Al-Mustaqbal University College
Department of Medical Instrumentation Techniques Engineering
Class: second stage
Subject: Mathematics II
Lecturer: Dr. Diyar Hussain Habbeb
Lecture: Lec7

Maclaurin Series (Power Series about $x=0$)

Let $f(x)$ be a function with derivatives of all orders at $x=0$ are exists.

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \cdots \quad \text{---(1)}$$

then by successive diffrentiations, we got:

$$f'(x) = a_1 + 2a_2 x + 3a_3 x^2 + 4a_4 x^3 + \cdots$$

$$f''(x) = 2a_2 + (3\times2)a_3 x + (4\times3)a_4 x^2 + \cdots$$

$$f'''(x) = (3\times2)a_3 + (4\times3\times2)a_4 x + \cdots$$

$$\vdots \qquad\qquad \vdots$$

Putting $x=0$ in each term above, yields:

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$$f(0) = a_0$$
$$f'(0) = a_1 \rightarrow a_1 = f'(0)/1!$$
$$f''(0) = 2a_2 \rightarrow a_2 = f''(0)/2!$$
$$f'''(0) = 3\times 2\, a_3 \rightarrow a_3 = f'''(0)/3!$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$f^{n}(0) = \text{-- --- - -} n!\, x_n \rightarrow a_n = f^{n}(0)/n!$$

by sub. the coefficients $(a_1, a_2, \ldots, a_n)$ in eq. ①, got:

$$\left[f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots + \frac{x^n}{n!} f^{n}(0) + \cdots \right] \quad \text{—} ②$$

the above eq. is a power series about $(x=0)$ and known as Mac laurin Series for function $f(x)$.

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Ex) Find the Maclaurin's Series for the function $f(x)=\sin(mx)$?

Sol.: $f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots + \frac{x^n}{n!} f^{n}(0)$ — (2)

then;

$f(x) = \sin mx \rightarrow f(0) = 0$

$f'(x) = m \cos mx \rightarrow f'(0) = m$

$f''(x) = -m^2 \sin mx \rightarrow f''(0) = 0$

$f'''(x) = -m^3 \cos mx \rightarrow f'''(0) = -m^3$

$f^{4}(x) = m^4 \sin mx \rightarrow f^{4}(0) = 0$

$f^{5}(x) = m^5 \cos mx \rightarrow f^{5}(0) = m^5$

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Ex Find Maclaurin's Series for $f(x) = \ln(1+x)$?

Sol.) $\because f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \cdots$

$f(x) = \ln(1+x) \longrightarrow f(0) = \ln(1+0) = 0$

$f'(x) = 1/(1+x) \longrightarrow f'(0) = 1$

$f''(x) = (-1)/(1+x)^2 \longrightarrow f''(0) = -1$

$f'''(x) = (2)/(1+x)^3 \longrightarrow f'''(0) = 2$

$f^{iv}(x) = (-6)/(1+x)^4 \longrightarrow f^{iv}(0) = -6 \longrightarrow$

$\ln(1+x) = 0 + x(1) + \frac{x^2}{2!}(-1) + \frac{x^3}{3!}(2) + \frac{x^4}{4!}(-6) + \cdots \longrightarrow$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^{n+1}}{n} x^n + \cdots$

by sub. in ② above $\longrightarrow$ get $\therefore (f(x) = \sin mx) \rightarrow$

$\sin mx = 0 + xm + \frac{x^2}{2!}(0) + \frac{x^3}{3!}(-m^3) + \frac{x^4}{4!}(0) + \frac{x^5}{5!} m^5 + \cdots$

$\sin mx = mx - m^3 \frac{x^3}{3!} + m^5 \frac{x^5}{5!} - m^7 \frac{x^7}{7!} + m^9 \frac{x^9}{9!} - \cdots$

Now putting $(m=1) \rightarrow$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

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Ex) Find Maclaurin's Series for $f(x) = e^{ax}$, then compute (e^2)?

Sol.)

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) + \cdots + \frac{x^n}{n!} f^{n}(0) + \cdots$$

$$f(x) = e^{ax} \longrightarrow f(0) = 1$$

$$f'(x) = a e^{ax} \longrightarrow f'(0) = a$$

$$f''(x) = a^2 e^{ax} \longrightarrow f''(0) = a^2$$

$$f'''(x) = a^3 e^{ax} \longrightarrow f'''(0) = a^3$$

$$\vdots \qquad\qquad \vdots$$

$$f^{n}(x) = a^n e^{ax} \longrightarrow f^{n}(0) = a^n \longrightarrow$$

$$e^{ax} = 1 + ax + a^2 \frac{x^2}{2!} + a^3 \frac{x^3}{3!} + \cdots + a^n \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{a^n x^n}{n!}$$

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Putting $a = 1 \rightarrow$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$\rightarrow$ Putting $x = 2 \rightarrow$ for example take up $(n = 6)$

$$e^2 = 1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} + \frac{2^6}{6!} = 7.35555$$

e^2 (calculator) $= 7.389$

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