} + \cdots \right] \quad (3)$$

It's clear that, like in Maclaurin's series, in Taylor's series $f(x)$ can be represented as follows :

$$f(x) = a_0 + a_1(x-a) + a_2(x-a)^2 + a_3(x-a)^3 + \cdots + a_n(x-a)^n + \cdots \quad (4)$$

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Ex) Find Taylor's Series for $f(x)=\sqrt{x}$ about $x=4$?

Sol.) using eq. 5 $\longrightarrow$

$$f(x) = f(4) + (x-4)f'(4) + \frac{(x-4)^2}{2!}f''(4) + \frac{(x-4)^3}{3!}f'''(4) + \cdots$$

hence,

$$f(x) = \sqrt{x} \quad \longrightarrow f(4) = 2$$

$$f'(x) = 1/2\sqrt{x} \quad \longrightarrow f'(4) = 1/4$$

$$f''(x) = -1/4x^{3/2} \quad \longrightarrow f''(4) = -1/32$$

$$f'''(x) = 3/8x^{5/2} \quad \longrightarrow f'''(4) = 3/256 \quad \longrightarrow$$

$$f(x) = \sqrt{x} = 2 + (x-4)\left(\frac{1}{4}\right) + \frac{(x-4)^2}{2!}\left(-\frac{1}{32}\right) + \frac{(x-4)^3}{3!}\left(\frac{3}{256}\right) + \cdots$$

$$\therefore \sqrt{x} = 2 + \frac{(x-4)}{4} - \frac{(x-4)^2}{64} + \frac{(x-4)^3}{512} + \cdots$$

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Ex/ Find Taylor series for $f(x) = \sin x$ about $x = \frac{\pi}{6}$?

Sol/ $f(x) = f(\frac{\pi}{6}) + (x-\frac{\pi}{6}) f'(\frac{\pi}{6}) + \frac{(x-\pi/6)}{2!} f''(\frac{\pi}{6}) + \cdots$

$f(x) = \sin x \rightarrow f(\pi/6) = \sin \pi/6 = \frac{1}{2}$

$f'(x) = \cos x \rightarrow f'(\pi/6) = \sqrt{3}/2$

$f''(x) = -\sin x \rightarrow f''(\pi/6) = -1/2$

$f'''(x) = -\cos x \rightarrow f'''(\pi/6) = -\sqrt{3}/2 \rightarrow$ by Sub. in eq. above

$$\sin x = \frac{1}{2} + (x-\frac{\pi}{6})\frac{\sqrt{3}}{2} + \frac{(x-\frac{\pi}{6})^2}{2!}(-\frac{1}{2}) + \frac{(x-\frac{\pi}{6})^3}{3!}(-\sqrt{3}/2) + \cdots$$

$$= \frac{1}{2} + (x-\frac{\pi}{6})\frac{\sqrt{3}}{2} - \frac{(x-\pi/6)^2}{2 * 2!} - \frac{\sqrt{3}}{2 * 3!}(x-\frac{\pi}{6})^3 + \cdots$$

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EX/ Find The Taylor Series for $f(x) = \cos x$ at $x = \frac{\pi}{2}$

Sol:

$$\sum_{n=0}^{\infty} \frac{f(a)}{n!}(x-a)^n = f(a) + (x-a)\cdot f'(a) + \frac{(x-a)^2}{2!}\cdot f''(a) + \frac{(x-a)^3}{3!}\cdot f'''(a) + \cdots$$

$f(x) = \cos x \rightarrow \cos\frac{\pi}{2} = 0$

$f'(x) = -\sin x \rightarrow -\sin\frac{\pi}{2} = -1$

$f''(x) = -\cos x \rightarrow -\cos\frac{\pi}{2} = 0$

$f'''(x) = \sin x \rightarrow \sin\frac{\pi}{2} = 1$

$$\sum_{n=0}^{\infty} f\left(\frac{\pi}{2}\right) = 0 + \left(x-\frac{\pi}{2}\right)\cdot(-1) + \frac{\left(x-\frac{\pi}{2}\right)^2}{2!}\cdot(0) + \frac{\left(x-\frac{\pi}{2}\right)^3}{3!}\cdot(1) + \cdots$$

$$\sum_{n=0}^{\infty} f\left(\frac{\pi}{2}\right) = -\left(x-\frac{\pi}{2}\right) + \frac{\left(x-\frac{\pi}{2}\right)^3}{6} + \cdots$$