### 3.6 Column Within a Rigid Frames

The factor K will depend on the ratio of the column to girder stiffness at each end.

$$
G=\frac{\left(\sum \frac{E I}{L}\right)_{\text {Columns }}}{\left(\sum \frac{E I}{L}\right)_{G i r d e r s}}=\frac{\left(\sum \frac{I}{L}\right)_{\text {Columns }}}{\left(\sum \frac{I}{L}\right)_{\text {Girders }}}
$$



Notes:

1. For pinned column base, use $G=10$.
2. For fixed column base, use $G=1$.

Also it is depending on the relative movements between the columns ends. i.e. frame without side sway (inhibited side sway) and with side sway or (uninhibited side sway). See Fig. C-C2.3 and Fig. C-C2.4.


Fig. C-C2.3. Alignment chart-sidesway inhibited (braced frame).


Fig. C-C2.4. Alignment chart-sidesway uninhibited (moment frame).

The frame is called braced frame (sides way inhibited), if it has one of the following:


Other else the frame is called unbraced frame (sides way uninhibited).

Example No. 1: The rigid frame shown in Figure is unbraced. Each member is oriented so that its web is in the plane of frame (bending about x -axis). Determine the effective length factor for columns AB and BC .


Solve:

## 1) Section Properties:

| Section | $\boldsymbol{I}_{\boldsymbol{x}}$ (in $^{\mathbf{4}}$ ) | Length $(\boldsymbol{f t})$ |
| :---: | :---: | :---: |
| $W 12 \times 96$ | 833 | 15 |
| $W 12 \times 120$ | 1070 | 15 |
| $W 24 \times 55$ | 1350 | 20 |
| $W 24 \times 68$ | 1830 | 18 |

2) Column AB

For joint $A: G_{A}=\frac{\left(\Sigma \frac{I}{L}\right)_{\text {Columns }}}{\left(\sum \frac{I}{L}\right)_{\text {Girders }}}=\frac{833 / 15}{1350 / 20+1830 / 18}=\frac{55.53}{169.17}=0.328$
For joint B: $G_{B}=\frac{\left(\sum \frac{I}{L}\right)_{\text {Columns }}}{\left(\sum \frac{I}{L}\right)_{\text {Girders }}}=\frac{833 / 15+1070 / 15}{1350 / 20+1830 / 18}=\frac{126.87}{169.17}=0.75$
From the alignment chart for sides way uninhibited (AISC Figure C-C2.4). use $G_{A}=0.328$ and $G_{B}=0.75$.


Fig. C.C2.4. Alignment chart-sidesway uninhibited
H.W: Determine the effective length factor for columns BC. (Ans: $K_{x}=1.8$ )

Example No. 2: For the two-story moment frame shown down, the column and girder sizes have been determined as shown. Assume in-plane bending about the strong axes for the columns and girders. Determine the effective length factor, $\boldsymbol{K}$, for columns $\boldsymbol{B F}$ and $\boldsymbol{F} \boldsymbol{J}$ using the alignment charts.


## Solve:

1) Section Properties:

| Member | Section | $\boldsymbol{I}_{\boldsymbol{x}}\left(\right.$ in $\left.^{\mathbf{4}}\right)$ | Length $(\boldsymbol{f t})$ |
| :---: | :---: | :---: | :---: |
| FJ | $W 12 \times 50$ | 391 | 20 |
| BF | $W 12 \times 72$ | 597 | 15 |
| EF, IJ | $W 18 \times 35$ | 510 | 20 |
| JK | $W 18 \times 40$ | 612 | 30 |
| FG | $W 18 \times 50$ | 800 | 30 |

## 2) Column BF

For joint B: $G_{A}=10$ (pinned Support)
For joint $\boldsymbol{F}: G_{B}=\frac{\left(\sum \frac{I}{L}\right)_{\text {Columns }}}{\left(\sum \frac{I}{L}\right)_{\text {Girders }}}=\frac{391 / 20+597 / 15}{510 / 20+800 / 30}=\frac{59.35}{52.167}=1.14$

From the alignment chart for sides way uninhibited (AISC Figure C-C2.4), use $G_{A}=10$ and $G_{B}=1.14$. Get: $K_{x}=1.93$.

## 3) Column FJ

For joint $\boldsymbol{F}: G_{A}=1.14$
For joint $\boldsymbol{J}: G_{B}=\frac{\left(\Sigma \frac{I}{L}\right)_{\text {Columns }}}{\left(\Sigma \frac{I}{L}\right)_{\text {Girders }}}=\frac{391 / 20}{510 / 20+612 / 30}=\frac{19.55}{45.9}=0.43$
From the alignment chart for sides way uninhibited (AISC Figure C-C2.4), use $G_{A}=1.14$ and $G_{B}=0.43$. Get: $K_{x}=1.25$.


## Problem:

1. The frame shown in Figure is unbraced, and bending is about the $x$-axis of the members. All beams are $\mathrm{W} 16 \times 40$, and all columns are $\mathrm{W} 12 \times 58$. Determine the effective length factor $K_{x}$ for column AB and BC .


Ans: $a . K x=1.98, \quad b . K x=1.42$
2. The frame shown in Figure is unbraced against sides way. Relative moments of inertia of the members have been assumed for preliminary design purposes. Use the alignment chart and determine $K_{x}$ for members $\mathrm{AB}, \mathrm{BC}, \mathrm{DE}$, and EF .


Ans: Member $A B: K x=2.00, \quad B C: K x=1.40, \quad D E: K x=1.20, \quad E F: K x=1.28$.

